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Cross-section sum rules and inequalities in a quark model for light and heavy flavour productions

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Abstract. The $qq\bar{q}$ model for inclusive processes is reformulated to consider the production of heavy flavours (c, b and t) and higher order flavour exchange effects. Predictions are made in terms of sum rules and inequalities for various inclusive cross-sections. Plausible parametrization of flavour symmetry breaking is also suggested.

Keywords. Quark model; inclusive reactions; light and heavy flavours.

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1. Introduction

A quark model with charm and colour, based on the model proposed by Mitra earlier [5], has been pursued by us in recent years with success [1–4]. However, one shortcoming of the model was that it does not allow production of ψ's and Υ's and $(t\bar{t})$ means which need double quark transition. This deficiency can be overcome if higher order flavour exchange effects are taken into the model which were neglected in the earlier versions. Also, recently considerable theoretical and experimental studies have been made on top and bottom quarks [6–8].

In the present paper we have reformulated the model to take into account these heavy quarks. In § 2, we summarize the reformulation of the model to consider the higher order flavour exchange effects as well as the heavy flavours. In § 3, we discuss the results for light and charm quarks while § 4 is devoted to bottom and top production. Section 5 deals with summary and conclusion.

2. The model with $SU(N)_f$ flavour symmetry

In the present model, meson baryon processes are assumed to be dominated by the $qq\bar{q} \rightarrow qq\bar{q}$ transition (figure 1). We denote the quark indices by 1 and 2 and the antiquark index by 3. The meson quark amplitude has the form (Choudhury and Goswami [3] hereafter referred as CG),

$$\tilde{O} = \pi_\beta^+ \pi_\alpha [\tilde{A}(\tfrac{1}{2}\partial_{\beta\alpha} + U_{\beta\alpha}^{(+)}) + \tilde{B} U_{\beta\alpha}^{(-)} + \tilde{C} E_{\alpha\beta}] \tag{1}$$

where π_α and π_β^+ are the incident and final meson fields and $\tilde{A}, \tilde{B}, \tilde{C}$ are the space-cum-spin-cum-colour factors. The operators $U_{\beta\alpha}$ and $E_{\alpha\beta}$ respectively generate the leading order (LO) and higher order (HO) flavour exchange effects in the model.

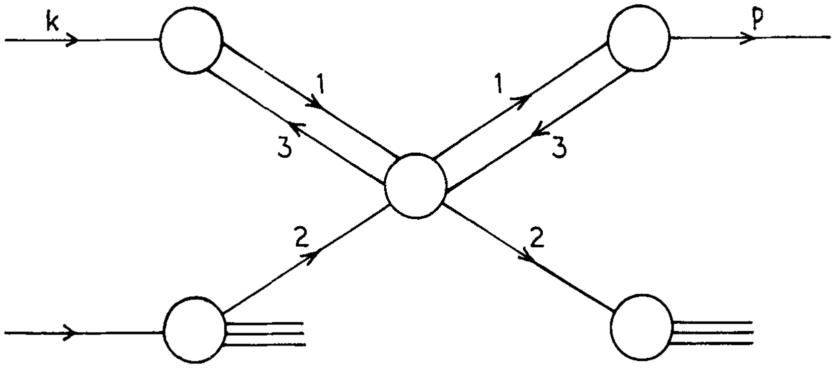

Figure 1. Meson baryon inclusive process in the present model.

They are defined as

$$U^{(+)}_{\beta\alpha} = (if_{\beta\alpha\gamma} + d_{\beta\alpha\gamma})\lambda^{(1)}_{\gamma}, \tag{2}$$

$$U^{(-)}_{\beta\alpha} = (-if_{\beta\alpha\gamma} + d_{\beta\alpha\gamma})\lambda^{(2)}_{\gamma}, \tag{3}$$

$$E_{\alpha\beta} = \frac{1}{N_f}\lambda^{(1)}_{\alpha}\lambda^{(2)}_{\beta} + \tfrac{1}{2}\lambda^{(1)}_{\varepsilon}\lambda^{(2)}_{\gamma}(D^{\beta\gamma}_{\alpha\varepsilon} - \bar{D}^{\beta\gamma}_{\alpha\varepsilon} + F^{\beta\gamma}_{\alpha\varepsilon}), \tag{4}$$

with

$$\bar{D}^{\beta\gamma}_{\alpha\varepsilon} = d_{\beta\gamma\delta}d_{\alpha\varepsilon\delta} \tag{5}$$

$$D^{\beta\gamma}_{\alpha\varepsilon} = f_{\beta\gamma\delta}f_{\alpha\varepsilon\delta} \tag{6}$$

$$F^{\beta\gamma}_{\alpha\varepsilon} = i(f_{\beta\gamma\delta}d_{\alpha\varepsilon\delta} + d_{\beta\gamma\delta}f_{\alpha\varepsilon\delta}) \tag{7}$$

where $f_{\beta\alpha\gamma}$'s and $d_{\beta\alpha\gamma}$'s are the structure factors in $SU(N)_f$ while N_f denotes the number of flavours. Here we define λ-matrices for $SU(N)_f$ to be

$$[\lambda_i, \lambda_j] = 2if_{ijk}\lambda_k, \tag{8}$$

$$\{\lambda_i, \lambda_j\} = \frac{4}{N_f}\delta_{ij} + 2d_{ijk}\lambda_k. \tag{9}$$

For our future use, it will be more convenient to abbreviate the flavour factors as

$$f^{\beta\alpha}_{A} = \tfrac{1}{2}\partial_{\beta\alpha} + U^{(+)}_{\beta\alpha}, \tag{10}$$

$$f^{\beta\alpha}_{B} = U^{(-)}_{\beta\alpha} \tag{11}$$

and

$$f^{\beta\alpha}_{C} = E_{\alpha\beta}, \tag{12}$$

instead of notation of CG [3].

