PRAMANA
— journal of
physics

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Supplement to Vol. 41
December 1993
pp. 217–233

Flavour-spin symmetry and weak decays of heavy hadrons

M P KHANNA

Centre of Advanced Study in Physics, Department of Physics, Panjab University, Chandigarh 160 014, India

Abstract. Recent developments in the theory of heavy quarks have increased the prospects for the study of non-perturbative QCD in the weak decays of heavy mesons and baryons and for the reliable determination of some of the parameters of the Standard Model. It has been made possible due to the discovery of a spin-flavour symmetry for heavy quarks which arises in QCD when quark mass is taken to infinity. Certain properties in hadrons containing a heavy quark then become independent of its mass and spin. These ideas have tremendous impact on the phenemenology of heavy hadrons. In particular, these symmetries give rise to restrictive relations among weak decay amplitudes and reduce the number of independent form factors. By relating various matrix elements and fixing normalization of some matrix elements, the heavy quark symmetry has enhanced our predictive ability, allowing in some cases to bypass the difficulties of understanding hadronic structure.

1. Introduction

The quark contribution to the QCD Lagrangian

$$L_{\text{quark}} = \overline{q}(i\gamma \cdot D - m_q)q + \overline{Q}(i\gamma \cdot D - M_Q)Q$$

separates into two pieces: one from the light quarks (q=u,d,s) and the other from the heavy quarks (Q=c,b,t). Each of these has adistinct symmetry. The light quark sector exhibits an appromimate chiral $SU(3)_L \times SU(3)_R$ symmetry due to the fact that the current quark masses for u, d, and s are very small compared with the typical hadron energy scale. This symmetry is spontaneously broken to the usual $SU(3)_F$ (FLAVOUR). Since at the energy scale of heavy hadrons the mass difference of the strange and up (down) constituent quarks is negligible, this $SU(3)_F$ symmetry is expected to work very well for the heavy hadrons. With this in mind, several authors [1] have used this symmetry to relate different decay amplitudes of charmed and b-hadrons.

On the other hand, in the limit of infinite quark mass, the other side of the scale, the dynamics of a heavy quark in QCD depends only on its velocity and is independent of its mass and spin. As a consequence, two additional symmetries [2] appear in the sector of hadrons containing one heavy quark. Since all the heavy quarks are treated on the same footing, we have an $SU(N_h)_{HF}$ (HEAVY FLAVOUR) symmetry; N_h being the number of heavy quarks.

The second symmetry is due to the fact that the spin degree of freedom of the heavy quark decouples from the light QCD degree of freedom in this limit. As a result, there is an additional SU(2) spin symmetry corresponding to the spin rotations of the heavy quark moving with a fixed velocity.

For N_h heavy quarks, therefore, there is an $SU(2N_h)$ flavour-spin symmetry. This new symmetry is known as heavy quark symmetry and has seen much progress during the last few years. More precisely, this symmetry ($SU(2N_h)$) implies that spectrum of heavy mesons and heavy baryons is independent of the flavour and spin of the quark, so are the transition form factors which describe the weak decays of these heavy hadrons. The ideas of the heavy quark limit are most easily implemented [3-10] in an effective field theory [3-5] call the Heavy Quark Effective Theory (HQET). Since the heavy hadrons contain both the heavy and light quarks, both the $SU(3)_F$ symmetry of the light quarks and the heavy quark $SU(2N_h)$ symmetry are expected to have implications for the low energy dynamics of heavy hadrons.

A simple result of the spin symmetry of the heavy quark is that 0^--mesons made up of a heavy quark and a light antiquark (e.g. D) and the corresponding vector mesons (e.g. D^*) should be degenerate. comparing this prediction with the experimental data we may conclude that the b and c quarks may certainly be treated as heavy and heavy flavour symmetry is thus an SU(2) symmetry denoted by $SU(2)_{HF}$.

The consequences of the spin-flavour symmetries (referred to as Isgur wise symmetry) for the weak semi-leptonic decays of the B-mesons to D and D^* mesons were obtained by Isgur and wise [4], Georgi [5] and others [6] who showed that all the form factors are expressible in terms of single universal function of velocity transfer which is normalized to unity at zero recoil. The implications of the heavy quark symmetry for the semileptonic decays of baryons [4,7,10] are that the six real, generally independent form factors are expressible in terms of single universal function of velocity transfer which is normalized to unity at zero recoil. The implications of the heavy quark symmetry for the semileptonic decays of baryons [4,7,10] are that the six real, generally independent form factors that describe the semleptonic decays of b- lambda baryons to charmed lambda baryon are expressible in terms of a single universal function of velocity transfer if we assume that b and c quarks are heavy enough to satisfy the requirements of HQET. For the transition in which a heavy quark c is changed to a light quark s, only two form factors are needed [10]. The corresponding theoretical description of decays of heavy hadrons is thus greatly simplified and model dependence is greatly reduced. Further, the reduction in the number of form factors leads to testable experimental predictions. One of the first applications of the ideas of heavy quark symmetry was to semileptonic B decays and to extract b to c mixing angle. It was made possible because at zero recoil the matrix element for $B \rightarrow D^*$ depends solely on the uncorrected form factor [9,12] and so could be used to determine V_{CB}. The intial analysis was made by Neubert [13] and gave

$$| V_{cb} | \left(\frac{\tau_B}{1.18 \text{ ps}}\right) = 0.045 \pm 0.007 .$$

In addition, if we assume that the matrix elements for the nonleptonic decays factor- rize, the same form factors that determine the properties of semileptonic decays also describe the nonleptonic ones [14]. The naive predictions show that HQET performs well when confronted with the experimental data. In particular, a strong prediction of the HQET is that the asymmetry parameter α for the decay $\Lambda_c \to \Lambda\pi^+$ is -1. Recent experimental results from CLEO [15] and ARGUS [16] seem to confirm this prediction. It is also observed that the decay $\Xi_c^0 \to \Xi^-\pi^+$ may provide a direct test for the factorization hypothesis. However, more stringent tests require better experimental data as also the incorporation of $1/m_c$ effects which have recently been studied upto order $1/M_Q^2$. New data from charm and beauty factories will allow us to test the heavy quark symmetry and to determine reliably some Standard Model parameters. In Sec.2 an elementary introduction is given on the spin-flavour symmetry. Sec.3 and Sec.4 deal with the matrix elements of weak currents for the meson and baryon states respectively. In Sec.5, we discuss the non-leptonic decays. We refer to the corrections in section 6. Finally, Sec.7 contains conclusions.

