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Inflation driven by energy and curvature dependent bulk viscosity

C WOLF
Department of Physics, North Adams State College, North Adams, MA 01247, USA

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Abstract. We study the relative rates of expansion in cosmologies admitting curvature dependent and energy dependent bulk viscosity. It is conjectured that curvature dependant bulk viscosity may be a phenomenological way of representing gravitational vacuum polarization around the time of the Planck era.

Keywords. Inflation; Planck era; bulk viscosity.

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1. Introduction

Inflationary cosmology has become a practical way of resolving the flatness and horizon problems of observational cosmology [1]. It was Guth's [2] great insight in observing that the radiative corrected Coleman and Weinberg potential [3] could serve as an effective cosmological constant to drive the early universe to a period of exponential expansion. Old inflation [2], new inflation [4], chaotic inflation [5], and extended inflation [6] are all confronted with the problem of just how large scale structure can be seeded from primordial fluctuations during the inflationary period. Extended inflation offers us the most plausible solution to the problem of how the false vacuum makes the transition to the true vacuum by allowing sufficient inflation for small times to resolve the horizon and flatness problems and a slowing down of the expansion rate at greater time to allow the true vacuum to percolate. In theories of extended inflation a scalar field of a Brans–Dicke type theory provides a catalyst for the above cosmological transition. In addition to the above theories of inflation driven basically by a scalar field displaced from the minimum of the potential, there are other mechanisms to drive inflation such as bulk viscosity driven inflation [7] and higher curvature driven inflation [8]. Bulk viscosity in particular has been shown to be a phenomenological way of representing the conversion of massive string modes in the early universe to massless modes as well as representing a general form of dissipation associated with particle creation [9]. In addition to stringy interconversions, curvature dependent bulk viscosity has been shown to be equivalent to particle creation through gravitational vacuum polarization [10]. In a separate note [11] we have shown that curvature dependent bulk viscosity leads to inflation, and in this note, we extend these studies to the case when the bulk viscosity coefficient depends on the curvature squared and the energy density. After a brief discussion based on microphysics justifying the functional dependence of the bulk viscosity coefficient on the energy density and curvature squared, we demonstrate that both of these cases lead to inflation in a spatially flat universe and derive the relative expansion rates in each case. It could very well be that extended inflation can be realized as a combination of these two cases with rapid inflation at the outset and a diminished inflation rate for later times. With the interest in demonstrating that inflation exists under a variety of circumstances and the speculation that there exists a cosmic-no hair theorem [12, 13, 14] for inflation, the two cases of inflation discussed in this paper add to the list of inflationary like cosmologies. Also, since inflation might be independent of initial conditions and be an attractor in initial condition space [15], bulk viscosity driven inflation might suggest that quantum effects through vacuum polarization washes out any dependence of the initial conditions. Certainly if all models of inflation contain a portion of the true initial state of the universe then the cases in this paper only substantiate our belief in a cosmic no-hair theorem and the "attractor" theory of inflation.

2. Bulk viscosity driven inflation

We begin our analysis of assuming a homogeneous, isotropic Robertson–Walker metric

$$(\mathrm{d}S)^2 = \mathrm{d}t^2 - R^2\left(\frac{\mathrm{d}r^2}{1-Kr^2} + r^2(\mathrm{d}\theta)^2 + r^2\sin^2\theta(\mathrm{d}\phi)^2\right) \tag{1}$$

where $K = 0, +1, -1$, $C = 1$ and R is the scale factor.

For the Ricci components we have

$$R_{00} = 3\frac{R''}{R} \tag{2}$$

$$R_{ij} = \left[\frac{R''}{R} + 2\left(\frac{R'}{R}\right)^2 + \frac{2K}{R^2}\right]g_{ij}.$$

The matter content we assume is described by a viscous fluid [16] wherein

$$T_{\mu\nu} = (\bar{P} + \varepsilon)U_\mu U_\nu - g_{\mu\nu}\bar{P} \tag{3}$$

where

$$\bar{P} = P - \xi U^a_{;a} \tag{4}$$

Here ξ is the bulk viscosity coefficient and $U^a_{;a}$ is the expansion.

For the Einstein equations we have

$$R_{\mu\nu} = -k\left[T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}\right]\left(k = \frac{8\pi G}{C^4}, C = 1\right). \tag{5}$$

The components of (3) are

$$T_{00} = \varepsilon, \tag{6}$$

$$T_{ij} = -\bar{P}g_{ij} = -\left(P - \frac{\xi 3R'}{R}\right)g_{ij}.$$

For the trace of $T_{\mu\nu}$ we have

$$T = T_{\mu\nu}g^{\mu\nu} = \varepsilon - 3\bar{P} = 9\xi\frac{R'}{R}. \tag{7}$$

Here we have assumed the radiative equation of state to be

$$P = \frac{\varepsilon}{3}.$$

The Einstein equations corresponding to (5) are for $K = 0$ (spatially flat)

$$3\frac{R''}{R} = -k\left[\varepsilon - \frac{1}{2}\left(9\xi\frac{R'}{R}\right)\right] \tag{8}$$

$$\frac{R''}{R} + 2\left(\frac{R'}{R}\right)^2 = -k\left[-P + 3\xi\frac{R'}{R} - \frac{1}{2}\left(9\xi\frac{R'}{R}\right)\right] \tag{9}$$