In order to apply the model to inclusive processes, we need the structure of the hadronic wave function. In the valence approximation, it reads as

$$\psi = \psi^s\eta^a\left[\frac{\chi'\phi' + \chi''\phi''}{\sqrt{2}}\right] \tag{13}$$

where the symbols have the same significance as in CG [3]. The inclusive cross-section for the pseudoscalar meson production $A + B \rightarrow C + X$ then takes the form

$$\begin{aligned} \sigma(A + B \rightarrow C + X) & \\ &= A'U' + A''U'' + B'V' + B''V'' + C'W' + C''W'' + D'X' + D''X'' \\ &\quad + E'Y' + E''Y'' + F'Z' + F''Z'' \end{aligned} \tag{14}$$

where

$$\langle \chi' | \tilde{A}^+ \tilde{A} | \chi' \rangle = A', \tag{15}$$

$$\langle \chi'' | \tilde{A}^+ \tilde{A} | \chi'' \rangle = A'', \tag{16}$$

$$\langle \chi' | \tilde{B}^+ \tilde{B} | \chi' \rangle = B', \tag{17}$$

$$\langle \chi'' | \tilde{B}^+ \tilde{B} | \chi'' \rangle = B'', \tag{18}$$

$$\langle \chi' | \tilde{C}^+ \tilde{C} | \chi' \rangle = C', \tag{19}$$

$$\langle \chi'' | \tilde{C}^+ \tilde{C} | \chi'' \rangle = C'', \tag{20}$$

$$\langle \chi' | \tilde{A}^+ \tilde{B} + \tilde{B}^+ \tilde{A} | \chi' \rangle = D', \tag{21}$$

$$\langle \chi'' | \tilde{A}^+ \tilde{B} + \tilde{B}^+ \tilde{A} | \chi'' \rangle = D'', \tag{22}$$

$$\langle \chi' | \tilde{B}^+ \tilde{C} + \tilde{C}^+ \tilde{B} | \chi' \rangle = E', \tag{23}$$

$$\langle \chi'' | \tilde{B}^+ \tilde{C} + \tilde{C}^+ \tilde{B} | \chi'' \rangle = E'', \tag{24}$$

$$\langle \chi' | \tilde{C}^+ \tilde{A} + \tilde{A}^+ \tilde{C} | \chi' \rangle = F', \tag{25}$$

$$\langle \chi'' | \tilde{C}^+ \tilde{A} + \tilde{A}^+ \tilde{C} | \chi'' \rangle = F'', \tag{26}$$

and

$$\langle \phi' | \tfrac{1}{2} f_A^{\beta\alpha} f_A^{\beta\alpha^+} | \phi' \rangle = U', \tag{27}$$

$$\langle \phi'' | \tfrac{1}{2} f_A^{\beta\alpha} f_A^{\beta\alpha^+} | \phi'' \rangle = U'', \tag{28}$$

$$\langle \phi' | \tfrac{1}{2} f_B^{\beta\alpha} f_B^{\beta\alpha^+} | \phi' \rangle = V', \tag{29}$$

$$\langle \phi'' | \tfrac{1}{2} f_B^{\beta\alpha} f_B^{\beta\alpha^+} | \phi'' \rangle = V'', \tag{30}$$

$$\langle \phi' | \tfrac{1}{2} f_C^{\beta\alpha} f_C^{\beta\alpha^+} | \phi' \rangle = W', \tag{31}$$

$$\langle \phi'' | \tfrac{1}{2} f_C^{\beta\alpha} f_C^{\beta\alpha^+} | \phi'' \rangle = W'', \tag{32}$$

$$\langle \phi' | \tfrac{1}{2} f_B^{\beta\alpha} f_B^{\beta\alpha^+} | \phi' \rangle = \langle \phi' | \tfrac{1}{2} f_A^{\beta\alpha^+} f_B^{\beta\alpha} | \phi' \rangle = X', \tag{33}$$

$$\langle \phi'' | \tfrac{1}{2} f_A^{\beta\alpha} f_B^{\beta\alpha^+} | \phi'' \rangle = \langle \phi'' | \tfrac{1}{2} f_A^{\beta\alpha^+} f_B^{\beta\alpha} | \phi'' \rangle = X'', \tag{34}$$

$$\langle \phi' | \tfrac{1}{2} f_B^{\beta\alpha} f_C^{\beta\alpha^+} | \phi' \rangle = \langle \phi' | \tfrac{1}{2} f_B^{\beta\alpha^+} f_C^{\beta\alpha} | \phi' \rangle = Y', \tag{35}$$

$$\langle \phi'' | \tfrac{1}{2} f_B^{\beta\alpha} f_C^{\beta\alpha^+} | \phi'' \rangle = \langle \phi'' | \tfrac{1}{2} f_B^{\beta\alpha^+} f_C^{\beta\alpha} | \phi'' \rangle = Y'', \tag{36}$$

$$\langle \phi' | \tfrac{1}{2} f_C^{\beta\alpha} f_A^{\beta\alpha^+} | \phi' \rangle = \langle \phi' | \tfrac{1}{2} f_A^{\beta\alpha^+} f_A^{\beta\alpha} | \phi' \rangle = Z', \tag{37}$$

$$\langle \phi'' | \tfrac{1}{2} f_C^{\beta\alpha} f_A^{\beta\alpha^+} | \phi'' \rangle = \langle \phi'' | \tfrac{1}{2} f_C^{\beta\alpha^+} f_A^{\beta\alpha} | \phi'' \rangle = Z''. \tag{38}$$

We note that the terms $C'W'$, $C''W''$, $E'Y'$, $E''Y''$, $F'Z'$ and $F''Z''$ have their origin in the higher order (HO) term detailed in (1) and (2).

The flavour coefficients U', U'', V', V'', W', W'', X', X'', Y', Y'', Z' and Z'' are given in table 1 for various processes.

In table 1, mesons having a top quark are denoted by T's [9] while T_t^0 denotes $\delta(t\bar{t})$ meson.

Table 1. The flavour coefficients as occurred in eq. (14) of the text.

