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Kinetic properties of an acoustic-like mode in a two-ion quasi-neutral plasma

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Abstract. Kinetic analysis of an acoustic-like mode in a plasma with hot and cold ion components has been carried out. Under the short wevelength approximation ($k\lambda_{De} \gg 1$), electrons are assumed to form a dynamic neutralising background and their contribution to the perturbation is neglected. The significant role of the hot ions to Landau damping of the acoustic-like mode is highlighted and a novel concept of plasma experiment is suggested.

Keywords. Kinetic property; acoustic-like mode; Landau damping; Maxwellian distribution; quasi-neutral.

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1. Introduction

It is well established that the ion-acoustic wave (IAW) is the only low frequency mode in an unmagnetized plasma system wherein the dynamics of both, the electrons and ions, is involved. The IAW propagates as a normal mode when $T_e \gg T_i$ i.e. electrons are hotter than ions, otherwise the mode is heavily Landau damped. Moreover, only longer wavelengths ($k\lambda_{De} \ll 1$) of IAW propagates whereas the shorter wavelengths ($k\lambda_{De} \gg 1$) transform into an ion plasma oscillation. However, recently Dwivedi *et al* [1] (hereafter called as paper I) have shown that an acoustic-like mode (ALM) appears to propagate as a normal mode even for the shorter wavelength case in a plasma consisting of hot and cold ions with different masses, number densities and temperatures. Under the short wavelength approximation the role of electrons in the perturbation has been neglected and the linear and nonlinear behaviour of the aforesaid mode has been described using fluid model. The plasma system considered in paper I may be anticipated in thermalisation of the counter streaming ion beams [2] of appropriate energies with hot electrons or in fusion plasmas with selective heating of one ion species. Similar situation is likely to be achieved in dusty plasmas in laboratory. The practical realisation of required conditions for the existence of ALM may not be very easily achieved due to highly restrictive theoretical constraints. The present conceptual plasma model may provide a base to the experimentalists to produce ALM in the laboratory [3].

From the linear dispersion relation derived for ALM in paper I, one finds that the relative ion density and temperature are crucial parameters for deciding the existence of the mode. Fluid treatment is found to be valid for $\varepsilon_T \gg \varepsilon_n \gg 1$ and $\varepsilon_n \gg \varepsilon_m$ with

$\varepsilon_m > 1$ and $\varepsilon_z > 1$, where all the notations are defined in paper I. For $\varepsilon_m < 1$, the fluid approximations are very well satisfied. Unlike the usual IAW where only electron and ion temperature ratio is a responsible parameter for kinetic effects to be important, ALM involves more free parameters to decide its kinetic propreties. This provides more freedom to play with these parameters so as to allow the existence of the mode. Under the fluid approximations, the required conditions for the existence of a normal mode cannot be specified quantitatively. The fluid treatment ignores the wave-particle interaction which cannot always be true. This interaction becomes important when the phase velocity of a mode is of the order of thermal velocity of a plasma component and leads to the collisionless Landau damping. Consideration of the wave-particle interaction cannot be described within the framework of a fluid model and hence the kinetic approach becomes necessary. The Landau damping being a kinetic phenomenon decides the quantitative limitation of the fluid model. The basic motivation behind the present analysis is to look for the quantitative estimation of the plasma parameters for the existence of the ALM as a normal mode. Wide range variability of more free parameters creates curiosity to analyse their effect on the Landau damping and find appropriate values of these parameters for the normal mode behaviour of the ALM. With this idea in mind, the kinetic analysis of the ALM has been performed and the contribution of the hot ions to its Landau damping has been highlighted.

It is concluded that the hot ion component causes finite Landau damping with sensitive dependence on the charge states of the ions. The mass ratio of hot and cold ions also constitutes an important candidate to decide the normal mode behaviour of the ALM. It is emphasized that the hot and cold ions considered in the present plasma model are not the manifestation of double humped distribution with two maxima but these represent the two different species of ions with Maxwellian distribution at their respective temperatures.