To study the implications of (8) and (9) using the radiative question of state we now insert the functional dependence of ξ on factors that are phenomenological representations of the underlying microphysics. In this regard Barrow [17] has pointed out that particle production due to the non-adiabatic decaying of the field driving slow rollover inflation can macroscopically be described by the viscous cosmological model of Murphy [18] with a bulk viscosity coefficient proportional to the energy density ($\xi \propto \varepsilon$). Along a somewhat similar line of reasoning Turok [19] has proposed a bulk viscosity coefficient proportional to $\varepsilon^{3/2}$ to model string loop production in the early universe and Barrow [20] has discussed the general case $\xi = C\varepsilon^n$ with the result that for $n > 1/2$ the solutions for a Friedman–Walker flat universe start out in a deSitter phase and evolve into a power law expansion in t for late times. As mentioned in [10], Gurovich and Starobinsky have shown that vacuum polarization in a gravitational field can be modelled by a bulk viscosity coefficient dependent on the curvature squared. Such a process would be significant when the curvature is large, in particular near the Planck era. We will in what follows study the two cases (energy density dependent, curvature squared dependent bulk viscosity) and derive the inflation rates in each case for a spatially flat universe. We thus write for the two cases

$$\xi = C_1(R_S)^2, \quad \xi = C_2\varepsilon \text{ for a spatially flat universe } (K = 0)$$

and study the evolution of the scale factor for a flat universe for each case.

(Here R_s = curvature scalar.)

Case I

From (8) and (9) we have

$$6\frac{R''}{R} + 6\left(\frac{R'}{R}\right)^2 = \frac{k9\xi R'}{2R} - \frac{9\xi kR'}{R} + \frac{27k\xi R'}{2R} = 9\xi k\frac{R'}{R} \tag{10}$$

From (2) we have

$$R_S = R_{\mu\nu}g^{\mu\nu} = 6\frac{R''}{R} + 6\left(\frac{R'}{R}\right)^2. \tag{11}$$

Setting $\xi = C_1(R_s)^2$ (C_1 = constant) and inserting the inflationary solution $R = R_0e^{\alpha t}$ into (10) we have

$$12\alpha^2 = 9kC_1(144\alpha^4)\alpha \tag{12}$$

$$\alpha = \left(\frac{1}{108C_1k}\right)^{1/3} \tag{13}$$

Case II
Upon dividing (9) by (10) we have using

$$P = \frac{\varepsilon}{3}, \quad \xi = C_2 \varepsilon,$$

$$\frac{\frac{3R''}{R}}{\frac{R''}{R} + 2\left(\frac{R'}{R}\right)^2} = \frac{1 - \frac{1}{2} 9 C_2 \frac{R'}{R}}{-\frac{1}{3} + 3C_2 \frac{R'}{R} - \frac{9}{2} C_2 \frac{R'}{R}}. \tag{14}$$

Inserting $R = R_0 e^{\alpha t}$ into (14) we have

$$\frac{3\alpha^2}{3\alpha^2} = \frac{1 - \frac{9C_2\alpha}{2}}{-\frac{1}{3} - \frac{3}{2} C_2 \alpha},$$

$$\alpha = \frac{4}{9C_2}. \tag{15}$$

Equations (13) and (15) can be written in CGS units as

$$\alpha_1 = \left[\frac{C^7}{8\pi G(108 C_1)}\right]^{1/3}, \quad \alpha_2 = \frac{4C}{9C_2}. \tag{16}$$

The dimensional dependence of C_1, C_2 on the fundamental units of mass, length and time can be computed using

$$[R_S] = \left[\frac{1}{L^2}\right], \quad [\varepsilon] = \left[\frac{mL^2}{T^2 L^3}\right], \quad [U^{\alpha}_{;\alpha}] = \left[\frac{1}{L}\right],$$

as

$$[C_1]\left[\frac{1}{L^5}\right] = \left[\frac{mL^2}{T^2 L^3}\right], \quad [C_1] = \left[\frac{mL^4}{T^2}\right], \tag{17}$$

in a like manner

$$[C_2] = [L]. \tag{18}$$

Numerically we have no way to date of calculating C_1, C_2. We would expect however that if the bulk viscosity coefficients could be calculated from a quantum gravitational microphysical model that C_1, C_2 would differ greatly due to the fact that the energy scale is so high (Planck scale) and any microphysical calculation based on quantum gravity may generate large corrections due to possible wormhole effects [21] and instanton effects that would cause large differences in C_1, C_2.

3. Conclusion

The above analysis has demonstrated that the two cases ($\xi = C_1 R_S^2, \xi = C_2 \varepsilon$) possess inflationary-like solution for a radiative equation of state for a spatially flat universe

($K = 0$). Actually the existence of such solutions can be traced to the homogeneous nature of the equations in the spatially flat case. As mentioned above, it would be of interest to ask if C_1 C_2 could be calculated from a quantum gravitational model of particle creation, string creation and gravitational vacuum polarization around the Planck era. The problem with any such calculation would be that it would be subject to the same scrutiny that all quantum gravitational calculations are subjected to because of the lack of a firm basis of quantum gravity based on unitarity, renormalizability and convergence of perturbative approach [22]. Actually, Padmanabhan and Chitre [23] have previously studied a bulk viscous universe with $K = 0$ and demonstrated that inflation results for a constant bulk viscosity coefficient. They also make the point, that in a microphysical sense, bulk viscosity can phenomenologically represent the viscous drag on superconducting strings in a magnetic field, the drag brought about in monopole-monopole interactions, the dissipative force due to particle creation, and the viscous drag from photon viscosity and neutrino viscosity. It would also be of interest to ask if the above inflationary solutions that we derived are stable to higher order curvature corrections to the action since certainly the curvature is high near the Planck era. [24] Lastly the subject of 'fragility' has recently surfaced in general relativity as a probe to the general problem of general relativistic stability and it would be of interest to ask if the above inflationary solutions are stable in this sense [25].

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