Processes	U'	U''	V'	V''	W'	W''	X'	X''	Y'	Y''	X'	Z''
$\pi^- P \to \pi^- X$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{32}$	$\frac{19}{96}$	$\frac{731}{20736}$	$\frac{329}{20736}$	$-\frac{1}{16}$	$\frac{11}{48}$	$\frac{1}{288}$	$-\frac{49}{864}$	$-\frac{1}{576}$	$-\frac{85}{1728}$
$K^- P \to \pi^- X$	0	0	0	$\frac{1}{3}$	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$\pi^+ P \to \pi^+ X$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{9}{32}$	$\frac{11}{96}$	$\frac{83}{20736}$	$\frac{977}{20736}$	$\frac{3}{16}$	$\frac{1}{16}$	$\frac{1}{288}$	$-\frac{53}{864}$	$\frac{1}{192}$	$-\frac{89}{1728}$
$\pi^+ P \to \pi^0 X$	$\frac{1}{8}$	$\frac{1}{24}$	0	$\frac{1}{6}$	$\frac{1}{32}$	$\frac{1}{96}$	0	$\frac{1}{12}$	$\frac{1}{16}$	$\frac{1}{48}$	0	$\frac{1}{24}$
$\pi^- P \to \pi^0 X$	$\frac{1}{8}$	$\frac{5}{24}$	$\frac{1}{4}$	$\frac{1}{12}$	$\frac{1}{32}$	$\frac{5}{96}$	0	$\frac{1}{12}$	$-\frac{1}{16}$	$\frac{1}{16}$	0	$\frac{1}{24}$
$K^- P \to \pi^0 X$	0	0	$\frac{1}{42}$	$\frac{1}{12}$	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$K^+ P \to K^+ X$	0	0	$\frac{9}{32}$	$\frac{11}{96}$	$\frac{1}{41472}$	$\frac{1}{41472}$	0	0	0	$\frac{1}{1728}$	$-\frac{1}{576}$	$-\frac{1}{1728}$
$K^- P \to K^- X$	$\frac{1}{4}$	$\frac{5}{12}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{41472}$	$\frac{1}{41472}$	$-\frac{1}{16}$	$-\frac{5}{48}$	$-\frac{1}{576}$	$-\frac{1}{432}$	$\frac{1}{576}$	$-\frac{1}{1728}$
$\pi^+ P \to K^+ X$	$\frac{1}{4}$	$\frac{1}{12}$	0	0	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$K^- P \to K^0 X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	0	0	0	0	0	0	0	0
$\pi^- P \to K^0 X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	$\frac{1}{64}$	$\frac{5}{192}$	0	0	0	0	0	0
$K^+ P \to K^0 X$	0	0	0	$\frac{1}{3}$	0	0	0	0	0	0	0	0
$\pi^- P \to \eta X$	$\frac{1}{24}$	$\frac{5}{72}$	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{17}{3456}$	$\frac{29}{10368}$	0	$-\frac{1}{36}$	$\frac{1}{96}$	$\frac{11}{864}$	$\frac{1}{72}$	$\frac{1}{432}$
$K^- P \to \eta X$	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{1}{12}$	$\frac{1}{36}$	$\frac{19}{6912}$	$\frac{91}{20736}$	0	0	$-\frac{1}{48}$	$-\frac{5}{144}$	0	0
$\pi^+ P \to \eta X$	$\frac{1}{24}$	$\frac{1}{72}$	0	$\frac{1}{18}$	$\frac{1}{384}$	$\frac{1}{10368}$	0	$-\frac{1}{36}$	$\frac{1}{96}$	$-\frac{1}{864}$	0	$\frac{1}{432}$
$\pi^- P \to \eta' X$	$\frac{1}{12}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{99}{6912}$	$\frac{155}{20736}$	0	$-\frac{1}{18}$	$\frac{7}{288}$	$\frac{25}{864}$	$\frac{5}{144}$	$-\frac{1}{216}$
$\pi^+ P \to \eta' X$	$\frac{1}{12}$	$\frac{1}{36}$	0	$\frac{1}{18}$	$\frac{49}{6912}$	$\frac{1}{2304}$	0	$-\frac{1}{18}$	$\frac{7}{288}$	$-\frac{1}{288}$	0	$\frac{1}{144}$
$K^- P \to \eta' X$	$\frac{1}{12}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{1}{18}$	$\frac{321}{27648}$	$\frac{805}{82944}$	0	0	$\frac{11}{576}$	$\frac{55}{1728}$	$\frac{5}{144}$	$\frac{5}{432}$
$\pi^- P \to \bar{D}^0 X$	$\frac{1}{4}$	$\frac{1}{12}$	0	0	$\frac{1}{64}$	$\frac{5}{192}$	0	0	0	$\frac{1}{24}$	0	0
$K^- P \to \bar{D}^0 X$	0	0	0	0	$\frac{1}{32}$	$\frac{1}{96}$	0	0	0	0	0	0
$\pi^- P \to D^- X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	$\frac{1}{64}$	$\frac{5}{192}$	0	0	0	$\frac{1}{24}$	0	0
$K^- P \to D^- X$	0	0	0	0	0	$\frac{1}{24}$	0	0	0	0	0	0
$\pi^+ P \to D_S^+ X$	0	0	0	0	$\frac{1}{256}$	$\frac{1}{768}$	0	0	0	0	0	0
$\pi^- P \to D_S^+ X$	0	0	0	0	$\frac{1}{256}$	$\frac{1}{768}$	0	0	0	0	0	0
$\pi^+ P \to D_S^- X$	0	0	0	0	$\frac{1}{256}$	$\frac{1}{768}$	0	0	0	0	0	0
$K^- P \to D_S^- X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	0	0	0	0	0	0	0	0
$\pi^- P \to \psi X$	0	0	0	0	$\frac{1}{2304}$	$\frac{5}{6912}$	0	0	0	0	0	0
$\pi^+ P \to \psi X$	0	0	0	0	$\frac{1}{2304}$	$\frac{1}{6912}$	0	0	0	0	0	0
$K^- P \to \psi X$	0	0	0	0	$\frac{1}{1152}$	$\frac{1}{3456}$	0	0	0	0	0	0
$\pi^+ P \to B_U^+ X$	$\frac{1}{4}$	$\frac{1}{12}$	0	0	$\frac{1}{100}$	$\frac{29}{1200}$	0	0	0	$\frac{1}{24}$	0	0
$K^- P \to B_U^+ X$	0	0	0	0	$\frac{41}{1600}$	$\frac{41}{4800}$	0	0	0	0	0	0
$\pi^- P \to B_d^0 X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	$\frac{1}{64}$	$\frac{89}{4800}$	0	0	0	$\frac{1}{24}$	0	0
$\pi^+ P \to \bar{B}_S^0 X$	0	0	0	0	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$K^- P \to B_S^0 X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	0	0	0	0	0	0	0	0
$\pi^+ P \to B_C^- X$	0	0	0	0	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$\pi^- P \to \Upsilon X$	0	0	0	0	$\frac{1}{1250}$	$\frac{1}{3750}$	0	0	0	0	0	0
$K^- P \to \Upsilon X$	0	0	0	0	$\frac{1}{1250}$	$\frac{1}{3750}$	0	0	0	0	0	0
$\pi^+ P \to \Upsilon X$	0	0	0	0	0	$\frac{1}{1875}$	0	0	0	0	0	0
$\pi^+ P \to T_U^0 X$	$\frac{1}{4}$	$\frac{1}{12}$	0	0	$\frac{1}{144}$	$\frac{5}{216}$	0	0	0	$\frac{1}{24}$	0	0
$\pi^- P \to T_U^0 X$	0	0	0	0	$\frac{13}{576}$	$\frac{13}{1728}$	0	0	0	0	0	0
$K^- P \to T_U^- X$	0	0	0	0	$\frac{13}{576}$	$\frac{13}{1728}$	0	0	0	0	0	0
$\pi^- P \to T_d^- X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	0	$\frac{1}{108}$	0	0	0	0	0	0
$K^- P \to T_d^- X$	0	0	0	0	0	$\frac{1}{108}$	0	0	0	0	0	0
$\pi^+ P \to T_S^- X$	0	0	0	0	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0

