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${}^{6}_{\Lambda\Lambda}$He hypernucleus in $\alpha + 2\Lambda$ cluster model

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Abstract. The binding energy of the double hypernucleus ${}^{6}_{\Lambda\Lambda}$He is calculated in $\alpha + 2\Lambda$ cluster model using the method of translation invariant basis TIMO. As regards the required interaction potentials we use a density dependent effective ΛN force and a gaussian form for $\Lambda\Lambda$ potential. With these interactions a very reasonable value of $B_{\Lambda\Lambda}$ is obtained if the oscillator states up to the excitation quantum number $N = 12$ are taken into account in the expansion of wavefunction of the hypernucleus. This value of N is much smaller than that obtained in an earlier study. This lowering in N value is attributed to a much better choice of ΛN potential used in the present study.

Keywords. Double hypernucleus; translation invariant basis; ΛN interaction.

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The binding energies of two double hypernuclei ${}^{10}_{\Lambda\Lambda}$Be and ${}^{6}_{\Lambda\Lambda}$He have been the only source of information on $\Lambda\Lambda$ interaction. A large number of binding-energy calculations [1–7] have been made to obtain information about $\Lambda\Lambda$ interaction which is essential for a general understanding of baryon–baryon interactions. The determination of $\Lambda\Lambda$ interaction obviously needs an accurate method of investigating the structure of hypernuclei and a better knowledge of ΛN and NN interactions. The required interactions [8,9] are now known to some accuracy. Dzhibuti *et al* have studied the $\Lambda\Lambda$ hypernucleus ${}^{6}_{\Lambda\Lambda}$He in $\alpha + 2\Lambda$ model using the method of translational-invariant oscillator basis TIMO and employing the gaussian form for both the $\Lambda\Lambda$ and the ΛN interaction potentials. Their conclusion is that the wavefunction of this system converges much faster than for nuclei and the states up to the excitation quantum number $N = 20$ are needed to obtain the correct value of $B_{\Lambda\Lambda}$. The fast convergence of the wavefunction is attributed to much shorter range of ΛN interaction. We do agree with this conclusion but we strongly feel that in order to check the convergence of the basis, a more realistic ΛN interaction should be used.

Keeping this in mind we calculate the binding energy of ${}^{6}_{\Lambda\Lambda}$He in the same approach and with the same $\Lambda\Lambda$ potential. For ΛN interaction, however, we use a more realistic density dependent effective ΛN potential. The mathematical formulation is exactly the same as in ref. [7] and is presented here again in the subsequent paragraphs. Even our notations are the same as those in ref. [7].

The mutual motion of three particles of unequal masses m_1, m_2 and m_3 in the field of a harmonic oscillator of frequency ω is described by the Jacobi coordinates

$$X_1^{(i)} = (-1)^{i+k}\left[\frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j}{m_i + m_j} - \mathbf{r}_k\right],$$
$$X_2^{(i)} = (-1)^{i+j}(\mathbf{r}_j - \mathbf{r}_i), \quad (i,j,k) = (1,2,3). \tag{1}$$

These coordinates are converted into dimensionles Jacobi coordinates:

$$x_1^{(i)} = X_1^{(i)} \Big/ \sqrt{\frac{\hbar}{\mu_1^{(i)}\omega}} \quad x_2^{(i)} = X_2^{(i)} \Big/ \sqrt{\frac{\hbar}{\mu_2^{(i)}\omega}} \tag{2}$$

where

$$\mu_1^{(i)} = \frac{m_i(m_i + m_j)}{M}, \quad \mu_2^{(i)} = \frac{m_i m_j}{m_i + m_k}$$

$$M = m_i + m_j + m_k. \quad (i,j,k) = (1,2,3).$$

We can go from set $(x_1^{(i)}, x_2^{(i)})$ to the set $(x_1^{(j)}, x_2^{(j)})$ by means of the following unitary transformation:

$$x_1^{(i)} = a_{ij} x_1^{(j)} + b_{ij} x_2^{(j)},$$
$$x_2^{(i)} = (-1)^{i+j+1} b_{ij} x_1^{(j)} + (-1)^{i+j} a_{ij} x_2^{(i)}, \tag{3}$$

where $a_{ij}^2 + b_{ij}^2 = 1$, and a_{ij} and b_{ij} are related with the particle masses by following relations

$$a_{ij} = (-1)^{j+k+1} \left[\frac{m_i m_k}{(m_i + m_j)(m_j + m_k)}\right]^{1/2},$$

$$b_{ij} = \left[\frac{m_j M}{(m_i + m_j)(m_j + m_k)}\right]^{1/2}, \quad (i,j,k) = (1,2,3). \tag{4}$$

The Hamiltonian of the system is

$$H = \frac{\hbar\omega}{2} \sum_{s=1}^{2} [\mathbf{p}_{x_s^{(i)}}^2 + (x_s^{(i)})^2]$$