2. The Heavy Quark Approximation

2.1. Physical intuition

The central idea is simple. The success of the constituent quark model demonstrates that, inside hadrons, strongly bound quarks exchange momentum of magnitude a few hundred MeV. We can think of a typical amount Λ by which the quarks are off-shell in the nucleon with $\Lambda = M_p/3 = 330$MeV. In a heavy hadron again the light quarks may be off-shell by an amount Λ. But if the mass M_Q of the heavy quark Q is large ($M_Q \gg \Lambda$) then, in fact, this quark is almost on-shell. Moreover, interactions with a light quark typically change the momentum by Λ so that the change in the velocity of Q is negligible, being of the order of $\Lambda/M_Q \ll 1$. One can, therefore, think of Q as moving with almost constant velocity and this velocity is, of course, the velocity of the hadron. The velocity of the heavy quark can be modified only by perturbative processes involving very hard gluons or electroweak interactions. In the rest frame of the heavy hadron the heavy quark is practically at rest and acts as a static source of gluons.

An analogy can be drawn from the picture of an atom in the non-relativistic quantum mechanics. The heavy quark is similar to the atomic nucleus, QCD to electromagnetic interactions, and the light quark degrees of freedom to the electron; an obvious distinction being that in the hadronic system, the strong interactions of the light degrees of freedom are not calculable because of their nonperturbative nature. The spin of the heavy quark decouples because the (spin- spin) hyperfine magnetic interaction is inversely proportional to the mass of the heavy quark and so is a good quantum number. In the infinite limit, therefore, low energy hadronic dynamics becomes independent of the mass and the spin of the heavy quark resulting in an SU(4) symmetry of $b_\uparrow$, $b_\downarrow$; $c_\uparrow$, $c_\downarrow$. The observation that the two new symmetries of flavour and spin of the heavy quark arise in the heavy quark approximation can also be seen more explicitly when we consider the propagator and vertex of the QCD in the infinite mass limit. The heavy quark spinor in the heavy quark approximation satisfies

$$\gamma \cdot v\, h_v^{(C)}(x) \quad = \quad h_v^{(C)}(x).$$

Consider the usual fermion propagator

$$\frac{i}{\gamma \cdot p' - M_Q}.$$

For a heavy quark moving with a velocity v having a residual mementum k,

$$p'^{\mu} = M_Q v^{\mu} + k^{\mu}.$$

Looking at the limit $M_Q \to \infty$, we see that

$$\begin{aligned}\frac{i}{\gamma \cdot p' - M_Q} &= \frac{i(\gamma \cdot p' + M_Q)}{(p'^2 - M_Q^2)} \\ &= \frac{i(M_Q \gamma \cdot v + M_Q + \gamma \cdot k)}{(2M_Q v \cdot k + k^2)} \\ &= \frac{i(\gamma \cdot v + 1 + \gamma \cdot k/M_Q)}{2v \cdot k + k^2/M_Q} \\ &\to \frac{i}{v \cdot k},\end{aligned}$$

so that we can take the propagator to be simply

$$\frac{i}{v \cdot k}.$$

Gluon coupling with a heavy quark is represented by the vertex

$$-i\, g\, T^a \gamma^{\mu}$$

This vertex for the heavy quark is sandwiched between spinors and so between the on shell projection operator $\frac{1}{2}(1 + \gamma \cdot v)$ giving

$$\frac{1 + \gamma \cdot v}{2} \gamma^{\mu} \frac{1 + \gamma \cdot v}{2} = v^{\mu},$$

so that the gluon vertex is replaced by

$$(-ig)\, T^a v^{\mu}.$$

New Feynman rules have no heavy quark mass dependence and so exhibit explicit symmetry under the change of heavy quark flavour. Also there are no γ-matrices in the Feynman rules, so the heavy quark spin symmetry is also apparent. These rules can be obtained in the heavy quark effective theory with an effective Lagrangian.

2.2. Heavy quark effective theory

The idea of the heavy quark limit can be easily formulated in terms of an effective field theory, termed as Heavy Quark Effective Theory (HQET) [3-11]. It is useful as a procedure for making explicit calculations.

The heavy quarkeffective Lagrangian is

$$L = \bar{h}_v (v^{\mu} D_{\mu}) h_v, \tag{1}$$

which with the QCD lagrangian for the gluon and light quarks describes the structure of hadrons. The heavy quark effective field h_v^i is a Dirac spinor that satisfies the constraint

$$\gamma \cdot h_v = h_v \qquad \text{or} \qquad \left(\frac{1+\gamma\cdot v}{2}\right) h_v = h_v. \tag{2}$$

It destroys a heavy quark of type i and four-velocity v and is related to the original field operator Q_i by

$$\text{or} \qquad h_v^i = e^{iM_Q v\cdot x} P_+(v) Q(x)_i. \tag{3}$$

$P_+(v) = \frac{1}{2}(1+\gamma\cdot v)$ is an on-shell projection operator. The Lagrangian in (1) has the symmetries associated with a quark of infinite mass and is derived from Dirac Lagrangian

$$L = \overline{Q}(\gamma\cdot D - M_Q)Q \tag{4}$$

in the infinite mass limit with the help of (3).

We have done this for the heavy quark. We can do a similar thing for the heavy antiquark. In the infinite mass limit, there is not enough energy to create heavy quark-antiquark pairs, so quark and antiquark fields are independent in HQET.

3. Matrix Elements of Currents betwen Meson States

A heavy meson consists of a heavy quark Q and a light antiquark $\overline{q}$. If we denote by P_1 and P_1^* the pseudoscalar (0^-) and vector (1^-) mesons, their quantun numbers are represented in the following interpolating fields

$$\begin{aligned} P_i(v) &= \bar{q}\gamma_5 h_v^i \sqrt{M_p}, \\ P_i^*(v) &= \bar{q}\gamma\cdot\varepsilon h_v^i . \sqrt{M_{p^*}} \end{aligned} \tag{5}$$

M_p and M_p^* are the masses of the pseudoscalar and vector mesons. The light quark q stands for a column vector in flavour SU(3)

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}. \tag{6}$$

Thus each of $P_i(v)$ and $P_i^*(v,\varepsilon)$ is an SU(3) antitriplet. Consider for example, the matrix element

$$\begin{aligned} \langle P_j(v') \mid J_\lambda \mid P_i(v)\rangle/\sqrt{M_p M_{p'}} &= \langle 0 \mid (\overline{q}_{v'}\gamma_5 h_{v'}^j) J_\lambda (\overline{h}_v^i \gamma_5 q_v) \mid 0\rangle \\ &= \langle 0 \mid (\overline{q}_{v'}\gamma_5 h_{v'}^j)(\overline{h}_{v'}^j \gamma_\lambda h_v^i)(\overline{h}_v^i \gamma_5 q_v) \mid 0\rangle \\ &= \mathrm{Tr}\left[\gamma_5 \left(\frac{\gamma\cdot v' + 1}{2}\right)\gamma_\lambda \right. \\ &\qquad \left. \left(\frac{\gamma\cdot v+1}{2}\right)\gamma_5 M\right] \end{aligned} \tag{7}$$

for the vector current. We may write

$$\begin{aligned} M &= \langle 0 \mid q_{v'}\overline{q}_v \mid 0\rangle \\ &= A + B\gamma\cdot v + C\gamma\cdot v' + D\gamma\cdot v\,\gamma\cdot v' \end{aligned} \tag{8}$$