(*Continued*)

Table 1. (*Continued*)

Processes	U'	U''	V'	V''	W'	W''	X'	X''	Y'	Y''	X'	Z''
$K^- P \to T_s^- X$	$\frac{1}{4}$	$\frac{5}{12}$	0	0	0	$\frac{1}{48}$	0	0	0	0	0	0
$\pi^+ P \to T_C^0 X$	0	0	0	0	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$\pi^+ P \to T_b^+ X$	0	0	0	0	$\frac{1}{64}$	$\frac{1}{192}$	0	0	0	0	0	0
$\pi^- P \to T_t^0 X$	0	0	0	0	$\frac{169}{259200}$	$\frac{169}{777600}$	0	0	0	0	0	0
$K^- P \to T_t^0 X$	0	0	0	0	$\frac{169}{518400}$	$\frac{169}{311040}$	0	0	0	0	0	0
$\pi^+ P \to T_t^0 X$	0	0	0	0	0	$\frac{169}{388800}$	0	0	0	0	0	0

Equation (14) is our main result which will be applied to study light and heavy flavour production in various inclusive processes.

3. Prediction of the model

We now use (14) to obtain relations among various inclusive cross-sections. Here we record a few of them to be tested with experimental data.

3.1 *Sum rules: Sum rules involving pure light-quark transition*

We have,

$$
\begin{aligned}
&3\sigma(\pi^- P \to \pi^- X) + \frac{5}{16}\sigma(K^+ P \to K^0 X) + \frac{575}{432}\sigma(K^- P \to \pi^- X) \\
&\quad + \frac{17}{24}\sigma(\pi^- P \to \pi^0 X) + 26\sigma(K^+ P \to K^+ X) \\
&= 2\sigma(K^- P \to \bar{K}^0 X) + 2\sigma(\pi^+ P \to \pi^+ X) + \frac{7033}{216}\sigma(K^+ P \to K^0 X) \\
&\quad + \frac{17}{24}\sigma(\pi^+ P \to \pi^0 X) + \frac{17}{96}\sigma(\pi^- P \to K^0 X).
\end{aligned} \tag{39}
$$

$$
\begin{aligned}
&3\sigma(\pi^- P \to \pi^- X) + \frac{5}{16}\sigma(K^- P \to \pi^- X) + \frac{355}{216}\sigma(K^- P \to \pi^- X) \\
&\quad + \frac{17}{24}\sigma(\pi^- P \to \pi^0 X) + 26\sigma(K^+ P \to K^+ X) \\
&= 2\sigma(\pi^+ P \to \pi^+ X) + 2\sigma(K^- P \to K^0 X) + \frac{14201}{432}\sigma(K^+ P \to K^0 X) \\
&\quad + \frac{17}{96}\sigma(\pi^- P \to K^0 X) + 2\sigma(K^- P \to \bar{K}^0 X)
\end{aligned} \tag{40}
$$

$$\sigma(\pi^- P\to\pi^0 X)+\frac{1}{2}\sigma(K^- P\to\pi^- X)+\sigma(\pi^- P\to\pi^0 X)$$

$$=\frac{1}{4}\sigma(K^- P\to\bar{K}^0 X)+\frac{1}{2}\sigma(K^+ P\to K^0 X)+\sigma(\pi^+ P\to\pi^0 X)$$

$$+\sigma(K^+ P\to K^0 X)+\sigma(\pi^+ P\to\pi^0 X)+\frac{1}{4}\sigma(\pi^- P\to K^0 X) \tag{41}$$

$$\sigma(\pi^- P\to\pi^0 X)+\frac{1}{2}\sigma(K^- P\to\pi^- X)+\sigma(\pi^- P\to\pi^0 X)$$

$$=\frac{8}{19}\sigma(K^+ P\to K^+ X)+\frac{1}{4}\sigma(K^- P\to\bar{K}^0 X)+\sigma(\pi^+ P\to\pi^0 X)$$

$$+\frac{1}{4}\sigma(\pi^- P\to K^0 X)+\frac{1}{2}\sigma(K^+ P\to K^0 X)+\sigma(\pi^+ P\to\pi^0 X)$$

$$+\frac{8}{19}\sigma(K^+ P\to K^+ X) \tag{42}$$

$$3\sigma(\pi^- P\to\pi^- X)+\frac{5}{19}\sigma(K^+ P\to K^+ X)+\frac{10925}{8208}\sigma(K^- P\to\pi^- X)$$

$$+\frac{323}{456}\sigma(\pi^- P\to\pi^0 X)+\frac{499}{19}\sigma(K^+ P\to K^+ X)$$

$$=2\sigma(K^- P\to\bar{K}^0 X)+2\sigma(\pi^+ P\to\pi^+ X)+\frac{269819}{8208}\sigma(K^+ P\to K^0 X)$$

$$+\frac{323}{456}\sigma(\pi^+ P\to\pi^0 X)+\frac{323}{1824}\sigma(\pi^- P\to K^0 X) \tag{43}$$