2. Vlasov equation and Landau damping

Using standard Vlasov equation and following the usual treatment of collisionless Landau damping [4], the expression for the Landau damping of ALM has been derived and analysed. The hot and cold ions can be described by Vlasov equation;

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v}_\alpha \cdot \nabla f_\alpha + \frac{q_\alpha}{m_\alpha} \mathbf{E} \cdot \frac{\partial f_\alpha}{\partial v_\alpha} = 0. \tag{1}$$

Here f_α is the distribution function for the αth plasma species. Subscript α denotes different ion components, hot ions (ih) and cold ions (ic). Let us assume that the ambient electric field $\mathbf{E}_0 = 0$, $f_\alpha = f_{\alpha 0} + \tilde{f}_\alpha$ such that $\tilde{f}_\alpha / f_{\alpha 0} \ll 1$, where $f_{\alpha 0}$ is the equilibrium and $\tilde{f}_\alpha$ the perturbed distribution functions of αth plasma species. Now the linearized form of (1) can be written as

$$\frac{\partial \tilde{f}_\alpha}{\partial t} + \mathbf{v}_\alpha \cdot \nabla \tilde{f}_\alpha + \frac{q_\alpha}{m_\alpha} \tilde{\mathbf{E}} \cdot \frac{\partial f_{\alpha 0}}{\partial v_\alpha} = 0, \tag{2}$$

where v_α is an independent variable and $q_\alpha = Z_\alpha q$, q being the charge on the ions and Z_α the charge multiplicity of αth plasma species. Considering one dimensional case and assuming the perturbation to vary as $\exp(-i\omega t + ikx)$, (2) can be Fourier analysed to yield,

$$\tilde{f}_\alpha = -\frac{i}{\omega - kv_\alpha}\frac{q_\alpha}{m_\alpha}\tilde{\mathbf{E}}\cdot\frac{\partial f_{\alpha 0}}{\partial v_\alpha}. \tag{3}$$

Associated perturbed density can be written as,

$$n_\alpha = \int \tilde{f}_\alpha \mathrm{d}v_\alpha = -i\frac{q_\alpha}{m_\alpha}\tilde{E}\int \frac{\partial f_{\alpha 0}/\partial v_\alpha}{\omega - kv_\alpha}\mathrm{d}v_\alpha. \tag{4}$$

Let us consider the equilibrium to be characterized by a Maxwellian velocity distribution $f_{\alpha 0}$;

$$f_{\alpha 0} = \frac{n_{\alpha 0}}{\pi^{1/2} v_{t\alpha}}\exp(-v_\alpha^2/v_{t\alpha}^2), \tag{5}$$

where $v_{t\alpha} = (2T_\alpha/m_\alpha)^{1/2}$ is the thermal velocity of αth species of plasma. Substituting the perturbed density $\tilde{n}_\alpha$ from (4) in the Poisson's equation one can, after some algebraic manipulation, derive the dispersion relation as,

$$1 - \sum_\alpha (\omega_{p\alpha}^2/k^2 v_{t\alpha}^2) Z'(\zeta_\alpha) = 0, \tag{6}$$

where $Z(\zeta_\alpha)$ is the plasma dispersion function defined by

$$Z(\zeta_\alpha) = \pi^{-1/2}\int \frac{\exp(-s_\alpha^2)}{s_\alpha - \zeta_\alpha}\mathrm{d}s_\alpha. \tag{7}$$

Here $\zeta_\alpha = (\omega/k)/v_{t\alpha}$, $s_\alpha = v_\alpha/v_{t\alpha}$ and $\omega_{p\alpha}^2 = 4\pi n_{\alpha 0} q_\alpha^2/m_\alpha$. Equation (6) is the required dispersion relation of the ALM with $Z'(\zeta_\alpha) = \partial Z(\zeta_\alpha)/\partial\zeta_\alpha$.