from Lorentz invariance. Now, because

$$\gamma \cdot v' \gamma_5 \left(\frac{\gamma \cdot v' + 1}{2}\right) = -\gamma_5 \left(\frac{\gamma \cdot v' + 1}{2}\right) \quad (9)$$

and

$$\left(\frac{\gamma \cdot v + 1}{2}\right) \gamma_5 \gamma \cdot v = -\left(\frac{\gamma \cdot v' + 1}{2}\right) \gamma_5, \quad (10)$$

we obtain from (7)

$$\langle P_j(v') \mid J_\lambda \mid P_i(v)\rangle / \sqrt{M_P M_{P'}} = \xi(v \cdot v')(v + v')_\lambda, \quad (11)$$

where

$$\xi(v \cdot v') = A - B - C + D. \quad (12)$$

$\xi(v \cdot v')$ contains the non-perturbative information on the response of the light quark(s) to a change in the velocity of the colour source from v to v′. Further, since the integral of the time component of the vector current is the generator of the heavy flavour group, we have the normalization condition

$$\xi(v \cdot v' = 1) = 1. \quad (13)$$

Similarly, we can calculate other marix elements of currents between mesonic states. For the ground state mesons, they turn out be

$$\begin{aligned}
\langle P_j(v') \mid J_\lambda \mid P_i(v)\rangle / \sqrt{M_P M_P} &= c_{ji}\xi(v \cdot v')(v + v')_\lambda, \\
\langle P_j^*(v') \mid J_\lambda^V \mid P_i(v)\rangle / \sqrt{M_P M_{P^*}} &= c_{ji}\xi(v \cdot v')\varepsilon_{\lambda\mu\nu\sigma}\varepsilon'^{\mu} v'^{\nu} v^{\sigma}, \\
\langle P_j^*(v') \mid J_\lambda^A \mid P_i(v)\rangle / \sqrt{M_P M_{P^*}} &= c_{ji}\xi(v \cdot v')\left[\varepsilon'_\lambda(1 + v' \cdot v) - v'^{\lambda}\varepsilon' \cdot v\right], \\
\langle P_j^*(v') \mid J_\lambda^V \mid P_i^*(v)\rangle / \sqrt{M_P M_{P'^*}} &= -c_{ji}\xi(v \cdot v')\left[\varepsilon' \cdot \varepsilon(v + v')_\lambda - (\varepsilon' \cdot v)\varepsilon_\lambda \right. \\
&\quad \left. -(\varepsilon \cdot v')\varepsilon'_\lambda\right], \\
\langle P_j^*(v') \mid J_\lambda^A \mid P_i^*(v)\rangle / \sqrt{M_P M_{P'^*}} &= -c_{ji}\xi(v \cdot v') \imath \varepsilon_{\lambda\mu\nu\sigma}\varepsilon'^{\mu}\varepsilon^{\nu}(v + v')^{\sigma}, \quad (14)
\end{aligned}$$

where c_{ji} is a measure of the renormalization effect [1] and is given by

$$c_{ji} = \left[\frac{\alpha_s(M_i)}{\alpha_s(M_j)}\right]^{a_I} \left[\frac{\alpha_s(M_j)}{\alpha_s(\mu)}\right]^{a_L(v \cdot v')}, \quad (15)$$

where, in the b→c transition for example

$$a_I = -\frac{6}{25}, \quad a_L(w) = \frac{8}{27}\left[w\, r(w) - 1\right] \quad \text{and}$$

$$r(w) = \frac{1}{\sqrt{w^2 - 1}} \ln\left(w + \sqrt{w^2 - 1}\right). \quad (16)$$

The semileptonic decays of B mesons requre the matrix elements of the vector and axial vector currents between beauty and charm mesons which can be described in terms of fourteen form factors. In general, they are defined by

$$\begin{aligned}
\langle D(v') \mid V_\lambda \mid B(v)\rangle / \sqrt{M_B M_D} &= [h_+(v \cdot v')(v + v')_\lambda + \\
&\quad h_-(v \cdot v')(v - v')_\lambda], \\
\langle D^*(v') \mid V_\lambda \mid B(v)\rangle / \sqrt{M_B M_{D^*}} &= h_v(v \cdot v')\varepsilon_{\lambda\mu\nu\sigma}\varepsilon'^{\mu} v'^{\nu} v^{\mathrm{sigma}}, \\
\langle D^*(v') \mid A_\lambda \mid B(v)\rangle / \sqrt{M_B M_{D^*}} &= h_{A1}(v \cdot v')\left[\varepsilon'_\lambda(1 + v' \cdot v) - \right. \\
&\quad \left. h_{A2}(v \cdot v')\varepsilon'^* \cdot v v_\lambda - h_{A3}\varepsilon'^* \cdot v v'_\lambda\right],
\end{aligned}$$

$$\begin{aligned}
\langle D^*(v') \mid V_\lambda \mid B^*(v)\rangle/\sqrt{M_{B^*}M_{D^*}} &= [-\varepsilon'^* \cdot \varepsilon\{h_1(v\cdot v')(v+v')_\lambda + \\
&\quad h_2(v\cdot v')(v-v')_\lambda\} \\
&\quad + h_3(\varepsilon'^* \cdot v)\varepsilon_\lambda - h_4(\varepsilon \cdot v')\varepsilon'^*_\lambda \\
&\quad - \varepsilon\cdot v'\, \varepsilon^* \cdot v\{h_5(v\cdot v')v_\lambda + \\
&\quad h_6(v\cdot v')v'_\lambda\}], \\
\langle D^*(v') \mid A_\lambda \mid B^*(v)\rangle/\sqrt{M_{B^*}M_{D^*}} &= i\varepsilon_{\lambda\mu\nu\sigma}\varepsilon'^{\mu}\varepsilon'^{\nu}\,[h_7(v\cdot v')(v+v')^\sigma \\
&\quad + h_8(v\cdot v')(v-v')^\sigma].
\end{aligned}$$

At the leading order in the heavy quark expansion, therefore,

$$h_+ = h_v = h_{A1} = h_{A3} = h_1 = h_3 = h_4 = h_7 = \xi,$$

while the remaining six form factors vanish. Using these relations, predictions can be made on the decay rates and asymmetry parameters. Since in an actual decay, the Isgur-Wise function is needed at points away from zero recoil, it is parmetrized in a variety of forms.