$$\sigma(K^- P\to\eta X)+\frac{35}{108}\sigma(K^+ P\to K^0 X)+\frac{2}{3}\sigma(\pi^- P\to\pi^0 X)$$

$$=\frac{2}{3}\sigma(K^- P\to\bar{K}^0 X)+\frac{1}{3}\sigma(K^+ P\to K^0 X)+\frac{71}{108}\sigma(K^- P\to\pi^- X)$$

$$+\frac{2}{3}\sigma(\pi^+ P\to\pi^0 X)+\frac{1}{6}\sigma(\pi^- P\to K^0 X) \tag{44}$$

$$4\sigma(\pi^+ P\to\eta X)+\frac{4}{3}\sigma(\pi^+ P\to\pi^0 X)+\frac{547}{216}\sigma(K^- P\to\pi^- X)$$

$$+\frac{710}{27}\sigma(K^+ P\to K^0 X)+\frac{95}{36}\sigma(\pi^+ P\to\pi^0 X)+\frac{95}{144}\sigma(\pi^- P\to K^0 X)$$

$$=\sigma(K^- P\to\pi^- X)+\sigma(K^- P\to\eta X)+\frac{95}{36}\sigma(\pi^- P\to\pi^0 X)$$

$$+28\sigma(K^+ P\to K^+ X) \tag{45}$$

$$\sigma(K^+ P\to K^+ X)=\frac{19}{16}\sigma(K^+ P\to K^0 X) \tag{46}$$

$$\sigma(K^- P \to K^- X) + 2\sigma(\pi^+ P \to \pi^0 X) + \frac{3197}{432}\sigma(K^- P \to \pi^- X)$$
$$+ \frac{8387}{216}\sigma(K^+ P \to K^0 X) + \frac{53}{24}\sigma(\pi^+ P \to \pi^0 X) + \frac{53}{96}\sigma(\pi^- P \to K^0 X)$$
$$= \frac{3}{2}\sigma(K^- P \to \bar{K}^0 X) + \sigma(K^+ P \to K^+ X) + \frac{53}{24}\sigma(\pi^- P \to \pi^0 X)$$
$$+ 38\sigma(K^+ P \to K^+ X) \tag{47}$$

$$\frac{1}{3}\sigma(K^- P \to K^- X) + \frac{1}{3}\sigma(\pi^- P \to \pi^- X) + \frac{1}{24}\sigma(K^+ P \to K^0 X)$$
$$+ \frac{169}{162}\sigma(K^- P \to \pi^- X) + \frac{7}{18}\sigma(\pi^+ P \to \pi^0 X)$$
$$+ \frac{7}{72}\sigma(\pi^- P \to K^0 X) + 7\sigma(K^+ P \to K^+ X)$$
$$= \sigma(K^- P \to \eta X) + \frac{11873}{1296}\sigma(K^+ P \to K^0 X) + \frac{7}{18}\sigma(\pi^- P \to \pi^0 X) \tag{48}$$

$$4\sigma(\pi^+ P \to \eta X) + \frac{4}{3}\sigma(\pi^+ P \to \pi^0 X) + \frac{1483}{216}\sigma(K^- P \to \pi^- X)$$
$$+ \frac{748}{27}\sigma(K^+ P \to K^0 X) + \frac{95}{36}\sigma(\pi^+ P \to \pi^0 X) + \frac{95}{144}\sigma(\pi^- P \to K^0 X)$$
$$= \sigma(K^+ P \to K^0 X) + \sigma(K^- P \to \eta X) + \frac{95}{36}\sigma(\pi^- P \to \pi^0 X)$$
$$+ 28\sigma(K^+ P \to K^+ X) \tag{49}$$

Without higher order term, the sum rules (39)–(49) reduce to the cross-section sum rules (3.1), (3.2) (3.9)–(3.17) of CG ([3]) respectively.

In table 2, we compare the sum rules with data [10, 11] at average energy $E_{cM} \sim 5.5$ GeV and average $P_{lab} \sim 15.0$ GeV/C. For completeness, we also record the similar comparison with the leading order (LO) term of CG ([3]) alone. From table 2, we observe that the higher order term makes the agreement better for eqs (42), (44), (45), (47), (48), (49) and worse for (39), (40), (41), (43) and remains same for (46).

3.2 *Sum rules involving charm quark transitions*

We now use (14) to record a few testable sum rules involving charm quark tansitions and including higher order term:

$$\sigma(\pi^- P \to \psi X) + \frac{1}{36}\sigma(K^- P \to \bar{K}^0 X) = \frac{1}{36}\sigma(\pi^- P \to K^0 X), \tag{50}$$

$$\sigma(\pi^+ P \to \psi X) + \frac{1}{36}\sigma(K^+ P \to K^0 X) = \frac{1}{36}\sigma(K^- P \to \pi^- X), \tag{51}$$

$$\sigma(K^- P \to \psi X) + \frac{1}{18}\sigma(K^+ P \to K^0 X) = \frac{1}{18}\sigma(K^- P \to \pi^- X). \tag{52}$$

Table 2. Tests of light quark sum rules at $E_{CM} \sim 5{\cdot}5$ GeV and $P_{lab} \sim 15{\cdot}0$ GeV/C.

Eqn. No.	LHS (mb) with L.O. and H.O.	RHS (mb) with L.O. and H.O.	LHS/ RHS	Eqn. No. from [3]	LHS (mb) with L.O.	RHS (mb) with L.O.	LHS/ RHS
(39)	487·48 ± 3.61	334·33 ± 8·87	1·45	(3·1)	123·68 ± 0·95	109·7 ± 1·64	1·13
(40)	502·1 ± 4·3	337·67 ± 7·65	1·49	(3·2)	130·00 ± 1·21	109·7 ± 1·64	1·2
(41)	89·6 ± 3.6	79·81 ± 4.18	1·12	(3·9)	40·78 ± 0·26	38·2 ± 1·5	1·07
(42)	89·6 ± 3·6	83·55 ± 4·01	1·07	(3·10)	38·2 ± 1·5	42·64 ± 0.18	0·89
(43)	491·7 ± 3·56	336·21 ± 8·38	1·46	(3·11)	124·85 ± 0·9	109·7 ± 1·64	1·14
(44)	35·22 ± 3·46	48·65 ± 2·3	0·72	(3·12)	6·98 ± 0·27	4·6 ± 1·8	1·52
(45)	367·8 ± 16·45	459·81 ± 9·56	0·8	(3.13)	53·87 ± 1·33	31·2 ± 0·28	1·73
(46)	23·2	14·32 ± 0·32	1·62	(3·14)	11·6	7·16 ± 0·2	1·62
(47)	581·86 ± 24·34	550·78 ± 3·61	1·06	(3·15)	80·1 ± 0·9	22·78 ± 0·48	3·52
(48)	138·74 ± 1·62	77·9 ± 6·54	1·78	(3·16)	16·55 ± 0·33	4·6 ± 1·8	3·6
(49)	490·68 ± 21·89	439·44 ± 8·53	1·12	(3·17)	53·87 ± 1·33	10·63 ± 1·97	5·1