Following Chen [4] or any other general book on plasma physics and considering power series expansion of $Z(\zeta_{ih})$ and asymptotic expansion of $Z(\zeta_{ic})$ for $\zeta_{ih} \ll 1$ and $\zeta_{ic} \gg 1$ respectively, the dispersion relation can be simplified to give

$$\omega^2/k^2 = (\varepsilon_T/\varepsilon_n\varepsilon_z^2)(1 + 3\varepsilon_n\varepsilon_z^2/\varepsilon_T)v_{tic}^2. \tag{8}$$

In deriving (8) the approximations $\varepsilon_T \gg \varepsilon_n \gg 1$ and $\varepsilon_n\varepsilon_z^2 \gg \varepsilon_m$ have been used. This is the dispersion relation for ALM [1] with $\gamma_{ic} = 3$. This is to note that by replacing hot ions with electrons and considering $\varepsilon_z = 1$, eq. (8) is reduced to the usual dispersion relation of an IAW.

Now retaining the Landau terms for hot and cold ions, (6) can be simplified to give;

$$\gamma = -(\mathrm{Im}\zeta_c/\mathrm{Re}\zeta_c) = -(\mathrm{Im}\omega/\mathrm{Re}\omega) = \left(\frac{\pi}{8}\right)^{1/2}(3+\theta)^{1/2}$$

$$\left(\theta + (\varepsilon_m/\varepsilon_T)^{1/2}\exp\left\{(1 - \varepsilon_m/\varepsilon_T)\frac{3+\theta}{2}\right\}\right)\exp\left(-\frac{3+\theta}{2}\right), \tag{9}$$

where

$$\theta = \left(\frac{1}{\varepsilon_n\varepsilon_z^2}\right)\varepsilon_T.$$

The second term inside the capital bracket arises due to the hot ion's contribution to the Landau damping. Replacement of hot ions by electrons reduces (9) to the usual expression of Landau damping for IAW wherein the contribution of electrons is negligibility small due to ε_m being a very small parameter. For very large values of θ ($\varepsilon_T \gg \varepsilon_n \gg \varepsilon_m$) the Landau damping is approximated by $\gamma \approx (\pi\varepsilon_m/8\varepsilon_n\varepsilon_z^2)^{1/2}$ and is governed by the balance between increase in hot ion contribution and decrease in cold ion contribution to the Landau damping. The increase (decrease) of the Landau damping due to hot (cold) ions is associated with the deviation of the phase velocity of ALM closer to (away from) the thermal velocity of hot (cold) ions. In other words, one can say that the deficiency in wave-particle interaction due to the cold ions is compensated by hot ions at higher values of θ and hence a constant (plateau) but finite level of Landau damping is achieved. All the features of the plateau can be described by the above approximate form of the Landau damping. The ratio of Landau dampings with and without hot ions is given as

$$-(\gamma_{h+}/\gamma_{h-}) = 1 + \frac{1}{\theta}\left(\frac{\varepsilon_m}{\varepsilon_T}\right)^{1/2} \exp[0{\cdot}5\{(1-\varepsilon_m/\varepsilon_T)(3+\theta)\}]. \tag{10}$$

It is obvious that for certain range of plasma parameters, the contribution of hot ions to the Landau damping of ALM can be significant and their role to decide its existence cannot be undermined. This can be attributed to the relative density of hot and cold ions which forms an important parameter to compete with their relative temperature to decide the kinetic effects (8). Charge multiplicity is also found to be a sensitive parameter.

3. Conclusions

In general the collisionless damping due to wave-particle interaction occurs if $\partial f_{\alpha 0}/\partial v_\alpha|\mathrm{v}_\alpha = w/k \ll 0$ otherwise, in the opposite limit, growth is obtained. A favourable situation for the latter arises in a plasma system of hot and cold ions associated with two maxima in double humped ion distribution. However, this paper considers a plasma model wherein the hot and cold ions form two different species of ions which are Maxwellian at their respective temperatures. Consequently the possible existence of an instability and its analysis lie beyond the scope of the proposed plasma model. Such configuration of ion components may be realised in dusty plasma systems provided that all the characteristic plasma lengths and perturbational wavelengths be larger than the dust size otherwise the present mathematical formalism will not be valid. Accordingly, the present analysis falls in the first category wherein slow particles are more than the fast ones. From numerical calculations, it is inferred that the acoustic-like mode propagates as a normal mode with weak Landau damping provided that the temperature ratio of hot and cold ions (ε_T) is larger than their density ratio (ε_n) by about an order of a magnitude (figures. 1–6). The weak Landau damping is clearly due to hot ions which contribute significantly for $\varepsilon_z < 1$ (figure 5) and $\varepsilon_m > 1$ (figure 3). The collisionless Landau damping is less for $\varepsilon_z > 1$ (figure 6) and $\varepsilon_m < 1$ (figure 4). The maximum peak occurs at $\varepsilon_n \approx \varepsilon_T$; the variation of damping rate at peak is of the order of 10%. In brief one can say that the lighter hot ions in majority ($\varepsilon_n > 10$) with higher charge state is favourable for the existence of the ALM in a three component plasma. However, the numerical analysis suggests that the