4. Baryon wave Functions

Similar ideas can also be applied to the case of baryons. A heavy baryon contains a heavy quark and two light quarks, which we refer to as diquark. The two quarks (or a diquark) are in a symmetric sextet or antisymmetric antitriplet state of $SU(3)_f$. Since heavy quark is a singlet in $SU(3)_f$, the transformation property of a heavy baryon is the same as that of light diquark it contains. spin $\frac{1}{2}$ baryons in the antitriplet state are

$$\begin{aligned}
\Lambda_Q &= \frac{1}{\sqrt{2}}(ud - du)\, Q\,, \\
\Xi_Q^{+\frac{1}{2}} &= \frac{1}{\sqrt{2}}(us - su)\, Q \qquad \text{(isospin up state)}\,, \\
\Xi_Q^{-\frac{1}{2}} &= \frac{1}{\sqrt{2}}(ds - sd)\, Q \qquad \text{(isospin down state)}
\end{aligned} \tag{17}$$

The light quarks in these are in isospin zero and spin zero state. spin $\frac{1}{2}$ baryons in the sextet state are

$$\begin{aligned}
\Sigma_Q^{+1} &= u\,u\,Q\,, \\
\Sigma_Q^{0} &= \frac{1}{\sqrt{2}}(ud + du)Q\,, \\
\Sigma_Q^{-1} &= d\,d\,Q\,, \\
\Xi_Q'^{+\frac{1}{2}} &= \frac{1}{\sqrt{2}}(us + su)\, Q \qquad \text{(isospin up state)}\,, \\
\Xi_Q'^{-\frac{1}{2}} &= \frac{1}{\sqrt{2}}(ds + sd)\, Q \qquad \text{(isospin down state)}\,, \\
\Omega_Q &= s\,s\,Q\,.
\end{aligned} \tag{18}$$

The light diquark in these baryons has isospin one and spin one.

4.1. Interpolating fields in terms of diquark fields of the light quarks

Since the ground state baryons have even parity, the diquark must also have even parity. So the diquark in B_3 has spin parity 0^+ and therefore, it is represented by a Lorentz scalar field ϕ. similarly, the diquark in B_6 multiplet has spin parity 1^+. It is represented by an axial vector field ϕ_μ.

The interpolating fields are

$$B_3(v,s) = \bar{u}(v,s)\phi_v h_v \,, \tag{19}$$
$$B_6(v,s) = \bar{B}_\mu(v,s,k)\phi_v^\mu h_v \qquad (k = 1,\ 2); \tag{20}$$

where $k = 1$ for spin $\frac{1}{2}$ baryon, $k = 2$ for spin $\frac{3}{2}$ baryon.

The wave functions, as given by Georgi [5], are

$$\bar{B}_\mu(v,s,k=1) = \frac{1}{\sqrt{3}}\bar{u}(v,s)\gamma_5(\gamma_\mu + v_\mu)\,, \tag{21}$$
$$\bar{B}_\mu(v,s,k=2) = \bar{u}_\mu(v,s)\,. \tag{22}$$

$u_\mu(v,s)$ and $u(v,\ s)$ are the Rarita-Schwinger vector spinor and the usual Dirac spinor respectively. The corresponding interpolating fields for the antibaryons are

$$\bar{B}_3(v,s) = \bar{h}_v\phi_v^+ u(v,s)\,,$$
$$\bar{B}_6(v,s) = \bar{h}_v\phi_v^{+\mu} B_\mu(v,s,k) \qquad (k = 1,\ 2); \tag{23}$$

where again $k = 1$ for spin $\frac{1}{2}$ baryon, $k = 2$ for spin $\frac{3}{2}$ baryon with

$$B_\mu(v,s,k=1) = -\frac{1}{\sqrt{3}}\gamma_5(\gamma_\mu + v_\mu)u(v,s)\,,$$
$$B_\mu(v,s,k=2) = u_\mu(v,s)\,. \tag{24}$$

The minus sign is the result of anticommutation relation

$$\{\gamma_5, \gamma_0\} = 0\,.$$

Further,

$$\gamma\cdot v\, B_\mu(v,s) = B_\mu(v,s)\,,$$
$$v^\mu B_\mu(v,s) = 0\,, \tag{25}$$
$$\bar{u}(v,s)\, u(v,s') = \delta_{ss'}\,,$$
$$\bar{u}_\mu(v,s)\, u_\mu(v,s') = -\delta_{ss'}\,. \tag{26}$$

4.2. Matrix elements of currents between baryon states

For the matrix elements of the vector and axial vector between heavy baryon states, we may write

$$\begin{aligned}\langle \Lambda_{Q_j}(v',s') \mid J^{ji}_{v \text{ or } A} \mid \Lambda_{Q_i}(v,s)\rangle &= \langle 0 \mid \bar{u}(v',s')\phi_{v'}h_{v'}\bar{h}_{v'} \left\{ \begin{array}{c}\gamma_\lambda \\ \gamma_\lambda\gamma_5\end{array} \right\} \\ &\quad h_v\bar{h}_v\phi_v^+ u(v,s) \mid 0\rangle \\ &= \langle 0 \mid \bar{u}(v',s')h_{v'}\bar{h}_{v'} \left\{ \begin{array}{c}\gamma_\lambda \\ \gamma_\lambda\gamma_5\end{array} \right\} \\ &\quad h_v\bar{h}_v u(v,s) \mid 0\rangle\langle 0 \mid \phi_{v'}\phi_v^+ \mid 0\rangle \\ &= \bar{u}(v',s') \left\{ \begin{array}{c}\gamma_\lambda \\ \gamma_\lambda\gamma_5\end{array} \right\} \phi_{v'}\phi_v^+ u(v,s).\end{aligned} \tag{27}$$

We parametrize $\langle 0 \mid \phi_{v'}\phi_v^+ \mid 0\rangle$ as before and write

$$\langle 0 \mid \phi_{v'}\phi_v^+ \mid 0\rangle \;=\; A + B\gamma\cdot v + C\gamma\cdot v' + D\gamma\cdot v\gamma\cdot v',$$

so that a Dirac matrix has to be inserted in (27).

In the limit of infinite mass, v is nearly equal to v′ and so $\gamma\cdot v$ and $\gamma\cdot v'$ can be replaced by 1, giving a single form factor as function of a Lorentz invariant velocity transfer

$$\omega \;=\; v\cdot v' \;=\; \frac{M^2_{\Lambda_{Q_j}} + M^2_{\Lambda_{Q_i}} - q^2}{2M_{\Lambda_{Q_j}}M_{\Lambda_{Q_i}}}. \tag{28}$$

So

$$\langle \Lambda_{Q_j}(v',s') \mid J^{ji}_{V \text{ or } A} \mid \Lambda_{Q_i}(v,s)\rangle \;=\; A(\omega = v\cdot v')\bar{u}(v',s') \left\{ \begin{array}{c} \gamma_\lambda \\ \gamma_\lambda\gamma_5 \end{array} \right\} u(v,s). \tag{29}$$

In general, the semileptonic decays are described by the six form factors defined by

$$\langle \Lambda_{Q_j}(v',s') \mid J^{ji}_{V-A} \mid \Lambda_{Q_i}(v,s)\rangle \;=\; \bar{u}_{Q_j}(v')[\{F_1(v\cdot v')\gamma_\mu + F_2(v\cdot v')v_\mu + F_3(v\cdot v')v'_\mu\} - \{G_1(v\cdot v')\gamma_\mu + G_2(v\cdot v')v_\mu + G_3(v\cdot v')v'_\mu\}\gamma_5]u_{Q_i}(v).$$