Table 3. Tests of sum rules involving charm quark transitions $E_{CM} \sim 8{\cdot}662$ GeV and $P_{lab} \sim 39{\cdot}5$ GeV/C.

Equation No.	LHS (mb)	RHS (mb)	LHS/RHS
(50)	0·2597367 ± ·0055565	0·1527778 ± 0·005	1·70
(51)	0·2194557 ± ·0083343	0·9722222 ± 0·0194444	·2252759
(52)	0·2597266 ± ·0055586	0·1527778 ± 0·025	1·7

In table 3, we compare these sum rules with data [10, 11] at average energy $E_{CM} \sim 8{\cdot}662$ GeV and average $P_{lab} \sim 39{\cdot}5$ GeV/C. It shows considerable flavour symmetry breaking.

Thus as discussed in our previous communication with leading order [3], here too, we observe that symmetries alone are not sufficient and dynamics of the underlying theory must be incorporated to accommodate such symmetry effects in the model.

3.3 *Quantitative estimates of flavour symmetry breaking in the model with higher order term*

In order to obtain the quantitative estimation of $SU(4)_f$ symmetry breaking we recast (50)–(52) as

$$\frac{\sigma(\pi^- P \to \psi X)}{(1/36)[\sigma(\pi^- P \to K^0 X) - \sigma(K^- P \to \bar{K}^0 X)]} = 1, \tag{50a}$$

$$\frac{\sigma(\pi^+ P \to \psi X)}{(1/36)[\sigma(K^- P \to \pi^- X) - \sigma(K^+ P \to K^0 X)]} = 1, \tag{51a}$$

$$\frac{\sigma(K^- P \to \psi X)}{(1/18)[\sigma(K^- P \to \pi^- X) - \sigma(K^+ P \to K^0 X)]} = 1. \tag{52a}$$

In table 4 we evaluate the experimental values of the above ratios, using eqs (50a)–(52a). We find that the relative strengths of the charm changing to the

Table 4. Experimental values of the ratios of various sum rules.

Equation No.	Experimental values with H.O. + L.O.
(50a)	$1{\cdot}36\times10^{-4}$
(51a)	$1{\cdot}5\times10^{-5}$
(52a)	$2{\cdot}92\times10^{-6}$
(41a)	4·65
(44a)	2·93
(39a)	1·53
(40a)	1·65
(42a)	0·62
(43a)	1·54
(45a)	1·28
(46a)	1·62
(47a)	0·93
(48a)	2·48
(49a)	0·86

charm zero transition are in the range $1{\cdot}36\times10^{-4}$ to $2{\cdot}92\times10^{-6}$. Such small numerical values suggest that the double quark transitions are much more suppressed than the single quark ones.

A similar analysis for light quark transitions, eqs (39)–(49) can be performed following the procedure discussed in ([3]). To that end, (41) and (44) can be recast as the ratios for strangeness changing to strangeness zero transition while (39), (40), (42), (43), (45)–(49) as those for diffractive transitions (no quantum number are exchanged) to non-diffractive ones.

$$\frac{(1/2)\sigma(K^-P\to\pi^-X)-(1/4)\sigma(\pi^-P\to K^0X)}{\begin{array}{c}[(1/4)\sigma(K^-P\to\bar{K}^0X)+(1/2)\sigma(K^+P\to K^0X)+\sigma(\pi^+P\to\pi^0X)\\ +\sigma(K^+P\to K^0X)+\sigma(\pi^+P\to\pi^0X)-2\sigma(\pi^-P\to\pi^0X)]\end{array}}=1 \quad (41a)$$

$$\frac{\sigma(K^-P\to\eta X)-(71/108)\sigma(K^-P\to\pi^-X)-(1/6)\sigma(\pi^-P\to K^0X)}{\begin{array}{c}[(2/3)\sigma(K^-P\to\bar{K}^0X)+(1/3)\sigma(K^+P\to K^0X)+(2/3)\sigma(\pi^+P\to\pi^0X)\\ -(35/108)\sigma(K^+P\to K^0X)-(2/3)\sigma(\pi^-P\to\pi^0X)]\end{array}}=1 \quad (44a)$$

$$\frac{3\sigma(\pi^-P\to\pi^-X)+26\sigma(K^+P\to K^+X)-2\sigma(\pi^+P\to\pi^+X)}{\begin{array}{c}[2\sigma(K^-P\to\bar{K}^0K)+(7033/216)\sigma(K^+P\to K^0X)+\\ (17/24)\sigma(\pi^+P\to\pi^0X)+(17/96)\sigma(\pi^-P\to K^0X)-(5/16)\sigma(K^+P\to K^0X)\\ -(575/432)\sigma(K^-P\to\pi^-X)-(17/24)\sigma(\pi^-P\to\pi^0X)]\end{array}}=1 \quad (39a)$$

$$\frac{3\sigma(\pi^-P\to\pi^-X)+26\sigma(K^+P\to K^+X)-2\sigma(\pi^+P\to\pi^+X)}{\begin{array}{c}[2\sigma(K^-P\to\bar{K}^0X)+(14201/432)\sigma(K^+P\to K^0X)+\\ (17/24)\sigma(\pi^+P\to\pi^0X)+(17/96)\sigma(\pi^-P\to K^0X)-(5/16)\sigma(K^-P\to\pi^-X)\\ -(355/216)\sigma(K^-P\to\pi^-X)-(17/24)\sigma(\pi^-P\to\pi^0X)]\end{array}}=1 \quad (40a)$$