The wavefunction $\psi(x_1^{(i)}, x_2^{(i)})$ which is the solution of equation

$$H\psi(x_1^{(i)}, x_2^{(i)}) = (N+3)\hbar\omega\psi(x_1^{(i)}, x_2^{(i)}) \tag{5}$$

where

$$N = 2(n_1 + n_2) + l_1 + l_2 \tag{6}$$

is the number of the oscillator exitation quanta, is characterized by L and m.

$$\psi(x_1^{(i)}, x_2^{(i)}) = [\langle x_1^{(i)} | n_1 l_1 m_1 \rangle \langle x_2^{(i)} | n_2 l_2 m_2 \rangle]_m^L$$
$$= |n_1 l_1(x_1^{(i)}) n_2 l_2(x_2^{(i)}) : Lm\rangle = [n_1 l_1 n_2 l_2]_m^L, \tag{7}$$

where $\langle x_s^{(i)} | n_s l_s m_s \rangle = R_{n_s l_s}(x_s^{(i)})\, Y_{l_s m_s}(\hat{x}_s^{(i)})$, $s = 1,2$, $i = 1,2,3$ are single particle oscillator functions.

The wavefunctions $\psi(x_1^{(i)}, x_2^{(i)})$ transform under unitary transformation (3) with Talmi–Moshinsky–Smirnov coefficients (TMSC) [10, 11]

$$|n_1 l_1(x_1^{(i)}) n_2 l_2(x_2^{(i)}) : Lm\rangle = \sum_{\gamma'} \left\langle n_1 l_1 n_2 l_2 : L \left| \frac{a_{ij}^2}{b_{ij}^2} \right| n_1' l_1' n_2' l_2' : L \right\rangle$$
$$|n_1' l_1'(x_1^{(j)}) n_2' l_2'(x_2^{(j)}) : Lm\rangle, \tag{8}$$

where

$$\left\langle n_1 l_1 n_2 l : L \left| \frac{a_{ij}^2}{b_{ij}^2} \right| n_1' l_1' n_2' l_2' : L \right\rangle$$

$$= \left\langle \gamma : L \left| \frac{a_{ij}^2}{b_{ij}^2} \right| \gamma' : L \right\rangle - \text{TMSC.}$$

These orbital functions are coupled to spin-isospin functions of the three particles to obtain the complete set of translation-invariant oscillator functions:

$$|NLSJTM_T\rangle = [[[n_1 l_1 n_2 l_2]^L [[\sigma_1 \sigma_2]^{S_{12}} \sigma_3]^S]]^J [[\tau_1 \tau_2]^{T_{12}} \tau_3]^T. \tag{9}$$

The wavefunction of hypernucleus designated by the quantum numbers J^Π, T and M_T can be expanded in terms of TIMO basis functions

$$\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = |J^\Pi T M_T\rangle = \sum_{\substack{NLSn_1 l_1 \\ n_2 l_2}} C_{NLS}^{n_1 l_1 n_2 l_2} |NLSJTM_T\rangle$$

$$= \sum_{NLS\gamma} C_{NLS}^{\gamma} |NLSJTM_T\rangle = \sum_i C_i |i\rangle. \tag{10}$$

The expansion coefficients C_{NLS}^{γ} are determined variationally. For the ground state of ${}^{6}_{\Lambda\Lambda}He$, $J^\Pi = 0^+$, $S = T = M = M_T = 0$.

To determine the binding energy we diagonalize the energy matrix constructed in the above basis:

$$\langle i|H|k\rangle = \langle i|T|k\rangle + \langle i|V|k\rangle \tag{11}$$

where T and V are the kinetic and potential energy operators, respectively.

The potential energy matrix elements is expressed as

$$\langle \Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)|V|\Psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)\rangle = \sum_{\substack{NLS\gamma \\ N'L'S'\gamma'}} C_{NLS}^{\gamma} C_{N'L'S'}^{\gamma'}$$

$$\times \sum_{\gamma''\gamma'''j} \left\langle \gamma : \left| L \left| \frac{a_{ij}^2}{b_{ij}^2} \right| \gamma'' : L \right\rangle \right\rangle \left\langle \gamma' : L' \left| \frac{a_{ij}^2}{b_{ij}^2} \right| \gamma''' : L' \right\rangle$$

$$\times \langle [|n_1'' l''(\mathbf{x}_1^{(j)}) n_2'' l_2''(\mathbf{x}_2^{(j)}) : Lm\rangle [S_{12}\sigma_3]^S]^J [T_{12}\tau_3]^T$$

$$\times V(r_0^{(j)} x_2^{(j)}) |[|n_1''' l_1'''(\mathbf{x}_1^{(j)}) n_2''' l_2'''(\mathbf{x}_2^{(j)}) : L'm'\rangle [S_{12}'\sigma_3]^S]^J [T_{12}\tau_3]^T\rangle. \tag{12}$$

We write the potential energy function as

$$V(r_0^{(j)} x_2^{(j)}) = \sum_{\lambda=0,1,2} V_\lambda^{l_3}(r_0^{(j)} x_2^{(j)})(L^{(\lambda)} S^{(\lambda)}). \tag{13}$$

