

Radiative electroweak parameters

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Abstract. The status of the present precision measurements of electroweak observables is reviewed with specific reference to the radiative parameters S, T, U or equivalently $\epsilon_1, \epsilon_2, \epsilon_3$. The significance of the obliqueness hypothesis is underlined and the importance of the “local fit” method of extracting these parameters from the data is emphasized. Possible new physics implications are briefly touched upon.

1. Radiative/Precision parameters

The precision-testing of a renormalizable relativistic quantum field theory is intrinsically connected with the accurate calculation of radiative corrections in it. This connection has a long tradition [1] in QED. The basic point there is extremely simple at the 1-loop-level. If one is measuring a quantity of order unity to the precision of the third decimal place, comparison with theory is meaningful only if there is a correct calculation taking all $O(\alpha)$ terms into account. For the electroweak processes, being studied on the Z mass-shell at LEP 1, the relevant couplings are semiweak. Since the accuracy in the LEP measurements has now reached the 10^{-3} level, the consideration [2] of 1-loop corrections to the tree-level contributions is very pertinent.

Of course, certain couplings, relevant to weak and electromagnetic processes, are known to an extraordinarily high accuracy. Specifically, we refer to two which form part of our reference frame : (1) the atomic fine structure constant α_{EM} , as measured in the A.C. Josephson effect, namely [3]

$$\alpha_{EM}^{-1} = 137.0359895(61);$$

(2) the Fermi constant G_μ , as measured in muon decay. The latter enters the muon lifetime τ_μ via the formula

$$\tau_\mu^{-1} = (192\pi^3)^{-1} G_\mu^2 m_\mu^5 f(m_e/m_\mu) \left(1 + \frac{3}{5} m_\mu^2 M_W^{-2} \right) \cdot [1 + (2\pi)^{-1} \alpha_{EM} (m_\mu^2) \left(\frac{25}{4} - \pi^2 \right)]. \quad (1)$$

In (1) m_ℓ is the mass of the lepton of type ℓ , M_W is the mass of the W ,

$$f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x, \quad (2)$$

and – accounting for the running α_{EM}^{-1} from eV energies to the electron mass and

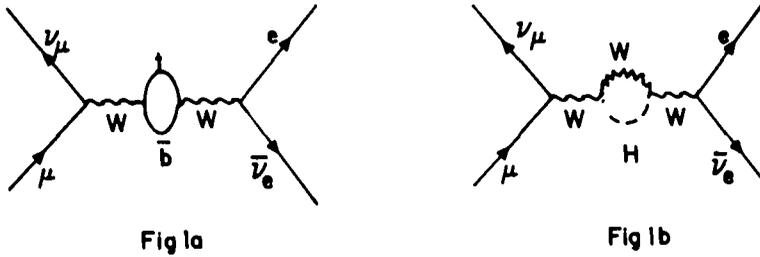


Figure 1. (a)[(b)] Top [Higgs] contribution at 1-loop level to muon decay

eventually to the muon mass -

$$\alpha_{EM}^{-1}(m_\mu^2) = \alpha_{EM}^{-1} - \frac{1}{3\pi} \ell n \frac{m_\mu^2}{m_e^2} + \frac{1}{6\pi} \simeq 136. \tag{3}$$

Using (1), (2) and (3) in conjunction with the experimental value of τ_μ , one can deduce that [3]

$$G_\mu = 1.166389(22) \times 10^{-5} \text{ GeV}^{-2}.$$

Evidently, our knowledge of α_{EM}^{-1} and G_μ is pretty precise. The unification of weak and electromagnetic interactions in the Standard Model (SM) implies a relation between these two quantities, namely

$$G_\mu = \pi \alpha_{EM} \left[\sqrt{2} M_W^2 (1 - M_W^2 M_Z^{-2}) (1 - \Delta r) \right]^{-1}. \tag{4}$$

In (4) Δr is a purely radiative constant, i.e. it vanishes in the tree approximation. To one loop, there are no QCD effects and a complete perturbative electroweak calculation of $(\Delta r)_{1\text{-loop}}$ has been done [4]. However, at present, that cannot be used for a precision test of the SM via (4). Thus is because the result depends on the two yet unknown masses $m_{t,H}$, i.e. those of the top quark and of the Higgs scalar. The former enters through the diagram of Fig. 1a and the latter (taking the single Higgs doublet Minimal Standard Model) through that of Fig. 1b. The precision testing of the MSM will be possible by use of (4) once $m_{t,H}$ get known accurately from direct measurements.

