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Symmetric scattering in $e^{\pm}$–H ionization collisions

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Abstract. Triple differential cross-sections for ionization of hydrogen atoms by electrons and positrons have been calculated for symmetric coplanar geometry following a multiple scattering method suggested and used earlier by the authors. Results show single binary peaks exactly at 45° and double binary peaks exactly at 135° for higher energies as are expected from an analysis of Briggs [3]. At lower energies there are certain deviations from these values. An analysis of scattering mechanism at peaks are also given. This supports Briggs' explanation.

Keywords. Cross-section; collision; ionization; peak; scattering.

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1. Introduction

In electron helium-atom ionization collisions with symmetric geometry, Pochat *et al* [1] first recorded a peak structure at a large scattering angle, after it was predicted in a theoretical calculation of Byron *et al* [2], over and above the binary peak around 45° for 200 eV incident energy. Since then this double peak structure in symmetric scattering in electron–atom ionization collision has attracted much attention ([3–7] for theoretical studies and [8, 9] for experimental results). The large angle peak was first interpreted by Briggs [3] as the result of a double collision process. The incoming electron is first back scattered from the atomic nucleus and is then scattered once again from the atomic electron. The first is a large-angle elastic scattering and the subsequent one is a small-angle ionization transition. The peak corresponds to the case when the two outgoing electrons proceed with equal velocities and at angles which are equal in magnitude but of opposite sign with reference to the incoming particle direction as the polar axis. If the binding energy is neglected, which is justified for high energy of incidence, the above scattering angle turns out to be exactly 135°. The other peak, which is more prominent, is the usual binary peak and should be at 45° for symmetric geometry and high incidence energy. First Born result describes this peak satisfactorily. For a description of the large angle peak a second order theory is needed. Second Born approximation and distorted wave Born approximation qualitatively describe the double peak structures for electron–helium ionization [5]. Brauner *et al* [6] recently reported one set of results for triple differential cross-sections for symmetric geometry in electron–hydrogen-atom ionization collisions. These results also show the above double peak structure. Compared to helium ionization their large angle peak appears to be more flat and less prominent. We report here the results of a similar calculation but with a different theoretical approach. We follow the multiple scattering approach of Das and Seal [10] in the analysis of the aforesaid structure. This method uses a first order correct three-particle wave-function of Das [11] multiplied by a suitable normalization factor. The resulting calculational method has recently been applied [10] in electron hydrogen-atom ionization problem for co-planar geometry. Our calculated results for TDCS for small momentum transfer cases are in good agreement with the absolute measurements of Ehrhardt *et al* [12]. It also describes well the relative experimental data of Weigold *et al* [13] and of McCarthy *et al* [14] for large momentum transfer cases. The method has been further used by the present authors for calculating the total, the single differential and the double differential cross-sections for electron–hydrogen-atom ionization cases [15]. It gives very good results for the total cross-section above 150 eV. The expected qualitative behaviour of the single and the double differential cross-section curves has also been reproduced. In the present article we have applied the same method in the analysis of TDCS for symmetric ionization of hydrogen atoms by electrons and positrons and obtained expected results with a double peak structure.

2. The scattering amplitude

The multiple scattering approach of Das and Seal [10] uses a first order correct three-particle final channel wave-function. Incidentally this wave-function is also the first order Faddeev wave-function [16] except for a normalization factor. The final form for the direct scattering amplitude in this method is (see [10] for details)

$$f(\bar{p}_1, \bar{p}_2) = N(-2f_{\mathrm{PWB}} + f_{eT} + f_{PT} + f_{Pe}), \tag{1a}$$

where the different terms within the braces are given by expressions of the form

$$-(2\pi)^{-1} \int \Phi^{(-)*}(\bar{r}_1, \bar{r}_2)(v_{12} + v_2)\exp(i\bar{p}_i\cdot\bar{r}_2)\varphi_{1S}(r_1)/(2\pi)^{3/2}. \tag{1b}$$