$$\frac{(16/19)\sigma(K^+P\to K^+X)}{\begin{array}{l}[2\sigma(\pi^-P\to\pi^0X)+(1/2)\sigma(K^-P\to\pi^-X)-(1/4)\sigma(K^-P\to\bar{K}^0X)\\ -2\sigma(\pi^+P\to\pi^0X)-(1/2)\sigma(K^+P\to K^0X)-(1/4)\sigma(\pi^-P\to K^0X)]\end{array}}=1 \tag{42a}$$

$$\frac{3\sigma(\pi^-P\to\pi^-X)+(504/19)\sigma(K^+P\to K^+X)-2\sigma(\pi^+P\to\pi^+X)}{\begin{array}{l}[2\sigma(K^-P\to\bar{K}^0X)+(269819/8208)\sigma(K^+P\to K^0X)\\ +(323/456)\sigma(\pi^+P\to\pi^0X)+(323/1824)\sigma(\pi^-P\to K^0X)\\ -(10925/8208)\sigma(K^-P\to\pi^-X)-(323/456)\sigma(\pi^-P\to\pi^0X)]\end{array}}=1 \tag{43a}$$

$$\frac{28\sigma(K^+P\to K^+X)}{\begin{array}{l}[4\sigma(\pi^+P\to\eta X)+(4/3)\sigma(\pi^+P\to\pi^0X)+(547/216)\sigma(K^-P\to\pi^-X)\\ +(710/27)\sigma(K^+P\to K^0X)+(95/36)\sigma(\pi^+P\to\pi^0X)+\\ (95/144)\sigma(\pi^-P\to K^0X)-\sigma(K^-P\to\pi^-X)-\sigma(K^-P\to\eta X)-(95/36)\sigma\\ \times(\pi^-P\to\pi^0X)]\end{array}}=1 \tag{45a}$$

$$\frac{\sigma(K^+P\to K^+X)}{(19/16)\sigma(K^+P\to K^0X)}=1 \tag{46a}$$

$$\frac{\sigma(K^-P\to K^-X)-39\sigma(K^+P\to K^+X)}{\begin{array}{l}[(3/2)\sigma(K^-P\to\bar{K}^0X)+(53/24)\sigma(\pi^-P\to\pi^0X)-2\sigma(\pi^+P\to\pi^0X)\\ -(3197/432)\sigma(K^-P\to\pi^-X)-(8387/216)\sigma(K^+P\to K^0X)\\ -(53/24)\sigma(\pi^+P\to\pi^0X)-(53/96)\sigma(\pi^-P\to K^0X)]\end{array}}=1 \tag{47a}$$

$$\frac{(1/3)\sigma(K^-P\to K^-X)+(1/3)\sigma(\pi^-P\to\pi^-X)+7\sigma(K^+P\to K^+X)}{\begin{array}{l}[\sigma(K^-P\to\eta X)+(11873/1296)\sigma(K^+P\to K^0X)\\ +(7/18)\sigma(\pi^-P\to\pi^0X)-(1/24)\sigma(K^+P\to K^0X)-\\ (169/162)\sigma(K^-P\to\pi^-X)-(7/18)\sigma(\pi^+P\to\pi^0X)-(7/72)\sigma(\pi^-P\to K^0X)]\end{array}}=1 \tag{48a}$$

$$\frac{28\sigma(K^+P\to K^+X)}{\begin{array}{l}[4\sigma(\pi^+P\to\eta X)+(4/3)\sigma(\pi^+P\to\pi^0X)+(1483/216)\sigma(K^-P\to\pi^-X)\\ (748/27)\sigma(K^+P\to K^0X)+(95/36)\sigma(\pi^+P\to\pi^0X)+\\ (95/144)\sigma(\pi^-P\to K^0X)-\sigma(K^+P\to K^0X)-\sigma(K^-P\to\eta X)-\\ (95/36)\sigma(\pi^-P\to\pi^0X)].\end{array}}=1 \tag{49a}$$

These ratios have also been recorded in table 4. We find that while the strangeness changing to strangeness zero transition are in the range 2·93–4·65, the diffractive to non-diffractive ones are in the range 0·62 – 2·48. The corresponding values with leading order term of CG[3] were 0·68–0·10 and 0·05–3·75 respectively.

3.4 *Pattern of symmetry breaking in the model*

In table 4, we observe that the typical ratios defined in (50a)–(52a) as well as (39a)–(49a) deviate away from unity. The results of (50a)–(52a) can be understood if the charm transitions cross-section parameters of (14) are much smaller than the corresponding parameters of light quark transitions. Besides, the light quark transition parameters themselves need to be subdivided into three categories to explain the deviation from unity in (39a)–(52a). They correspond to diagonal transitions, charge exchange and hypercharge/strangeness exchanges defined with subscripts P, Q and Y respectively. Hence the overall pattern of table 4 indicate that

$$\begin{aligned}&(A'_P, A''_P, B'_P, B''_P, C'_P, C''_P, D'_P, D''_P, E'_P, E''_P, F'_P, F''_P)\\ &(A'_Q, A''_Q, B'_Q, B''_Q, C'_Q, C''_Q, D'_Q, D''_Q, E'_Q, E''_Q, F'_Q, F''_Q)\\ &A'_Y, A''_Y, B'_Y, B''_Y, C'_Y, C''_Y, D'_Y, D''_Y, E'_Y, E''_Y, F'_Y, F''_Y)\\ &\gg (A'_C, A''_C, B'_C, B''_C, C'_C, C''_C, D'_C, D''_C, E'_C, E''_C, F'_C, F''_C)\end{aligned} \tag{53}$$

to be compared with (3.19) of (CG [3]). However with the enlargement of number of parameters with higher order term, one cannot get additional information of the parameters occurred in (53). Thus the underlying theory of quark dynamics should satisfy the phenomenological condition (53). However, as described in [3] it is possible to understand qualitatively, the relative suppression of charm production cross-section over others due to the negative intercept of the charmed trajectory [12].