Next we come to the ρ -parameter. For low energy measurements it can be obtained (in the approximation of neglecting momentum transfers and lepton masses) in terms of the charged current cross section $\sigma(\nu_\mu e \rightarrow \mu \nu_e)$ divided by the neutral current cross section $\sigma(\nu_\mu e \rightarrow \nu_\mu e)$ at the same energy:

$$\rho = \left[\frac{\sigma(\nu_\mu e \rightarrow \nu_\mu e) \left(\frac{1}{4} - s_\theta^2 + \frac{4}{3} s_\theta^4 \right)^{-1}}{\sigma(\nu_\mu e \rightarrow \mu \nu_e)} \right]^{1/2}. \tag{5}$$

In (5) $s_\theta^2 \equiv 1 - M_W^2/M_Z^2 = 1 - c_\theta^2 \simeq 0.23$. The present best experimental number for this parameter is [5]

$$\rho = 1.008 \pm 0.014.$$

In SM, for any number of Higgs doublets, ρ is unity at the tree level but changes radiatively. The theoretical calculation is scheme-dependent. In the on-shell renormalization scheme [6], the 1-loop expression contains a quadratic dependence on m_t . For $m_t \gg M_Z$ one can write

$$\rho \simeq 1 + \frac{3\alpha_{EM}}{16\pi} \frac{m_t^2}{s_\theta^2 c_\theta^2 M_Z^2}. \quad (6)$$

There is, however, a somewhat different ρ -parameter which pertains to the on-shell $Zf\bar{f}$ coupling as probed in LEP 1. The tree-level $Zf\bar{f}$ vertex of the SM, with T_{3f} referring to the $SU(2)_L$ 3rd-component of isospin of f , is

$$V_\mu^{\text{tree}} = \sqrt{\sqrt{2}G_\mu} M_Z \gamma_\mu [(T_{3f} - s_\theta^2 Q_f) - T_{3f} \gamma_5].$$

After 1-loop radiative effects it changes to

$$V_\mu^{1\text{-loop}} = \sqrt{\sqrt{2}G_\mu} M_Z \gamma_\mu \left[\sqrt{\rho^{EFF}} (T_{3f} - \sin^2 \theta_W^{EFF} Q_f) - \sqrt{\rho^{EFF}} T_{3f} \gamma_5 \right]. \quad (7)$$

High statistics measurements on the Z at LEP 1 imply [7]

$$\begin{aligned} \rho^{EFF} &= 1.000 \pm 0.0036, \\ \sin^2 \theta_W^{EFF} &= 0.2324(11). \end{aligned}$$

Allowing for the evolution of the fine structure constant α_{EM} from eV energies to LEP 1, where [7]

$$\alpha_{EM}^{-1}(M_Z^2) = 128.2 \pm 0.09_{-0.4}^{+0.0m_t},$$

(4) changes to

$$G_\mu = \pi \alpha_{EM} (M_Z^2) \left[\sqrt{2} M_W^2 (1 - M_W^2/M_Z^2)(1 - \Delta r_W) \right]^{-1}. \quad (8)$$

The parameter Δr_W , appearing in (8), is also fully known to 1-loop and has a quadratic dependence on m_t .

The radiative parameters $\epsilon_1, \epsilon_2, \epsilon_3$ or S, T, U can now be defined as

$$\epsilon_1 \equiv \rho^{EFF} - 1 \equiv \alpha_{EM} T, \quad (9a)$$

$$\begin{aligned} \epsilon_2 &\equiv c_\theta^2 (\rho^{EFF} - 1) + s_\theta^2 \Delta r_W (c_\theta^2 - s_\theta^2)^{-1} - 2 (\sin^2 \theta_W^{EFF})^{-1} \\ &\equiv -(4s_\theta^2)^{-1} \alpha_{EM} U, \end{aligned} \quad (9b)$$

$$\begin{aligned} \epsilon_3 &\equiv (c_\theta^2 - s_\theta^2) \{ s_\theta^2 \sin^2 \theta_W^{EFF} \}^{-1} - 1 \} + c_\theta^2 (\rho^{EFF} - 1) \\ &\equiv (4s_\theta^2)^{-1} \alpha_{EM} S. \end{aligned} \quad (9c)$$

In motivating these strange-looking combinations, one may say the following. The ϵ_1 parameter is just the radiative/new physics part of the effective ρ -parameter on the Z mass-shell, while the combinations standing for $\epsilon_{2,3}$ have been chosen [8] in such a way that the quadratic terms in m_t cancel and only an insensitive logarithmic dependence on the top mass survives. Moreover, as explained below, these combinations are the most natural 1-loop radiative parameter in the ‘‘obliqueness’’