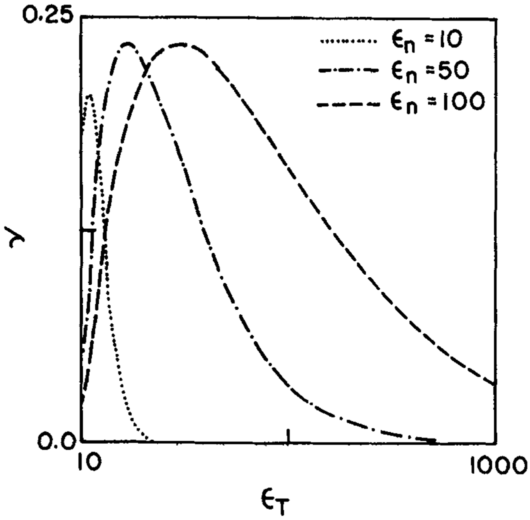

Figure 1. Variation of Landau damping (γ) with hot and cold ion temperature ratio (ε_T) without hot ions contribution for $\varepsilon_z = 1$, $\varepsilon_m = 1$.

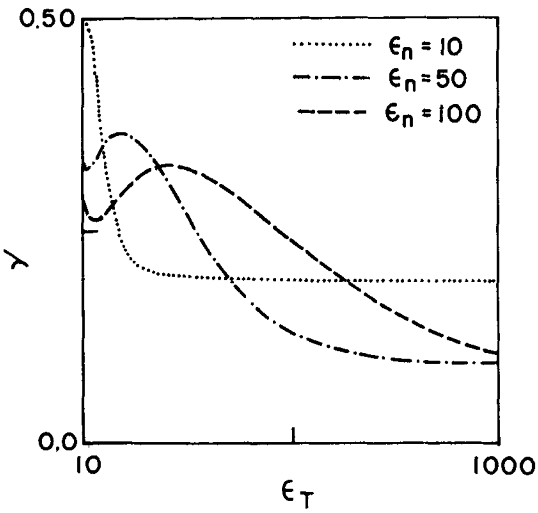

Figure 2. Variation of Landau damping with hot and cold ion temperature ratio with hot ions contribution for $\varepsilon_z = 1$, $\varepsilon_m = 1$.

mode will always be associated with finite but small Landau damping for any practical situation within the framework of the present theoretical constraints.

Thus the role of hot ions to the Landau damping of ALM in a two-ion species quasi-neutral plasma has been analysed and numerical calculations have been done for the appreciation of the results. Some calculations for the similar plasma parameters have been done by Farr and Budwine [5] for the study of high frequency flute like instability in multi-component plasmas. Their calculations involve electron's contribution to the perturbation whereas in this work electron's contribution is justifiably

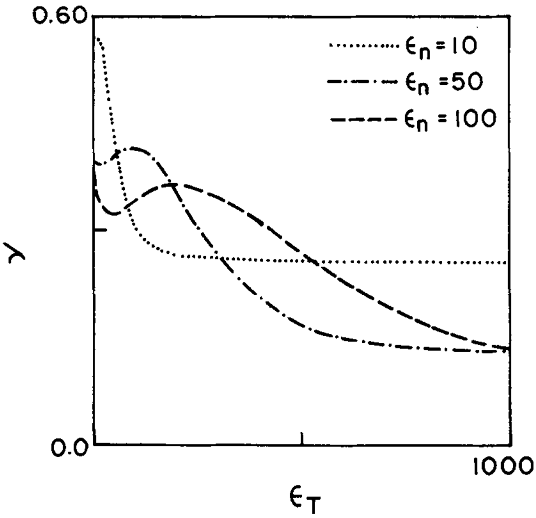

Figure 3. Variation of Landau damping with hot and cold ion temperature ratio with hot ions contribution for $\varepsilon_z = 1$, $\varepsilon_m = 2$.

