PRAMANA
— journal of
physics

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Vol. 40, No. 4,
April 1993
pp. 291–297

Strange solitons in a chiral quark-meson model

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MS received 28 December 1993

Abstract. We consider a $SU(3)$ quark soliton model based on chiral invariant quark-meson coupling. We find soliton solutions with nonzero strangeness and $B = 1$ in the model with nontrivial kaonic fields, for values of the coupling constant greater than the phenomenologically acceptable number. Hence they do not correspond to known strange baryons.

Keywords. Quarks; mesons; chiral; $SU(3)$; strange soliton: kaonic fields; strange baryon.

PACS Nos 11·30; 12·38; 12·40

1. Introduction

As it is not possible to derive the low energy hadron properties from the first principles of QCD, various models have been introduced to explain baryon properties in this energy region. In particular two simple models based on the chiral symmetry of hadronic interactions have been studied in recent times. In the Skyrme model [1, 2] one considers purely mesonic degrees of freedom and baryons are obtained as the solitonic configurations of the meson fields. The winding number of the soliton solution is identified with the baryon number. On the other hand, chiral quark soliton models [3–5] are hybrid in nature and both quark and mesonic degrees of freedom are present in the theory. These models are quite successful in explaining static properties of the nucleon to a good accuracy.

In the simplest version of the chiral quark meson model based on $SU(2) \times SU(2)$ symmetry, the Lagrangian is:

$$L = i\bar{\Psi}\partial\!\!\!/\Psi - gF_\pi(\bar{\Psi}_L U \Psi_R + \bar{\Psi}_R U^+ \Psi_L) + \frac{F_\pi^2}{4}\mathrm{tr}(\partial_\mu U \partial^\mu U^+) + \frac{F_\pi^2 m_\pi^2}{4}\mathrm{tr}[(U + U^+) - 2]. \quad (1)$$

Here Ψ denotes the iso-doublet of quark fields, u, d, $U = \exp(-i\boldsymbol{\tau}\cdot\boldsymbol{\Phi})$ where τ_i are the usual iso-spin matrices and Φ_i are the pionic degrees of freedom. F_π is the pion decay constant and g is the coupling constant (with our normalization, $F_\pi = 93$ MeV). For describing nucleons, one considers a hedgehog ansatz $\boldsymbol{\Phi} = \hat{r}\theta(r)$ (as in the Skyrme model) and $\Psi = \begin{pmatrix} G(r)\chi \\ i\boldsymbol{\sigma}\cdot\hat{r}F(r)\chi \end{pmatrix}$ where the hedgehog spinor:

$$\chi = \frac{1}{\sqrt{2}} (u\downarrow - d\uparrow) \tag{2}$$

satisfies $(\boldsymbol{\tau} + \boldsymbol{\sigma})\chi = 0$.

Also $\theta(0) = \pi$ and $\theta(\infty) = 0$. One considers a soliton solution of the Lagrangian with three quarks which will be identified with the nucleons after projecting to spin-isospin eigenstates. It is found that a good phenomenological fit is obtained for $gF_\pi = 500$ MeV or $g = 5{\cdot}367$. Here it should be mentioned that in the papers of Birse and Banerjee [4] and Banerjee *et al* [5], actually a linear σ model has been considered. However we have checked that there is very little change when we work with the nonlinear σ model corresponding to L in eq. (1). Also, just as in the Skyrme model [6–8], there are configurations in which meson fields are twisted n-times in the azimuthal plane corresponding to baryon number n in this model also [9].

In the Skyrme model, a naive extension to $SU(3)$ by embedding the $SU(2)$ soliton in the $SU(3)$ and treating the symmetry breaking as a first order perturbation leads to difficulties [10, 11]. In the Callan–Klebanov approach, the strange baryons are treated as excitations of a bound state of kaon and the hedgehog soliton [12, 13] and phenomenologically this is quite successful. In the quark soliton model, an equivalent of this method using a random phase approximation (RPA) has been used [14, 15]. Here the lowest energy mean-field soliton of the model is a nonstrange hedgehog as in the $SU(2)$ case and one considers strange excitations of the ground state. This approach gives stronger symmetry breaking in the baryon sector than is realistic.