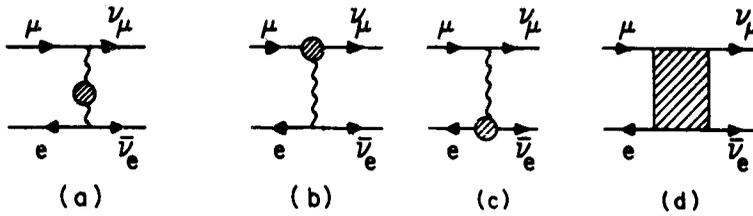


Figure 2. 1-loop contribution to muon decay

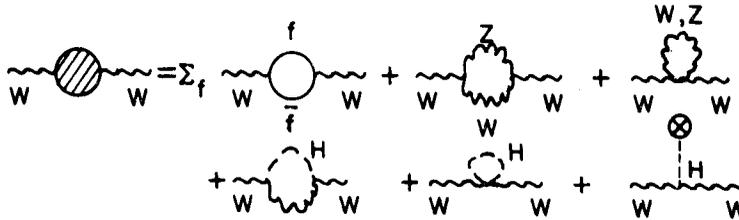


Figure 3. Enumeration of the W-boson vacuum polarization graphs

approximation [9] of retaining only vector boson polarization terms and neglecting vertex corrections and box graphs.

2. Obliqueness and oblique parameters

In order to understand the efficacy of the obliqueness approximation, it is instructive to look at the 1-loop terms in the muon decay amplitude. The tree diagram involves the exchange of a charge-carrying W between the muon and the electron converting them into ν_μ and $\bar{\nu}_e$ respectively. At the 1-loop level, separately there are the W vacuum polarization contributions (Fig. 2a), vertex corrections for the $\mu \rightarrow \nu_\mu$ transition (Fig. 2b), vertex corrections for the $\nu_e \rightarrow e$ transition (Fig. 2c) and box-type graphs (Fig. 2d). The vector boson propagator, to one loop, has in fact been enumerated in Fig. 3, though of course the last tadpole graph is absent in the unitary gauge and – in any event – drops out of renormalized on-shell amplitudes.

These vacuum polarization contributions form a gauge-invariant subset and will henceforth be called oblique corrections. They totally dominate over the vertex corrections and the box graphs (by nearly an order of magnitude) in their contributions to Δr . This numerical domination by the vacuum polarization terms is a generic feature of all 1-loop physically interesting radiative corrections considered to date with one important exception. The latter is the $Zb\bar{b}$ coupling where the vertex correction from a triangular loop with the two top and one longitudinally polarized W_L internal lines (Fig. 4) makes a numerically significant contribution on account of the top-antitop-Higgs coupling which enters in the $t\bar{t}W_L$ vertex.

Treating the $Zb\bar{b}$ vertex separately, one is then justified at least at the 10%

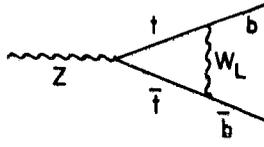
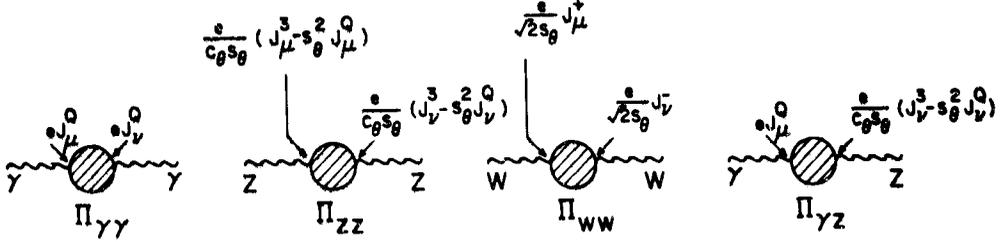

 Figure 4. The $Zb\bar{b}$ vertex at 1-loop


Figure 5. Electroweak vector boson self-energies

level in keeping only the oblique corrections and ignore the rest. This causes a tremendous simplification in the problem as detailed below. All radiative effects to 1-loop can now be described in terms of vacuum polarization terms that are gauge-independent Π -functions, i.e.

$$\int d^4x e^{iq \cdot x} \langle \Omega | J_\mu^A(x) J_\nu(0) | \Omega \rangle = -\Pi^{AB}(q^2) \eta_{\mu\nu} + q_\mu q_\nu \text{ terms.}$$

The parameters S, T, U are, in fact, linear combinations of appropriately defined Π -functions. As a result, they represent compact, model-independent parametrizations of 1-loop radiative corrections in the obliqueness approximation. They are not only gauge-independent but are renormalization scheme invariant since they appear [10] in the coefficients of higher dimensional operators of a 1-loop effective Lagrangian density. We shall see that there are two additional desirable features of these parameters. First, new physics contributions to them add linearly to those from the SM; this actually is a property of the Π -functions as can be seen by inserting a complete set of states. Second, they are optimal probes of any nondecoupled heavy new physics, if present. These points will be elaborated below.