For f_{PWB}, $\Phi^{(-)}(\bar{r}_1, \bar{r}_2)$ is given by $\exp(i\bar{p}_1\cdot\bar{r}_1 + i\bar{p}_2\cdot\bar{r}_2)$, corresponding to two plane waves for the ejected electron and the scattered particle, for f_{eT}, which is also the first Born amplitude, $\Phi^{(-)}(\bar{r}_1, \bar{r}_2) = \varphi^{(-)}_{\bar{p}_1}(\bar{r}_1)\exp(i\bar{p}_2\cdot\bar{r}_2)$ corresponding to the plane wave for the scattered particle and Coulomb wave (due to the Coulomb potential of the target nucleus) for the ejected electron. The term f_{PT} is similar to f_{eT} except for the fact that the role of the electron and the projectile is interchanged. For f_{Pe}, $\Phi^{(-)}(\bar{r}_1, \bar{r}_2)$ is given by $\varphi^{(-)}_{\bar{p}}(\bar{r})\exp(i\bar{P}\cdot\bar{R})$, where $\bar{r} = (\bar{r}_2 - \bar{r}_1)/2$, $\bar{R} = (\bar{r}_2 + \bar{r}_1)/2$, $\bar{p} = \bar{p}_2 - \bar{p}_1$ and $\bar{P} = \bar{p}_2 + \bar{p}_1$. In this last term the projectile electron interaction is taken into account exactly in the final channel. Now each of the scattering amplitudes contains two parts corresponding to those of v_{12} and v_2 of the interaction potential V_i, in the initial channel. Thus one has, for example,

$$f_{Pe} = f_{Pe}(v_{12}) + f_{Pe}(v_2) \tag{2}$$

where $v_{12} = z/r_{12}$, $v_2 = -z/r_2$, $z = 1$ for electron and $z = -1$ for positron.

Now, if in any situation, contribution from a single term becomes dominant, it will be indicative of the fact that in that kinematic region the particular potential whose effect is exactly taken into account in the corresponding final-channel wave function plays the prime role in the scattering process. Since in our formulation the final scattering amplitude is split, each representing a particular interaction mechanism, this approach is particularly suitable for an analysis of the scattering mechanism. Detailed results in the present context are presented in the next section and an analysis of the scattering mechanism is also given.

3. Results and discussion

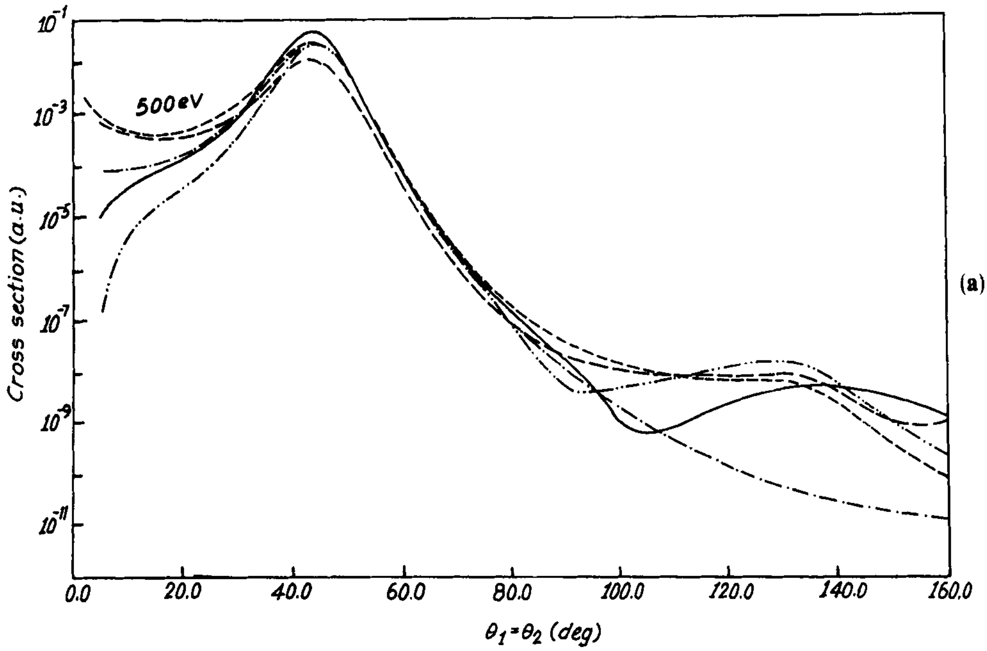
Results of our present calculation are presented in figures 1(a) and (b). Figure 1(a) shows results corresponding to an incident energy of 500 eV. For this energy all the calculations show single binary peak almost at 45°. The large angle double binary peak is prominent in the present calculations and also in Brauner *et al* [6]. In our case the peak is a little more prominent and the peak position is exactly at 135° while for the curve of Brauner *et al* the peak is at about 130°. For the positron also our results show a non-prominent peak at about 130° while the results of Brauner does not show any peak. This large angle peak is altogether absent in the first Born curve. This is expected since the coulomb potential of the target nucleus which plays a key role in this angular region is missing from the first Born amplitude because of orthogonality of the initial and the final states. Figure 1(b) gives the results for electron and positron and first Born results for electron for 200 eV and 1000 eV. For 1000 eV electron and positron the curves show peaks exactly at 135°. The large angle peaks are more steep compared to those for 500 eV curves. For 200 eV energy the large angle peak is somewhat broader, almost a plateaue-like structure. The position of the peak is at a greater value of θ_1. In the curves in figure 1(a) and 1(b) for electrons of 500 eV and 1000 eV energies there are dips at about 105°. For 200 eV energy the dip assumes the form of a flat U-shaped valley. For electrons our results show a third weak peak beyond 160° for 500 and 1000 eV energies. It may be recalled here that CBA [7] calculations also show such peaks beyond 160°. Due to observational difficulties such a peak could not be confirmed experimentally. However some absolute measurements over entire observable angular region may help in discriminating further between different theories.