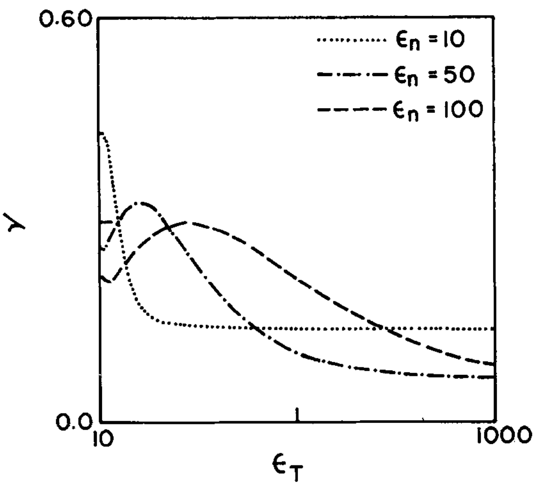

Figure 4. Variation of Landau damping with hot and cold ion temperature ratio with hot ions contribution for $\varepsilon_z = 1$, $\varepsilon_m = 0{\cdot}5$.

neglected. They argue that their calculations have some relevance for an experiment which involves injection of the hot ions into a cold background plasma like mirror confined plasmas.

Based on the present analysis a novel concept of plasma experiment is proposed wherein cold ions can be injected into a hot background plasma to produce a plasma system which can support ALM mode. This may provide an experimental idea to study the acoustic-like mode in the laboratory plasmas.

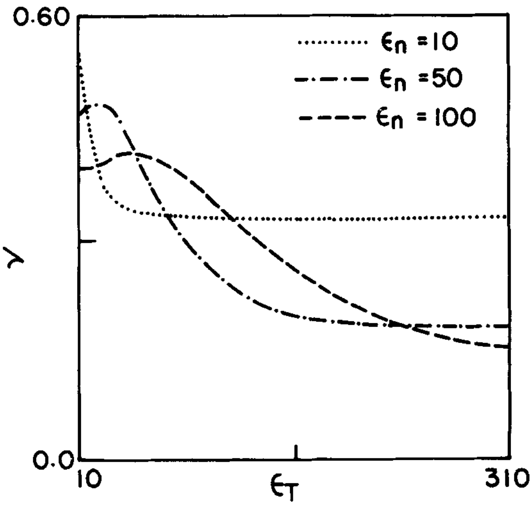

Figure 5. Variation of Landau damping with hot and cold ion temperature ratio with hot ions contribution for $\varepsilon_z = 0{\cdot}5$, $\varepsilon_m = 1$.

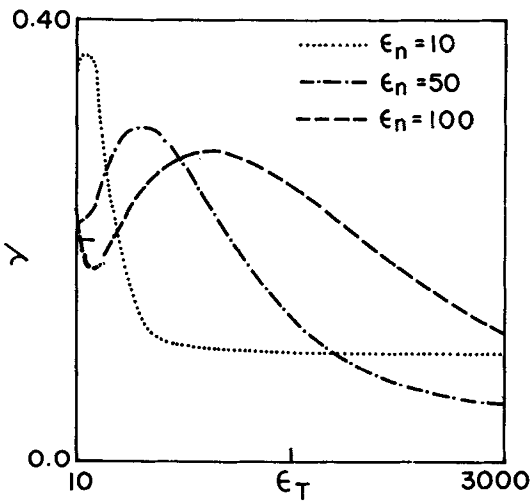

Figure 6. Variation of Landau damping with hot and cold ion temperature ratio with hot ions contribution for $\varepsilon_z = 2$, $\varepsilon_m = 1$.

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