3.5 *Phenomenological form of flavour symmetry breaking*

From table 4 and the previous analysis we find that the flavour symmetry is badly broken when more flavours are included. It implies that one should develop dynamical approaches taking into account the mass effects rather than symmetry alone. Such dynamical investigation needs knowledge of explicit $qq\bar{q}$ and qqq wave functions within the present model. Since it goes beyond the present symmetric model, we rather suggest plausible phenomenological form of flavour symmetry breaking as shown in table 4. Denoting the RHS of ratios (41a)–(52a) as the flavour symmetry breaking parameter f_{SB} we expect that asymptotically

$$\underset{E_{\mathrm{cm}}\to\infty}{\mathrm{Lt}}\, f_{\mathrm{SB}} = 1 \tag{54}$$

where the exact flavour symmetry will be restored. An analysis of table 4 then suggests that for the ratios involving the light quark transition, (41a)–(49a), f_{SB} can be parametrized as

$$f^{\mathrm{L}}_{\mathrm{SB}} \sim a_{\mathrm{L}}\left(\frac{m_u}{m_{\mathrm{S}}}\right)^{b_{\mathrm{L}}} \tag{55}$$

where $a_{\mathrm{L}} \sim 0{\cdot}81$–$0{\cdot}97$, and $b_{\mathrm{L}} \sim 1$–$-5{\cdot}5$, while for (50a)–(52a) involving charm quark

$$f^{H}_{\mathrm{SB}} \sim a_H\left(\frac{m_u}{m_{\mathrm{C}}}\right)^{b_H} \tag{56}$$

where $a_H \sim 0{\cdot}44$–$0{\cdot}54$, and $b_H \sim 5$–9.

In figure 2, we plot $f^{\mathrm{L}}_{\mathrm{SB}}$ and f^{H}_{SB} *vs* E_{CM} taking data from Flamino *et al* [10, 11].

Let us also consider our result in the context of the heavy flavour symmetry of the Isgur and Wise type [13, 14]. In the conventional $SU(N)_f$ flavour symmetry, the correction terms do not lend themselves to a reasonable perturbation estimates as

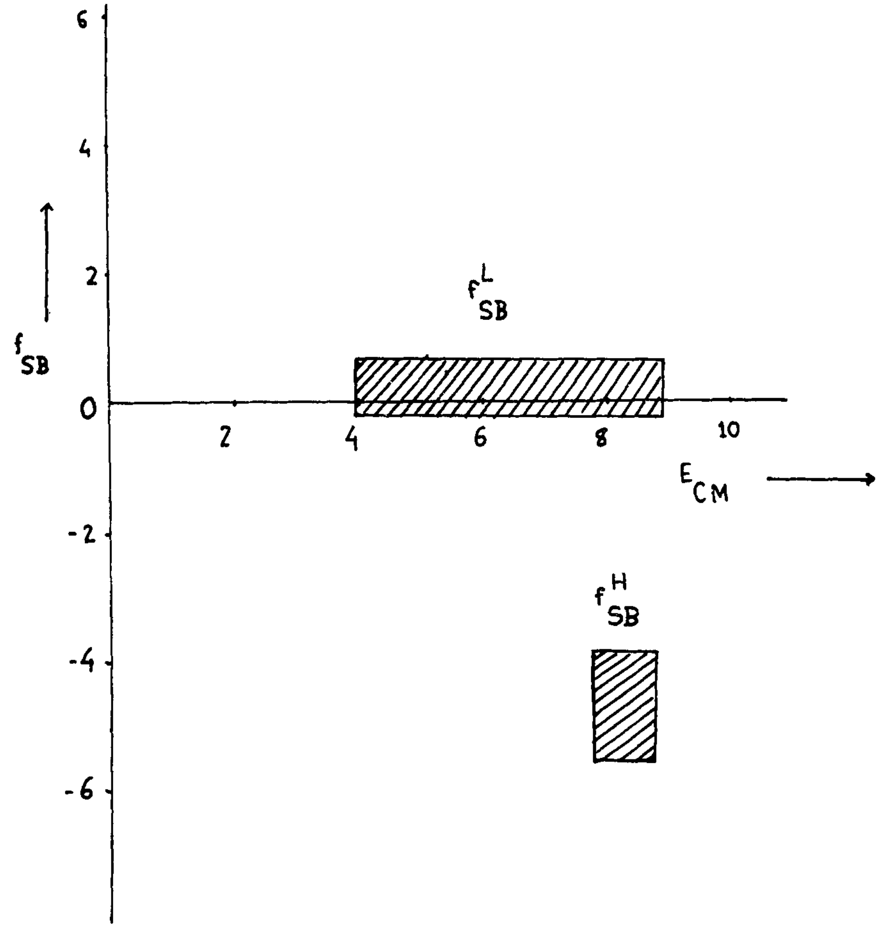

Figure 2. Flavour symmetry breaking functions f_{SB}^{L} and f_{SB}^{H} *vs* E_{CM} defined in (55) and (56) of text.

is evident from f_{SB}^{L} and f_{SB}^{H} defined in (55) and (56). However in Isgur-Wise symmetry, the correction terms are down by $1/m_H(Q)$ and hence small. In this context we also note that f_{SB}^{H} can have alternative parametrization

$$f_{SB}^{H} \sim \left(\frac{\Lambda_{QCD}}{m_C}\right)^{\alpha} \tag{57}$$

with $\Lambda_{QCD} = 0{\cdot}22\,\text{GeV}$ [15] and $\alpha \sim 4{\cdot}9$–7. This conforms to the present lore [16] that heavy flavour symmetry is broken by effects of order $\Lambda_{QCD}/m_H(m_H =$ heavy quark mass).

4. Bottom and top production in the model

$T_t^0(t\bar{t})$ and $\Upsilon(b\bar{b})$ are produced through double quark exchanges in the model as $\psi(C\bar{C})$. Hence the production processes for all the three varieties of heavy flavours can be related in the present model. But due to lack of data we desist from discussing them any further.

5. Summary and conclusion

We have studied the light and heavy flavour productions taking into account the double flavour exchange in a quark model pursued by us in recent years [1–4] which was proposed by Mitra earlier [5]. Such higher order effect is a necessity to produce heavy flavoured mesons like $\psi(c\bar{c})$, $\Upsilon(b\bar{b})$ and $T_t^0(t\bar{t})$ in the model. As flavour symmetry is found to be badly broken, we have attempted suitable parametrization of such symmetry breaking. The present model however falls short of their explicit evaluations as such an investigation needs knowledge of explicit $qq\bar{q}$ and qqq wave functions. Studies on such dynamical aspects of the model are in progress.

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