To unfold the mechanism of scattering in the vicinity of the two peaks we consider different components of the scattering amplitude in the vicinity of peaks given in table 1. First note that around the peak angle 45° each component attains a maximum in the modulus and all components have comparable magnitude. But around the other peak angle 135° the component f_{Pe} is dominating. This indicates that around this second peak position the projectile electron interaction is most important in the

Table 1. Different components of scattering amplitude for $e^- - \mathrm{H}$ symmetric ionization collisions for 500 eV incident electron energy.

$\theta_1=\theta_2$ (deg)	f_{PWB}	$f_{eT}(=f_B)$	f_{PT}	f_{Pe}
30	$-0{\cdot}26^{-1\dagger}$	$-0{\cdot}17^{-1}+0{\cdot}53^{-2}i$	$-0{\cdot}18^{-1}+0{\cdot}68^{-2}i$	$-0{\cdot}21^{-1}+0{\cdot}10^{-1}i$
35	$-0{\cdot}46^{-1}$	$-0{\cdot}33^{-1}+0{\cdot}13^{-1}i$	$-0{\cdot}34^{-1}+0{\cdot}14^{-1}i$	$-0{\cdot}40^{-1}+0{\cdot}19^{-1}i$
40	$-0{\cdot}85^{-1}$	$-0{\cdot}67^{-1}+0{\cdot}33^{-1}i$	$-0{\cdot}67^{-1}+0{\cdot}34^{-1}i$	$-0{\cdot}76^{-1}+0{\cdot}34^{-1}i$
45	$-0{\cdot}96^{-1}$	$-0{\cdot}85^{-1}+0{\cdot}45^{-1}i$	$-0{\cdot}85^{-1}+0{\cdot}45^{-1}i$	$-0{\cdot}85^{-1}+0{\cdot}30^{-1}i$
50	$-0{\cdot}43^{-1}$	$-0{\cdot}44^{-1}+0{\cdot}20^{-1}i$	$-0{\cdot}45^{-1}+0{\cdot}18^{-1}i$	$-0{\cdot}35^{-1}+0{\cdot}74^{-2}i$
120	$0{\cdot}61^{-4}$	$-0{\cdot}64^{-5}-0{\cdot}27^{-5}i$	$0{\cdot}43^{-4}+0{\cdot}19^{-4}i$	$0{\cdot}11^{-3}-0{\cdot}27^{-4}i$
130	$0{\cdot}57^{-4}$	$-0{\cdot}40^{-5}-0{\cdot}18^{-5}i$	$0{\cdot}44^{-4}+0{\cdot}21^{-4}i$	$0{\cdot}89^{-4}-0{\cdot}58^{-4}i$
135	$0{\cdot}55^{-4}$	$-0{\cdot}33^{-5}-0{\cdot}16^{-5}i$	$0{\cdot}44^{-4}+0{\cdot}22^{-4}i$	$0{\cdot}56^{-4}-0{\cdot}64^{-4}i$
140	$0{\cdot}53^{-4}$	$-0{\cdot}28^{-5}-0{\cdot}14^{-5}i$	$0{\cdot}44^{-4}+0{\cdot}23^{-4}i$	$0{\cdot}31^{-4}-0{\cdot}46^{-4}i$
150	$0{\cdot}50^{-4}$	$-0{\cdot}21^{-5}-0{\cdot}11^{-5}i$	$0{\cdot}43^{-4}+0{\cdot}23^{-4}i$	$0{\cdot}24^{-4}-0{\cdot}18^{-4}i$