Nothing apriori disallows a state with a strange quark even in the classical ground state. Indeed, it would be only natural to think of a strange baryon as corresponding to a soliton solution with a nontrivial strange quark component and kaonic fields, with the s quark providing the strangeness quantum number just as the iso-spin of the baryons comes from the u/d quarks in this model. This is our motivation for investigating classical solutions with non-zero strangeness in the quark soliton model.

2. Kaon field configurations in a quark-meson σ-model

The Lagrangian for the $SU(3)$ nonlinear σ-model we have considered is

$$L = i\bar{\Psi}\partial\!\!\!/\Psi - gF_\pi(\bar{\Psi}_L U \Psi_R + \bar{\Psi}_R U^+ \Psi_L) + \frac{F_\pi^2}{4}\mathrm{tr}(\partial_\mu U \partial^\mu U^+) + L_{SB} \tag{1}$$

where,

$$L_{SB} = \frac{(m_\pi^2 + 2m_k^2)}{12} F_\pi^2 \mathrm{tr}[(U + U^+) - 2] - \frac{(m_k^2 - m_\pi^2)}{2\sqrt{3}} F_\pi^2 \mathrm{tr}[\lambda_8(U + U^+)]. \tag{2}$$

Here Ψ represents the flavour triplet of quarks, $U = \exp(i\boldsymbol{\lambda}\cdot\boldsymbol{\phi}/F_\pi)$ describes the pseudoscalar mesons, g a coupling constant and other quantities have their usual significance.

For the hedgehog soliton corresponding to pionic configurations, U has the form:

$$U_\pi = \begin{pmatrix} \exp(-i\boldsymbol{\tau}\cdot\hat{r}\theta(r)) & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & 1 \end{pmatrix}$$

$$=\begin{pmatrix} \cos\theta - i\tau\cdot\hat{r}\sin\theta & \vdots & 0 \\ \cdots & \cdots & \cdots \\ 0 & \vdots & 1 \end{pmatrix} \tag{3}$$

We are interested in configurations in which kaonic fields are also present. A simple ansatz for U for such configurations is

$$U = U_\pi U_k$$

with

$$U_k = \exp\left(i \sum_{i=4}^{7} \lambda_i K_i \right). \tag{4}$$

A straightforward calculation gives

$$U_k = \begin{pmatrix} 1 + \dfrac{KK^+(\cos k - 1)}{k^2} & \vdots & i\dfrac{\sin k K}{k} \\ \cdots & \cdots & \cdots \\ i\dfrac{K^+ \sin k}{k} & \vdots & \cos k \end{pmatrix} \tag{5}$$

where

$$K = \begin{pmatrix} K_4 - iK_5 \\ K_6 - iK_7 \end{pmatrix} \text{ and } k^2 = K^+ K. \tag{6}$$

We have considered "s-wave kaons" for which $K = K(r)$. Let Ψ_1 denote the u/d quark wavefunction corresponding to eigenenergy ε_1 and Ψ_2 denote the s quark wave function corresponding to an eigenenergy ε_2 with

$$\Psi_i = \begin{pmatrix} G_i \chi_i \\ i\sigma\cdot\hat{r} F_i \chi_i \end{pmatrix} \exp(-i\varepsilon_i t), \quad i = 1, 2 \tag{7}$$

where χ_i denotes the spin-flavour wave function. Then, for a solution with n u/d-quarks and m s-quarks, the effective Lagrangian corresponding to this ansatz for U is

$$\begin{aligned} L = & \, in\bar{\Psi}_1 \partial\!\!\!/ \Psi_1 - \frac{F_\pi^2}{2}\left(\frac{\theta'^2}{2} + \frac{2\sin^2\theta}{x^2} \right) - \frac{F_\pi^2}{2} k'^2 \\ & + im\bar{\Psi}_2 \partial\!\!\!/ \Psi_2 - ngF_\pi[(G_1^2 - F_1^2)\cos\theta - 2G_1 F_1 \sin\theta] \frac{(1+\cos k)}{2} \\ & - mg(G_2^2 - F_2^2)\cos k + m_\pi^2 F_\pi^2 (\cos\theta - 1)\frac{(1+\cos k)}{2} + m_k^2 F_\pi^2 (\cos k - 1). \end{aligned} \tag{8}$$