In the leading order the results from HQET are

$$F_1(v\cdot v') \;=\; G_1(v\cdot v'),$$
$$F_2(v\cdot v') \;=\; G_2(v\cdot v') \;=\; F_3(v\cdot v') \;=\; G_3(v\cdot v') \;=\; 0.$$

4.3. Heavy to heavy transition

As an example of the method, we choose the semileptonic decays of the bottom baryon Λ_b , because it is especially simple :

$$\Lambda_b^0 \;\rightarrow\; \Lambda_c^+ e^- \bar{\nu}_e\ .$$

The matrix element is

$$\langle \Lambda_c;\ v' \mid J_\mu^{weak} \mid \Lambda_b;\ v\rangle \;=\; \bar{u}(v')\gamma_\mu(1-\gamma_5)u(v)F_\Lambda(w). \tag{30}$$

Furthermore, this universal form factor $F_\Lambda(w)$ is identical to the elastic form factor of Λ_b and is normalized to 1 at $\omega = 1$.

4.4. Heavy to light transition

In a heavy to light transition, such as

$$\Lambda_c \;\rightarrow\; \Lambda$$

we do not have such a large symmetry. However, the symmetry in the heavy baryon wave function does reduce the number of form factors even in this case. The matrix element of the weak current for a heavy baryon Λ_c into a light baryon B is given by [10]

$$\langle B(p') \mid \bar{q}L_\mu h \mid \Lambda_c\rangle \;=\; \bar{u}_B(p')[A_1(p'\cdot v) + \gamma\cdot vA_2(p\cdot v)]L_\mu u_{\Lambda_c}(v)\ . \tag{31}$$

Here, there are two independent form factors, $L_\mu = \gamma_\mu(1-\gamma_5)$ is the left handed current operator, $v = p/M$ is the four velocity of Λ_c and p' is the four momentum of the daughter baryon B.

5. Nonleptonic Decays

To lowest order in the weak interaction, the nonleptonic weak interaction Hamiltonian H_w in the Standard Model can effectively be written in the current x current form

$$H_w = \frac{G_F}{\sqrt{2}} J_\mu J^\mu + \text{h.c.},$$

where the weak current J_μ is given by

$$J_\mu = [\bar{u}\ \bar{c}\ \bar{t}]\gamma_\mu(1-\gamma_5)\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix}.$$

The weak eigenstates d', s' and b' are related to the mass eigenstates d, s and b through the cabibbo-Kobayashi-Maskawa mixing matrix

$$\begin{bmatrix} d' \\ s' \\ b' \end{bmatrix} = \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}\begin{bmatrix} d \\ s \\ b \end{bmatrix}.$$

5.1. The factorization hypothesis

In this approaaach for nonleptonic weak decays, one assumes that the long distance QCD effects occur largely in the initial and final state configurations. This is referred to as the factorization hypothesis as it allows [14] the decay amplitude to be written as a product of matrix elements of currents, e.g.

$$\begin{aligned} A(D^+ \to \bar{K}^0\pi^+) &= \frac{G_F}{\sqrt{2}}[a_1\langle \bar{K}^0 \mid J^\mu \mid D^+\rangle\langle \pi^+ \mid j_\mu \mid 0\rangle \\ &\quad + a_2\langle \bar{K}^0 \mid j\mu \mid 0\rangle\langle \pi^+ \mid J_\mu \mid D^+\rangle]. \end{aligned} \tag{32}$$

This is similar to the manner in which semileptonic decay amplitudes are constructed where factorization occurs automatically since no strong coupling occurs between the leptons and quarks.

In what follows we assume that the amplitudes for the decays factorize. Although this hypothesis, so far, lacks a good theoretical justification, yet it seems to work phenomenologically for the decays of heavy mesons [14]. More recently, it has been argued by Dugan and Grinstein [17] that this assumption is valid when the meson emitted in the decay of the heavy hadron by the virtual W is so energetic that it escapes the strong interaction effects of the surrounding matter. This corresponds to the production of a light meson. Fortunately, the decays we are considering fall into this category. The nonleptonic decays of heavy mesons have been analysed [2] in the heavy quark approximation.

5.2. Nonleptonic decays of heavy baryons

The factorized amplitude for the decay $\Lambda_c \to BX$, where X may be a pseudoscalar meson, a vector meson or anything else, can be written as

$$M_{\Lambda_c \to BX} = \frac{G_F}{\sqrt{2}} V_{cj} V^*_{kl} H_\mu X^\mu \tag{33}$$

with appropriate K-M matrix elements where

$$X^\mu = \langle X \mid J^\mu \mid 0 \rangle = f_P q^\mu, \tag{34}$$

with X as the pseudoscalar meson P, q as its fourth momentum and f_P its decay constant. H^μ is defined by

$$H_\mu = \langle B(p') \mid \bar{q} L_\mu h \mid \Lambda_c \rangle, \tag{35}$$

as given by Eq.(31). A_1 and A_2 are two real form factors describing the weak current. (Note that HQET predicts $G_A = -G_V$). A_1 and A_2, in general, depend on the flavor of the light spin $\frac{1}{2}$ baryon involved but not on X. Thus all decays of the type $\Lambda_c \to B$ are related to each other.

5.3. Heavy baryon to a light baryon and a pseudoscalar meson

Substituting

$$q = m_{\Lambda_c} v - p' \tag{36}$$

in (33), we obtain, the decay amplitude for $\Lambda_c \to B\ P$ as

$$M_{\Lambda_c \to BP} = \frac{G_F}{\sqrt{2}} f_P V_{cj} V^*_{kl} \bar{u}(p')[G_1 - G_2 \gamma_5] u_{\Lambda_c}(v), \tag{37}$$

with appropriate K-M matrix elements, where (c, k) and (j, l) stand for the $+\frac{2}{3}$ charge quarks and $-\frac{1}{3}$ quarks respectively. Moreover,

$$\begin{aligned} G_1 &= A_1 (m_{\Lambda_c} - m_\Lambda) + A_2 (m_{\Lambda_c} + m_\Lambda - 2\ E_\Lambda), \\ G_2 &= A_1 (m_{\Lambda_c} + m_\Lambda) - A_2 (m_{\Lambda_c} - m_\Lambda - 2\ E_\Lambda), \end{aligned} \tag{38}$$

E_Λ being the energy of the daughter baryon in the rest frame of Λ_c.