† number a^n stands for $a \times 10^n$



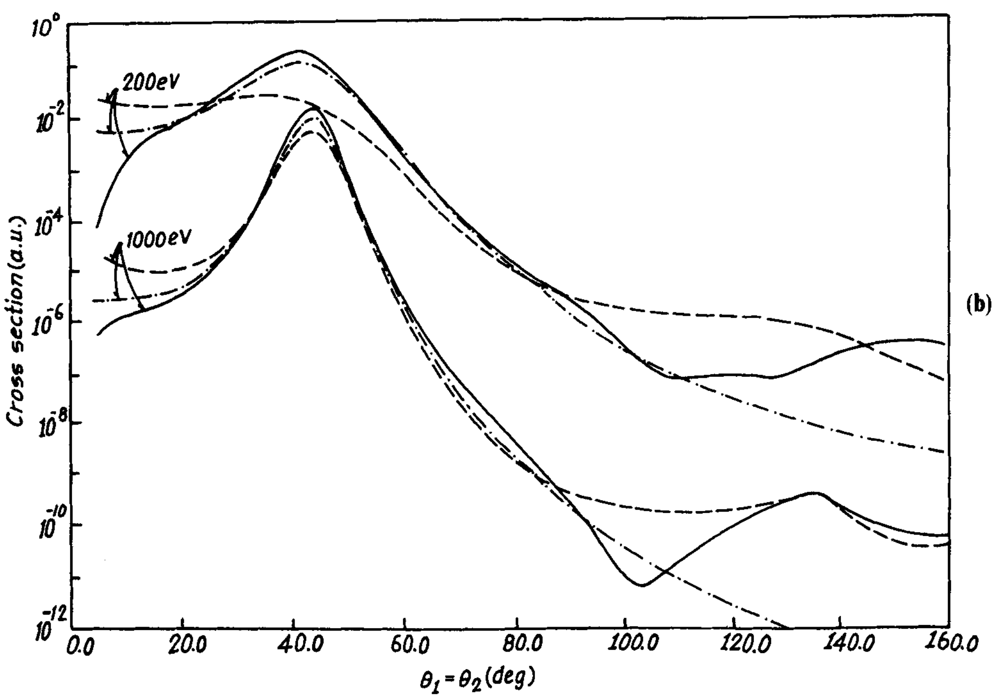


Figure 1. Triple differential cross-sections for symmetric scattering with equal energies and angles for ionization of hydrogen atoms by electrons and positrons for a) 500 eV energy, b) 200 and 1000 eV energies. ———, present results for electron; — — —, present results for positron; —·—, Born results for electron; —··—··, results of Brauner *et al* [5] for electron; ---- results of Brauner *et al* for positron.

Table 2. Contribution to the component f_{Pe} from projectilè electron (v_{12}) and projectile target (v_2) potentials.

$\theta_1 = \theta_2$ (deg)	$f_{Pe}(v_{12})$	$f_{Pe}(v_2)$
30	$-0.14^{-1\dagger} + 0.10^{-1}i$	$-0.07^{-1} + 0.00^{-1}i$
35	$-0.26^{-1} + 0.20^{-1}i$	$-0.14^{-1} - 0.01^{-1}i$
40	$-0.47^{-1} + 0.40^{-1}i$	$-0.29^{-1} - 0.06^{-1}i$
45	$-0.56^{-1} + 0.47^{-1}i$	$-0.29^{-1} - 0.17^{-1}i$
50	$-0.27^{-1} + 0.19^{-1}i$	$-0.08^{-1} - 0.12^{-1}i$
120	$-0.16^{-5} + 0.21^{-6}i$	$0.13^{-3} - 0.27^{-4}i$
130	$-0.88^{-6} + 0.12^{-6}i$	$0.90^{-4} - 0.58^{-4}i$
135	$-0.69^{-6} + 0.92^{-7}i$	$0.57^{-4} - 0.64^{-4}i$
140	$-0.55^{-6} + 0.75^{-7}i$	$0.32^{-4} - 0.46^{-4}i$
150	$-0.37^{-6} + 0.57^{-7}i$	$0.24^{-4} - 0.18^{-4}i$

† number a^n stands for a × 10^n

final channel while there is no such preference around the first peak position. Next we refer table 2 where separate contribution to f_{Pe} from potentials v_{12} and v_2 are presented. It is clear from this table that around 135° angle v_2 gives the major contribution while around 45° v_{12} gives the dominating contribution. Thus the dominating contribution to the scattering amplitude around 135° is $f_{Pe}(v_2)$. This result is in conformity with the ionization mechanism suggested by Briggs for the large angle peak. The other peak obviously corresponds to the binary collision. Interaction potentials in the final channel wave function does not have a major role to play there. These merely give higher order corrections to the scattering amplitude.

4. Conclusions

It is clear from our present calculation that the multiple scattering method of Das and Seal [10] gives results for the symmetric triple differential cross-sections for e^-–H ionization collisions at high energies with all the desired qualitative features. The calculation moreover confirms the ionization mechanism for the large angle peak as a double binary peak as suggested by Briggs. Future experimental results will further discriminate between the different theories of ionization.

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