The equations of motion derived from this Lagrangian are the following:

The u-d quark equations are:

$$\left(-\frac{\partial}{\partial x} - \frac{\sin\theta(1+\cos k)}{2} \right) G_1 - \left(-\bar{\varepsilon}_1 + \frac{\cos\theta(1+\cos k)}{2} \right) F_1 = 0. \tag{9}$$

$$\left(-\frac{\partial}{\partial x}-\frac{2}{x}+\frac{\sin\theta(1+\cos k)}{2}\right)F_1-\left(-\bar{\varepsilon}_1+\frac{\cos\theta(1+\cos k)}{2}\right)G_1=0. \quad (10)$$

The s quark equations are

$$\left(-\frac{\partial}{\partial x}\right)G_2-(-\bar{\varepsilon}_2-\cos k)F_2=0. \quad (11)$$

$$\left(-\frac{\partial}{\partial x}-\frac{2}{x}\right)F_2-(\bar{\varepsilon}_2-\cos k)G_2=0. \quad (12)$$

The equation for the 'pionic' function $\theta(x)$ is:

$$\theta''+\frac{2\theta'}{x}-\frac{\sin 2\theta}{x^2}-\alpha^2\frac{\sin\theta(1+\cos k)}{2}$$
$$+ng^2[(G_1^2-F_1^2)\sin\theta+2G_1F_1\cos\theta]\frac{(1+\cos k)}{2}=0. \quad (13)$$

and the 'kaonic' function $k(x)$ satisfies

$$k''+\frac{2k'}{x}+\frac{\sin k}{2}\left[-\frac{\alpha^2}{2}(\cos\theta-1)-2\beta^2+ng^2((G_1^2-F_1^2)\cos\theta-2G_1F_1\sin\theta)\right.$$
$$\left.+2mg^2(G_2^2-F_2^2)\right]=0. \quad (14)$$

In these equations we have used the following dimensionless, scaled variables,

$$x=gF_\pi r, G_i(x)=(gF_\pi)^{3/2}G_i(r), F_i(x)=(gF_\pi)^{3/2}F_i(r), \theta(x)=\theta(r),$$

$$K(x)=K(r), \bar{\varepsilon}_i=\frac{\varepsilon_i}{gF_\pi}, \alpha=\frac{m_\pi}{gF_\pi}\cdot\beta=\frac{m_k}{gF_\pi} \text{ with } i=1,2.$$

The energy of such a configuration with n u/d quarks and m s quarks in units of gF_π is

$$E=n\bar{\varepsilon}_1+m\bar{\varepsilon}_2+\int E_s 4\pi x^2\,\mathrm{d}x \quad (15)$$

$$E_s=\frac{1}{g^2}\left[\frac{\theta'^2}{2}+\frac{\sin^2\theta}{x^2}+\frac{k'^2}{2}+\alpha^2(1-\cos\theta)\frac{(1+\cos k)}{2}+\beta^2(1-\cos k)\right]. \quad (16)$$

The fields are subjected to the boundary conditions $F_i(0)=0$, $G_i(\infty)=0$, $\theta(0)=\pi$ and $\theta(\infty)=0$, $k'(0)=0$ and $k(\infty)=0$, with $i=1,2$. These equations are solved using the standard boundary value routine COLSYS [16].

3. Results and discussion

For bound states with three quarks, the total energy must be less than $3gF_\pi$. We find that there are soliton solutions corresponding to bound states of $3s$ quarks and $2s$,