Then the right hand side of the above equation becomes

$$\langle [|n_1 l_1(\mathbf{x}_1^{(j)}) n_2 l_2(\mathbf{x}_2^{(j)}) : Lm\rangle [S_{12}\sigma_3]^S]^J [T_{12}\tau_3]^T|$$

$$\times \sum_\lambda V_\lambda^{l_3}(r_0^{(j)} x_2^{(j)})(L^{(\lambda)} S^{(\lambda)}) |[|n_1' l_1'(\mathbf{x}_1^{(j)}) n_2' l_2'(\mathbf{x}_2^{(j)}) : L'm'\rangle$$

$$\times [S_{12}'\sigma_3]^S]^J [T_{12}\tau_3]^T\rangle$$

$$= \delta_{l_1 l_1'} \delta_{S_{11} S_{12}} \delta_{n_1 n_1'} [(2L+1)(2L'+1)(2S+1)(2S'+1)]^{1/2}$$

$$\times \sum_{\lambda=0,1,2} (-1)^{\lambda} W(JSL'\lambda|LS')\, W(l_1' l_2' \lambda|l_2 L')\, W(\sigma_3 S' S_{12} \lambda|S_{12}' S)$$

$$\times \langle l_2 \| L^{(\lambda)} \| l_2' \rangle \langle S_{12} \| S^{(\lambda)} \| S_{12}' \rangle$$

$$\times \langle n_2 l_2(\mathbf{x}_2^{(j)}) | V_{\lambda}^{l_s}(r_0^{(j)} x_2^{(j)}) | n_2' l_2'(\mathbf{x}_2^{(j)}) \rangle, \tag{14}$$

Where W (abcd|ef) are the Racah coefficients.

The ground state $B_{\Lambda\Lambda}$ of ${}^{6}_{\Lambda\Lambda}$He is obtained by diagonalizing the energy matrix. For $\Lambda\Lambda$ interaction potential we use the same gaussian potential which was used in [7]. For ΛN interaction however we use a density dependent effective ΛN force [8].

$$V_{\Lambda N}(|\mathbf{r}-\mathbf{r}'|;\rho) = \bar{V}_0 \frac{1}{(\Pi d^2)^{3/2}} \exp[-(\mathbf{r}-\mathbf{r}')^2/d^2] \left[1 - \beta\rho^{2/3}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right]$$

The $\Lambda - \alpha$ potential $V_{\Lambda}(r)$ is obtained by folding this potential into the density of α particle [8] $\rho(\mathbf{r})$

$$V_{\Lambda}(r) = \bar{V}_0 \int \frac{\exp[-(\mathbf{r}-\mathbf{r}')^2/d^2]}{(\Pi d^2)^{3/2}} \left[1 - \beta\rho^{2/3}\left(\frac{\mathbf{r}+\mathbf{r}'}{2}\right)\right] \rho(\mathbf{r}')\mathrm{d}\mathbf{r}'. \tag{15}$$

This integral is evaluated in momentem space. The $V_{\Lambda}(r)$, after reverting it back to coordinate space, reads as

$$V_{\Lambda}(r) = \frac{\bar{V}_0}{2\Pi^2} \int F(q) \exp\left[-\frac{q^2 d^2}{4}\right] q^2 j_0(q_0 r)\mathrm{d}q. \tag{16}$$

where $j_0(qr)$ is the spherical based function of order zero and

$$F(q) = \int \exp(i\mathbf{q}\cdot\mathbf{r})\rho(\mathbf{r})[1 - \beta\rho^{2/3}(\mathbf{r})]\mathrm{d}\mathbf{r}. \tag{17}$$

The best fitted values of parameters $\bar{V}_0$, β and d are $\bar{V}_0 = 297{\cdot}86$ MeV fm^3, $\beta = 1{\cdot}92$ fm^2, $d = 0{\cdot}729$ fm.

Using the above potentials we compute the ground state binding energy of ${}^{6}_{\Lambda\Lambda}$He by varying $\hbar\omega$ for a fixed value of N and then repeating the calculations for different values of N until the system becomes bound. We find that for smaller N (say $N = 6$) the system is unbound and the energy is dependent on $\hbar\omega$. With increasing N, however, this dependence of energy on $\hbar\omega$ becomes weaker and weaker and at $N = 12$, the system becomes bound and the energy of system becomes almost independent of $\hbar\omega$. The predicted energy at $N = 12$ is about 10·72 MeV which compares well with the experimental [refs 12–14] value $10{\cdot}9 \pm 0{\cdot}6$ MeV.

Thus we see that the TIMO basis converges much faster with a density dependent effective ΛN interaction than with a gaussian interaction. This study also indicates that in order to further investigate the features of this model one should use a range of phenomenological $\Lambda\Lambda$ potentials based on meson exchange models and take into account the internal structure of α particle.

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