In the limit of a negligible pion mass, the form factors G_1 and G_2 become

$$\begin{aligned} G_1 &= \frac{m_{\Lambda_c} - m_\Lambda}{m_{\Lambda_c}} (m_{\Lambda_c} A_1 + m_\Lambda A_2), \\ G_2 &= \frac{m_{\Lambda_c} + m_\Lambda}{m_{\Lambda_c}} (m_{\Lambda_c} A_1 + m_\Lambda A_2). \end{aligned} \tag{39}$$

Obviously the ratio G_2/G_1 is approximately independent of the ratio A_2/A_1 and is a fixed number :

$$\frac{G_2}{G_1} = \frac{m_{\Lambda_c} - m_\Lambda}{m_{\Lambda_c} + m_\Lambda} = .34\ . \tag{40}$$

Further, the asymmetry parameter α is defined as [18]

$$\alpha = \frac{2 \mathrm{Re} \kappa A^* B}{\mid A \mid^2 + \kappa^2 \mid B \mid^2}, \tag{41}$$

where

$$A = \frac{G_F}{\sqrt{2}} f_P V_{cj} V^*_{kl} G_1 \qquad \text{and}$$

$$B = -\frac{G_F}{\sqrt{2}} f_P V_{cj} V^*_{kl} G_2. \tag{42}$$

From (40) and (41) we see that $\alpha = -1$ and is independent of the value of A_1 and A_2. This is a very strong prediction of HQET. Recent experimental results [15] seem to confirm the prediction. For the decay $\Lambda_c \to \Lambda\pi^+$ the asymmetry parameter has been measured to be

$$\alpha = -1.0^{+0.4}_{-0.0} \text{ (CLEO) [15] and } -0.96 \pm +0.42 \text{ (ARGUS) [16].}$$

The associated decay rates can be calculated from

$$\Gamma(\Lambda_c^+ \to BP) = \frac{|p_B|}{4\pi m_{\Lambda_c}}(E_B + m_B)\left[|A|^2 + \frac{E_B - m_B}{E_B + m_B}|B|^2\right]. \tag{43}$$

Substituting the appropriate numbers for the decay $\Lambda_c^+ \to \Lambda\pi^+$ and using the experimental value of the decay rate as input, we obtain

$$1.23\{|G_1|^2 + (.1169)|G_2|^2\} = .39\,. \tag{44}$$

Then the use of Eq.(40) enables the determination of $|G_1|^2$ and $|G_2|^2$ as

$$|G_1|^2 = .1576 \quad , \quad |G_2|^2 = 1.364\,. \tag{45}$$

Invoking the SU(3) symmetry arguments we find that the form factors for the modes $\Lambda_c \to p\bar{K}^0$ and $\Xi_c^0 \to \Xi^-\pi^+$ are related to those of $\Lambda_c^+ \to \Lambda\pi^+$ through a factor of $-\sqrt{2/3}$. G_1 and G_2 are determined by Eq. (45). The computed values of the decay widths and asymmetry parameters are listed in Table 1. Also listed are the available experimental values. Note that the values of the decay rate and the asymmetry parameter compare reasonably well with the available experimental data. The decay rate for $\Lambda_c \to p\bar{K}^0$ is seen to lie within the observed range. This is in contrast with recent theoretical prediction [18,19], employing current algebra techniques and the factorization hypothesis, of a value that is a factor three higher than the measured one. Further, the decay width of $\Xi_c^0 \to \Xi^-\pi^+$ may be regarded as a good parameter for testing the factorization hypothesis. Since the contribution from the factorized amplitude is small, the decay width is lower than the prediction of Ref 18. A precise measurement of this decay width may provide a pertinent test for the factorization assumption. For absolute values of the form factors A_1 and A_2 one has to await the measurement of form factors in the process $\Lambda_c \to \Lambda e^+\nu$. The predicted values in Eq. (4) can then be confronted with data, when they become available. Before concluding this section, we would like to mention that we have ignored the terms suppressed by $1/m_c$ in our estimations. The associated corrections on the amplitudes in the b-sector are of the order of 4% while in the charm sector, they are of the order of 10%.

5.4. $\Lambda_c \to B\,V$ transitions

For the emission of a vector meson V in the weak decay of Λ_c, the matrix element has the form :

$$M_{\Lambda_c \to BV} = f_V\, H_\mu \epsilon^\mu, \tag{46}$$

Table 1: Values of decay width Γ in units of 10^{11} sec^{-1} and asymmetry parameter α : (a) predicted values, (b) experimental results

Decay Mode	Decay width Γ		Asymmetry Parameter α	
	(a)	(b)	(a)	(b)
$\Lambda_c^+ \to \Lambda\pi^+$	0.39 [21]	0.39 ± 0.17	-0.99	-0.96 ± 0.42 [18]
$\Lambda_c^+ \to p\bar{K}^0$	0.879	1.17 ± 0.42	-0.98	
$\Xi_c^0 \to \Xi^-\pi^+$	0.249		-0.92	
$\Lambda_c^+ \to \Lambda\rho^+$	0.21			
$\Lambda_c^+ \to p\bar{K}^{*0}$	0.27	0.3 ± 0.15		

where ε^μ is the polarization vector of the vector meson and f_V is its decay constant. Substituting for H_μ, the relevant matrix element becomes

$$M_{\Lambda_c\to BV} = \frac{G_F}{\sqrt{2}} V_{ij}V_{ck} f_V \bar{u}_B(p')\left[A_1(p'\cdot v) + \gamma\cdot v\, A_2(p'\cdot v)\right] L_\mu u_{\Lambda_c}(v)\varepsilon^\mu .$$

The decay width is then given by

$$\Gamma = \frac{|p_B|}{2\pi m_{\Lambda_c}} G_F^2 \,|V_{ij}|^2 |V_{ck}|^2 \times \left[E_B\left(A_1^2 - A_2^2 - 2A_1A_2\right) - 2M_B\left(A_1 - A_2\right)A_2\right]. \tag{47}$$

The form factors A_1 and A_2 are determined employing Eqns. (38), (39) and (45). The decay rates for the processes $\Lambda_c \to \Lambda\rho^+$ and $\Lambda_c^+ \to p\bar{K}^{*0}$ are then determined and listed in Table 1. Here, we have used $f_\rho = f_{K^*} = .11\,\mathrm{GeV}^2$ [20].

Using the experimental values of the lifetime of Λ_c, $\tau(\Lambda_c) = 1.91 \times 10^{-13}$ sec, we find the predicted branching ratios

$$\begin{aligned} B(\Lambda_c^+ \to \Lambda\rho^+) &= 4.8 \times 10^{-3}, \\ B(\Lambda_c^+ \to p\bar{K}^{*0}) &= 5.2 \times 10^{-3}. \end{aligned} \tag{48}$$

The branching ratio of $\Lambda_c \to p\bar{K}^{*0}$ is consistent with the observed value [22]. In line with this, we expect our results for the $\Lambda_c^+ \to \Lambda\rho^+$ to have better agreement with experimental observation than the results of Ref 19 and we look forward to our predictions being tested. Finally, in conjunction with the results of the previous section, we conclude that HQET my indeed constitute a benchmark for studying many of the long standing problems in high energy physics.

