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Gravitational field of a tachyon in a de Sitter universe

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Abstract. An exact solution of Einstein's equations is interpreted as describing the gravitational field of a tachyon in a de Sitter universe. Switching off the cosmological constant yields the gravitational field of a tachyon in flat spacetime background.

Keywords. Einstein's equations; Tachyon; de Sitter universe.

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Tachyons are hypothetical particles with spacelike four-momentum. They have velocities greater than that of light in vacuum. As opposed to the general notion, Bilaniuk *et al* [1] showed that the existence of tachyons does not violate special relativity. Many researchers (see [2–6]) have put much effort to resolve different conceptual paradoxes associated with their existence and to investigate their quantum mechanical properties. Narlikar and Sudarshan [7] studied the propagation of tachyons in an expanding universe. Narlikar and Dhurandhar [8] studied the tachyon trajectories in the Schwarzschild spacetime. Further Dhurandhar [9] extended this in the Kerr field. Much work has been carried out on tachyon and has been reviewed by Recami [6]. The attempts to detect such particles have, however, yielded null results so far.

Vaidya [10] obtained an exact solution of Einstein's equations for a tachyon in flat spacetime background. He argued that (a) the gravitational field produced by a non-rotating tachyon must have all the geometrical characteristics of the Schwarzschild field, (b) as there cannot be an observer (tardyonic) which can find a tachyon at rest, the gravitational field due to this should be axially symmetric and should contain an additional parameter, say v, such that $|v| > 1$ (velocity of light $c = 1$), and (c) it should not be possible to transform away the new parameter for $|v| > 1$. However, for $|v| < 1$ one should get coordinates such that the solution reduces to that of Schwarzschild. A solution of Einstein's equations obtained by Vaidya satisfies all the above conditions and therefore he interpreted that the solution gives the gravitational field of a tachyon. In recent years there has been much interest in obtaining solutions of Einstein as well as Einstein–Maxwell equations in a de Sitter background (see references [11–16]). In this note we present an exact solution of Einstein's equations for a tachyon in the de Sitter universe.

The Einstein field equations with cosmological constant Λ are

$$R_{ik} - \tfrac{1}{2} g_{ik} R + \Lambda g_{ik} = 8\pi T_{ik}. \tag{1}$$

A static and axially symmetric vacuum solution of the Einstein's equations is given by the line element:

$$d\tau^2 = B dt^2 - B^{-1} d\rho^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{2}$$

where

$$B = 1 - v^2 + \frac{m}{\rho} + \frac{\Lambda\rho^2}{3}, \tag{3}$$

and

$$r = \rho(1 - v\cos\theta)^{-1}, \tag{4}$$

m and v are constants. The above line element can be written as

$$d\tau^2 = B du^2 + 2 du\, d\rho - r^2(d\theta^2 + \sin^2\theta\, d\phi^2) \tag{5}$$

where

$$u = t - \int [B(\rho)]^{-1} d\rho \tag{6}$$

is the retarded time. For $|v| < 1$, one can transform the above line element to

$$d\tau^2 = B' du'^2 + 2du' dr' - r'^2(d\theta'^2 + \sin^2\theta' d\phi^2), \tag{7}$$

where

$$B' = 1 - \frac{2M}{r'} + \frac{\Lambda r'^2}{3}, \tag{8}$$

and

$$M = -\frac{m(1 - v^2)^{-3/2}}{2}, \tag{9}$$

by the following coordinate transformations:

$$u' = u(1 - v^2)^{1/2},$$

$$r' = \frac{r(1 - v\cos\theta)}{(1 - v^2)^{1/2}},$$

$$\cos\theta' = \frac{\cos\theta - v}{1 - v\cos\theta}. \tag{10}$$

The coordinate transformations given by (10) imply that the origin O′ moves with respect to O with a uniform velocity v along the common Z-axis in the flat spacetime background [10, 17]. As the line element (7) represents the Schwarzschild field in a de Sitter background, and the solution (2) satisfies the required conditions, one interprets this to give the gravitational field of a tachyon in the de Sitter background. However, this solution has a singularity at $r = 0$ and $\sec\theta = v$. It is obvious that the latter gives a right circular cone of semi-vertical angle arcsec (v) as the singularity surface. Switching off the cosmological constant one gets the gravitational field of a tachyon in the flat spacetime background which was earlier obtained by Vaidya.

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