The oblique parameters S, T, U of (9) theoretically emerge from the (generally divergent) γ, Z and W self-energies and the $\gamma - Z$ mixing amplitudes $\Pi_{\gamma\gamma}(q^2)$, $\Pi_{ZZ}(q^2)$ and $\Pi_{\gamma Z}(q^2)$ respectively (Fig. 5). The latter are defined as functions of the four momentum scale q of the relevant gauge bosons. Electromagnetic gauge invariance implies $\Pi_{\gamma\gamma}(0) = 0 = \Pi_{\gamma Z}(0)$. Denote the weak isospin currents as $J_{1,2,3}^\mu$ and the electromagnetic current as $J_Q^\mu = J_3^\mu + \frac{1}{2}J_Y^\mu$. Then the Z -current is $(e/s_\theta c_\theta)(J_3^\mu - s_\theta^2 J_Q^\mu)$ where $e^2 = 4\pi\alpha_{EM}$. Thus

$$\Pi_{\gamma\gamma} = e^2 \Pi_{QQ}, \quad (10a)$$

$$\Pi_{ZZ} = e^2 s_\theta^{-2} c_\theta^{-2} (\Pi_{33} - 2s_\theta^2 \Pi_{3Q} + s_\theta^4 \Pi_{QQ}), \quad (10b)$$

$$\Pi_{WW} = e^2 s_\theta^{-2} \Pi_{11}, \quad (10c)$$

$$\Pi_{\gamma Z} = e^2 c_\theta^{-1} s_\theta^{-1} (\Pi_{3Q} - s_\theta^2 \Pi_{QQ}) \quad (10d)$$

at all values of q^2 . The “theoretical” definitions of S, T and U are

$$\begin{aligned} S &= 16\pi M_Z^{-2} [\Pi_{33}(M_Z^2) - \Pi_{33}(0) - \Pi_{3Q}(M_Z^2)] \\ &= 8\pi M_Z^{-2} [\Pi_{3Y}(0) - \Pi_{3Y}(M_Z^2)], \end{aligned} \quad (11a)$$

$$T = 4\pi s_\theta^{-2} c_\theta^{-2} M_Z^{-2} [\Pi_{11}(0) - \Pi_{33}(0)], \quad (11b)$$

$$U = 16\pi M_W^{-2} [\Pi_{11}(M_W^2) - \Pi_{11}(0)] - 16\pi M_Z^{-2} [\Pi_{33}(M_Z^2) - \Pi_{33}(0)]. \quad (11c)$$

(11) and (9) match in the obliqueness approximation.

T and U receive nonzero contributions from the violation of weak isospin and are finite on account of the weak isospin symmetric nature of the divergence terms. S originates from the mixing between the weak hypercharge and the third component of weak isospin as a consequence of the spontaneous symmetry breakdown mechanism. Soft operators, involved in the latter, do not affect the leading divergence because of Symanzik’s theorem. The nonleading divergence cancels out in the difference between $\Pi_{3Y}(M_Z^2)$ and $\Pi_{3Y}(0)$, leaving a finite S . The Π_{AB} functions receive contributions from different sources additively. This enables us to define $\tilde{\Pi}_{AB} = \Pi_{AB} - \Pi_{AB}^{SM}$ where Π_{AB}^{SM} is the Standard Model contribution. Thus the twiddled pi function would arise purely from new physics beyond SM. The latter depends on the yet unknown Higgs mass m_H (logarithmically, on the strength of Veltman’s screening theorem) and the top mass m_T (with leading quadratic dependence). In terms of direct experimental searches as well as theoretical consistency in the perturbative calculation scheme, one can say that $91 \text{ GeV} < m_T < 200 \text{ GeV}$ and $60 \text{ GeV} < m_H < 1 \text{ TeV}$.