5.5. B-meson decays into heavy baryons

B-meson decays into heavy baryons have also been studied [23] in heavy quark formalism. For the decay,

$$B \to \Xi_c\bar{\Lambda}_c$$

for example, we may write

$$\langle \Xi_c(v_\Xi)\bar{\Lambda}_c(v_\Lambda) \mid \bar{s}\gamma_\mu c\bar{c}\gamma^\mu(1-\gamma_5)b \mid B(v)\rangle =$$
$$\bar{u}_{\Xi_c}[\gamma_\mu(1-\gamma_5)\frac{\sqrt{m_B}}{2}\gamma_5(\gamma\cdot v - 1)(\alpha + \beta\gamma\cdot v')\gamma^\mu(1-\gamma_5)]u_{\Lambda_c}(v_{\Lambda_c})$$

Here the symmetries of the HQET haave been used to express the four-quark weak current responsible for the decay as a product of two-quark currents.

6. Corrections

Heavy-quark symmetry makes many predictions in the $m_Q \longrightarrow \infty$ limit. But, because we want to apply these ideas to c and b quarks which are not really infinitely massive, we must include Λ_{QCD}/m_Q corrections. Fortunately, the effective field theory allows a systematic inclusion of corrections [25], so there is some predictive power from the symmetry even when it is broken.

7. Conclusions

It appears that it may be useful to study heavy hadrons in the heavy quark approximation. There are many advantages of this limit: the heavy quark acts as a pointlike probe of the light constituent quarks and the relativistic effects are simplified. The results of the symmetries of the heavy quark limit may be employed to act as consistency-checks on models. HQET coupled with the factorization hypothesis can be used to study nonleptonic decays of heavy hadrons. The naive predictions on charm baryon decays show that HQET performs well when confronted with the experimental data. It is also observed that the decay $\Xi_c^0 \rightarrow \Xi^- \pi^+$ may provide a direct test for the factorization hypothesis. However, more stringent tests require better experimental data as also the incorporation of $1/m_c$ effects. $1/M_Q$ and other corrections have already been studied [24] sytematically upto order $1/M_Q^2$. Eventually, a more detailed study of the heavy to light form factors is called for. For the knowledge of the universal form factors (Isgur-Wise function) we need nonperturbative QCD, but an understanding of these functions is ultimately necessary for any quantitative analysis based on heavy quark symmetry. In future, it may be possible to compute the functions from first principle, by using effective theory on the lattice. In the meantime QCD sum rules have recently been employed [25]. New data from charm and beauty factories will allow us to test heavy quark symmetry and to determine reliably some Standard Model parameters. Heavy quarks may well turn out to be a useful tool to understand the nature of QCD of the light quarks.

Appendix : $1/M_Q$ Corrections – (an introduction)

One of the main virtues of the HQET is that it allows a systematic study of the corrections that arise frin the approximations we have made.

The heavy quark equation of motion is

$$(i\gamma \cdot D - M_Q)Q(x) = 0 \tag{1}$$

We can put the quark almost on the mass shell by introducing the field $h_v(x)$:

$$Q(x) = e^{-iM_Q v \cdot x} h_v(x).$$

The equation of motion in terms of h_v is

$$[i\gamma \cdot D + (\gamma \cdot v - 1)M_Q] h_v(x) = 0 . \tag{2}$$

We separate $\frac{1}{2}(1+\gamma\cdot v)$ and $\frac{1}{2}(1-\gamma\cdot v)$ components of $h_v(x)$ as

$$h_v(x) = h_v^{(+)}(x)h_v^{(-)}(x),$$

with

$$h_v^{\pm}(x) = \frac{1\pm\gamma\cdot v}{2}h_v(x).$$

Multiply (2) by $\frac{1}{2}(1\pm\gamma\cdot v)$ to get

$$\imath v\cdot Dh_v^{(+)} = -\frac{1+\gamma\cdot v}{2}\imath\gamma\cdot Dh_v^{(-)}(x), \quad (3)$$

$$\imath v\cdot Dh_v^{(-)} + 2M_Q h_v^{(-)} = -\frac{1-\gamma\cdot v}{2}\imath\gamma\cdot Dh_v^{(+)}(x). \quad (4)$$

To solve these equations, we assume that h_v^{+} is of the order $(M_Q)^0$ and $h_v^{(-)}$ of the order $1/M_Q$.

From (4)

$$h_v^{(-)} = \frac{1}{2M_Q}\frac{1-\gamma\cdot v}{2}\imath\gamma\cdot Dh_v^{(+)} - \frac{1}{2M_Q}\imath v\cdot Dh_v^{(-)}. \quad (5)$$

Substituting (5) into (3) and dropping terms of the order $1/M_Q^2$, we obtain

$$\imath v\cdot Dh_v^{(+)} = -\frac{1+\gamma\cdot v}{2}\imath\gamma\cdot\left\{\frac{1}{2M_Q}\frac{1-\gamma\cdot v}{2}\imath\gamma\cdot Dh_v^{(+)}\right\}. \quad (6)$$

We can see that

$$\frac{1+\gamma\cdot v}{2}\gamma\cdot D\frac{1-\gamma\cdot v}{2}\gamma\cdot D\frac{1+\gamma\cdot v}{2} = \frac{1+\gamma\cdot v}{2}\left[D^2-(v\cdot D)^2+\frac{1}{2}g_s G_{\mu\nu}\sigma^{\mu\nu}\right]\frac{1+\gamma\cdot v}{2}. \quad (7)$$

Substituting (7) in (6) we obtain the equation of motion upto order $1/M_Q$. This equation of motion is obtained from the Lagrangian

$$L_{eff}^{v} = \bar{h}_v^{(+)}\imath v\cdot Dh_v^{(+)} - \frac{1}{2M_Q}\bar{h}_v^{(+)}\left[D^2-(v\cdot D)^2+\frac{1}{2}g_s G_{\mu\nu}\sigma^{\mu\nu}\right]h_v^{(+)}. \quad (8)$$

From this procedure it should be clear how to include into L_{eff}^{v} higher order terms in the $1/M_Q$ expansion. The $1/M_Q$ term in L_{eff}^{v} is treated as small and perturbation theory is used to calculate its effect :

$$\Delta L = \frac{1}{2M_Q}\bar{h}_v^{(+)}\left[D^2-(v\cdot D)^2+\frac{1}{2}g_s G_{\mu\nu}\sigma^{\mu\nu}\right]h_v^{(+)}. \quad (9)$$

This breaks the symmetries of HQET but includes all orders of QCD. It is often difficult to make precise calculations of $1/M_Q$ effects.

References

[1] R.L. Kingley er al., Phys. Rev. **D11** (1975) 1919;
A. Ali, J. G. Korner and G. Kramers, Z. Phys. **C11** (1981) 21;
D. Zeppenfeld, Z.Phys. **C8** (1981) 77;
M. J. Savage and M. B. Wise, Phys. Rev. **D39** (1989) 3346;
M. J. Savage and M. B. Wise, Nucl. Phys. **B326** (1989) 15;
M. J. Savage and M. B. Wise, Phys. Rev. **D42** (1990) 1527;
Rajesh Arora, Gurpreet Kaur Sidana and M.P. Khanna, Phys. Rev. **D45** (1992) 4121.
There are many other papers on the subject. For a review on charmed baryons see J. G. Körner and H. Siebert, Annual Rev. Nucl. Part. Sc. (1992) to be published.