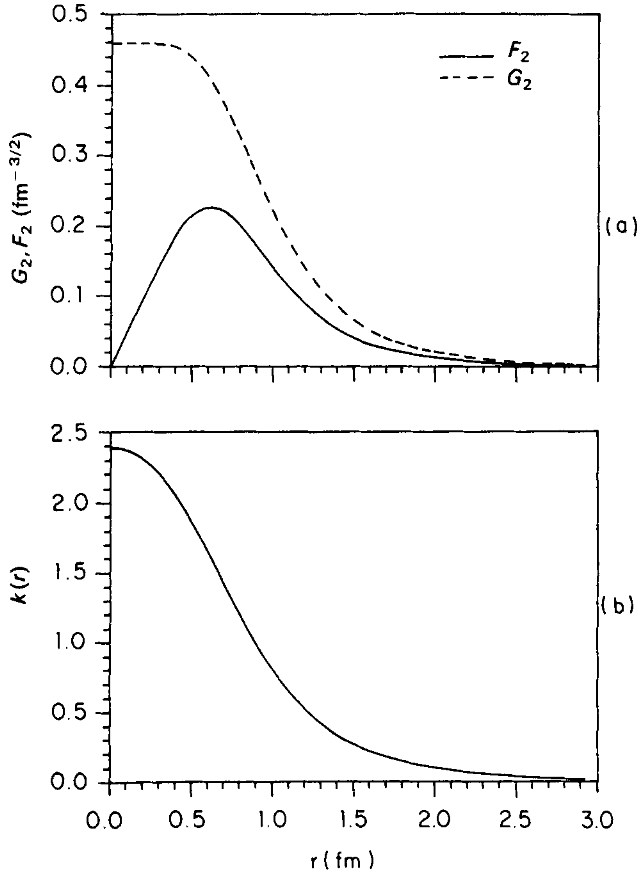

Figure 1. (**a**) Upper and lower components G_2 and F_2 of the quark wave function and (**b**) $k(r)$ as a function of r for $g = 9$ for the 3s-quark state.

Table 1. The spectrum of 3-s states.

g	$\bar{\varepsilon}_2$	Scalar energy(S)	Total energy (in units of gF_π)
8·70	0·683	0·954	3·00
9·00	0·656	0·951	2·91
10·0	0·580	0·919	2·66
11·0	0·520	0·874	2·43
12·0	0·471	0·825	2·24
13·0	0·431	0·778	2·07
14·0	0·397	0·733	1·92
15·0	0·368	0·691	1·79

1u/d quarks corresponding to $S = -3$ and -2 in this model for $g \geqslant 9{\cdot}0$ only. We do not find any soliton corresponding to a $S = -1$ state made up of 1s and 2u/d quarks. The energies for different values of g are reproduced in tables 1 and 2. The wave function components G_2, F_2 and the kaon function $k(r)$ are portrayed in figure 1 for a typical value of g for the 3s quark soliton. We do not find any soliton solution

Table 2. The spectrum of 2-u,d and 1-s states.

g	$\bar{\varepsilon}_1$	$\bar{\varepsilon}_2$	Total energy (in units of gF_π)
11·0	−0·0409	0·950	2·68
12·0	−0·127	0·958	2·52
13·0	−0·198	0·963	2·34
14·0	−0·261	0·968	2·23

ε_1 corresponds to the u-d eigenvalue and ε_2 corresponds to the s-quark eigenvalue.

for "P-wave kaons" for which K has the form:

$$K = \boldsymbol{\tau}\cdot\hat{r}k(r)\chi$$

where χ is a 2 component object (with no space dependence).

Now, from proton and neutron masses, one expects a value of $g \simeq 5{\cdot}4$ [4, 5]. Hence these solutions are unphysical on the face of it and do not correspond to the known strange baryons.

Now for phenomenological reasons, a symmetry breaking term of the form [17]:

$$L'_{\rm sb} = -\,\mathrm{tr}[(\beta' T + \beta'' S)\partial_\mu U \partial_\mu U^+ U + \mathrm{h.c}]$$

where β' and β'' are constants, T and S are the matrices

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{and } S = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

has been considered in the literature. For a good fit of the mesonic masses and decay constants, $\beta' = -26{\cdot}4\,\mathrm{MeV}^2$ and $\beta'' = -985\,\mathrm{MeV}^2$. We have included this term also and found that it has a negligible effect on the results (less than 1%).

To conclude, a simple ansatz for the meson fields in a quark soliton model with $SU(3) \times SU(3)$ symmetry does not lead to a satisfactory picture of strange baryons. The fault could be in the form of the ansatz and/or assumption of $SU(3) \times SU(3)$ invariance of the quark–meson coupling.

Acknowledgement

The authors acknowledge the support of Department of Atomic Energy, India through a project. We thank M V N Murthy and R Shankar for making COLSYS accessible to us.

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