Consistent with the practice in the recent literature [11,12], we choose the SM reference point at $m_T = 140 \text{ GeV}$, $m_H = 100 \text{ GeV}$ and the QCD fine structure constant on the Z $\alpha_S(M_Z) = 0.120$. Of course, shifts in these values can easily be incorporated [11]. The most reliable extraction of $\tilde{S}, \tilde{T}, \tilde{U}$ is now from current accelerator data. The parameters \tilde{S} and \tilde{T} are best obtained by performing a “local fit” [11] to various cross sections and asymmetries at LEP for processes $e^+e^- \rightarrow f\bar{f}$ as functions of the CM energy \sqrt{s} in the Z lineshape region. The original local fit was done [11] with 700,000 data points, but on updated analysis [13] with 1.5 million events around the Z peak yields $\tilde{S} = -0.42 \pm 0.45$, $\tilde{T} = -0.19 \pm 0.41$. If one combines these with the rather inaccurately known value of M_W , then the use of (8) and (9b) leads to $U = -0.12 \pm 0.90$. The errors will be significantly reduced once the W -mass is better known. In Fig. 6 we show the 90% confidence level allowed elliptic region in the \tilde{S}, \tilde{T} plane. It should be pointed out that most technicolor and walking technicolor scenarios (and in general condensate models of electroweak symmetry breaking), pertaining to nondecoupled new physics, predict large positive values of \tilde{S} and \tilde{T} outside this ellipse and are disfavored [14] by the data. In contrast, supersymmetric models, which stand for decoupled new physics, generally predict numerically small values of \tilde{S} and \tilde{T} close to zero which cannot be tested at the present level of accuracy.

In conclusion, the radiative electroweak parameters $\epsilon_{1,2,3}$ (or S, T, U) constitute compact, model-independent probes into new physics. The contributions from

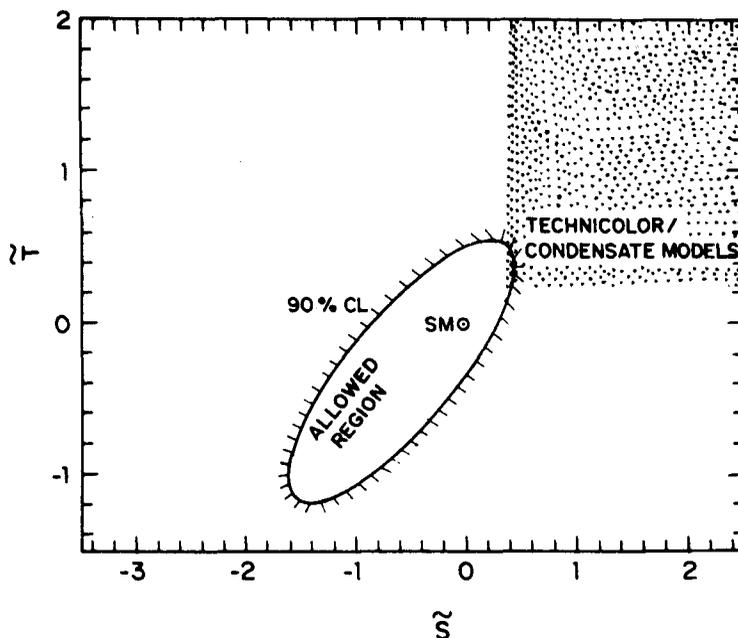


Figure 6. Experimentally allowed (90% CL) ellipse in the \tilde{S} , \tilde{T} plane

the latter (marked by twiddles) are linearly additive to those from the SM in the obliqueness approximation. The data tend to lie in the third quadrant of the \tilde{S} , \tilde{T} plane, disfavoring technicolor and related condensate models. The errors in \tilde{U} will remain large until the W -mass is determined to much better accuracy.

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Discussion

M. Drees : The final 2 loop calculation of EW quantities (2 point functions) have been performed by A. Diouadi.

Probir Roy : Yes, but the first comprehensive calculation of all large 2-loop EW corrections were done by Kniehl, Halzen and Stong.

M. Drees : Charginos and neutralinos do decouple in the limit where their masses are much larger than m_W .

Probir Roy : Yes. In fact, all SUSY particles fall in the decoupling theory category. However, because of mixings lighter charginos and neutralinos can be numerically somewhat sensitive to the oblique parameters.

R.P. Saxena : You made a statement that the vertex and box diagrams are small compared to the self energy diagrams. Is that a one loop phenomenon or is it generally true?

Probir Roy : It is largely true in one loop calculations. Complete two loop calculations have not yet been done, but large corrections have been computed and there this property does not hold.