[2] N. Isgur and M. B. wise, Phys. Lett. **B232** (1989) 113; **B237** (1990) 527;
There are now several lecture notes which discuss in detail the ideas, calcultional methods and results of the new (so called Isgur-Wise) symmetry. They are :
J. D. Bjorken, SLAC-Report 378 in Proc. 18th SLAC Summer Institute of Particle Physics (1990) ed. by J. F. Hawthorne.
H. Georgi, Lectures at TASI,HUTP-91-Aoel (1991).
B. Grinstein, HUTP-91-A040, SSCL-Preprint 17 (1991).
T Mannel, Heavy Quark Effective Theory, IKDA 92/6, Jan. 92. Inst. Kernphysik, Tech. Hochschule, Darmstadt.
M. B. Wise, CALT-68-1721 Lectures at Lake Louise Winter Institute (1991).

[3] M. Voloshin and M Shifman, Sov. J. Nucl. Phys. **45** (1987) 292; **47** (1988) 511;
E Eichten and B. Hill, Phys. Lett. **B234** (1990) 511;
H. Politzer and M. B. Wise, Phys. Lett. **B206** (1988) 681; **B208** (1988) 504;
H. Georgi, Phys. Lett. **B240** (1990) 447.

[4] N. Isgur and M. B. Wise, Nucl. Phys. **B348** (1991) 276.

[5] H. Georgi, Nucl. Phys. **B348** (1991) 293.

[6] B. Grinstein, Nucl. Phys. **B339** (1990) 253;
A. Falk, H. Georgi, B Grinstein and M. B. wise, Nucl. Phys. **B343** (1990) 1;
A. Falk and B. Grinstein, Phys. Lett. **B247** (1990) 406.

[7] J. Bjorken, SLAC preprint SLAB-PUB 5278 (1990);
A. Acker, S. Pakvasa, S.F. Tuan and S. P. Rosen, Phys. Rev.**D43** (1991) 3083.

[8] F. Hussain, J. G. Körner, M. Kramer and G. Thompson, Z. Phys.**C51** (1991) 321;
J.G. Körner and G. Thompson, Phys. Lett. **B264** (1991) 185.

[9] M. E. Luke, Phys. Lett **B252** (1990) 447;
A. F. Falk, B. Grinstein and M. E. Luke, Nucl. Phys. **B357** (1991) 185.

[10] T. Mannel, W. Roberts and Z. Ryzak, Nucl. Phys. **B355** (1991) 38.

[11] T. Mannel, W. Roberts and Z. Ryzak, Phys. Lett. **B248** (1990) 392; **B254** (1991) 274; **B255** (1991) 593; **B259** (1991) 359; **B259** (1991) 485.

[12] H. Georgi, B. Grinstein and M.B. Wise, Phys. Lett. **B252** (1990) 456;
A.F. Falk, B. Grinstein and M.E. Luke, Nucl. Phys. **B357** (1991) 185.

[13] M. Neubert, Phys. Lett.**B264** (1991) 455.

[14] M. Wirbel, B. Stech and M. Bauer, Z. Phys. **C29** (1985) 637;
M. Bauer, B. Stech and M. Wirbel, Z. Phys. **C3** (1987) 103;
W.A. Bardeen, A.J. Buras and J.M. Gerard, Phys. Lett. **B180** (1987) 133; Nucl. Phys. **B293** (1987) 787.

[15] P. Avery et. al., Phys. Rev. **D43** (1991) 3599.

[16] The ARGUS Collaboration, DESY 91-091, August 1991.

[17] M. Dugan and B. Grinstein, Phys. Lett. **B255** (1990) 583.

[18] S. Pakvasa, S.F. Tuan and S.P. Rosen, Phys. Rev **D42** (1990) 3746.

[19] Gurpreet Kaur and M.P. Khanna, Phys. Rev.**D43** (1991) 182.

[20] M. A. Shifman, in Lepton and Photon Interaction, proceedings of the International Symposium on Lepton and Photon Interactions at High Energies, Hamburg, Germany, 1987, ed. R. Ruckl and W. Bartel.

[21] Gurpreet Kaur and M.P. Khanna, Phys. Rev. **D45** (1992) R3024.

[22] Particle Data Group, J.J. Hernandez et. al., Phys. Lett. **B239** (1991) 1.

[23] M. Neubert, Phys. Rev. **D45** (1992) 2451; Phys. Rev **D46** (1992) 1076;
E. Bagan, P. Ball, V.M. Braun and H.G. Dosh, Phys. Lett. **B278** (1992) 457;
A.V. Radyushkin, Phys. Lett. **B271** (1991) 2189;
M. Neubert, V. Rieckert, B. Stech, and Q.P. Xu in *Heavy Flavours*, ed. by A.J. Buras and M. Linder, Advanced series on Directions in High Energy physics (world scientific, Singapore, 1992);
A.F. Falk and M. Neubert, Second Order Power Corrections in the Heavy Quark Effective Theory, SLAC-PUB-5897, 1992; SLAC-PUB-5898, 1992.

[24] T. Mannel, W. Roberts and Z. Ryzak, Phys. Rev. **D45** (1992) 875.

[25] M. Neubert, Z. Ligeti and Y. Nir, SLAC-PUB-5915, 1992;
B. Blok and M. Shifman, Santa Barbara Preprint NSF-IT-P-92-100, 1992;
For introduction to $1/M_Q$ corrections see the appendix.

Discussion

M.V. Purohit : Can you comment on the numerical precision to which form factors are known to be 1 at $q^2 = q^2_{max}$? In order to extract the CKM elements, at least $|V_{cs}V_{cd}|$, we need to know $|f_+^{D\to K}(q^2_{max})/ f_+^{D\to\pi}(q^2_{max})|$. How well is this known? (1%, 5%, 10% ?)

M.P. Khanna : For charm, the form factors are known to be approximately 10%, for beauty 1 or 2%. I don't know about the ratio of form factors.

M. Purohit : The corrections of zero recoil point are $\mathcal{O}(1/m^2)$ for weak currents. They have been estimated in potential and Skyrme models (e.g. Manohar and Wise) and found to be $\sim$ 1-2% for baryons.

M.P. Khanna : At zero recoil point certain form factors remain uncorrected by 1/M corrections. In those cases the correction have been estimated to be about 1%.

N. Nimai Singh : Is the Isgur-wise factorization technique applicable to the $q\bar{Q}$ system in the presence of a complicated confinement interaction (off-shell quarks)?

M.P. Khanna : Yes, provided one of the final particles comes out with a lot of momentum. Dugan and Grinstein have proved that. If all the particles in the final state are heavy, the factorization hypothesis is likely to become crude.