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Six quark cluster effects and Λ-binding energy difference between the mirror hypernuclei $^4_\Lambda$He and $^4_\Lambda$H

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Abstract. The lambda binding energy difference between two mirror hypernuclei $^4_\Lambda$He and $^4_\Lambda$H is studied by incorporating the quark structure of nucleons, specially the six quark cluster effects. A small contribution in the binding energy difference (59 keV in the non-relativistic quark model and 29 keV in the bag model) is obtained in the present work.

Keywords. Λ-binding energy; mirror hypernuclei; six-quark cluster.

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1. Introduction

The study of binding energy difference between mirror nuclei began quite a long time ago to understand the effect of charge symmetry breaking in NN interaction in nuclear structure and to account for the missing Coulomb energy problem (Nolen and Schiffer (1969)). Some recent studies in this connection are Blunden and Iqbal (1987); Brandenburg *et al* (1988) and Hatsuda *et al* (1990), among others. Such studies have been extended to mirror hypernuclei also (Friar and Gibson 1978; Gibson and Lehman 1979). The lightest pair of mirror hypernuclei $^4_\Lambda$He and $^4_\Lambda$H shows a noticeable difference of 350 keV in the ground state lambda binding energies (Juric *et al* 1973). Coulomb contribution to $\Delta B_\Lambda = B_\Lambda(^4_\Lambda\text{He}) - B_\Lambda(^4_\Lambda\text{H})$ is small (10–20 keV) and of opposite sign (Friar and Gibson 1978). When this contribution is added, the ΔB_Λ amounts to some 360–370 keV i.e. lambda particle is more bound in $^4_\Lambda$He than in $^4_\Lambda$H. This points towards the existence of charge symmetry breaking (CSB) ΛN interaction. Biswas *et al* (1980) calculated Λ-binding energy difference between $^4_\Lambda$He and $^4_\Lambda$H hypernuclei as 0·53 MeV on the basis of a theoretically derived charge symmetry breaking ΛN interaction. It is generally believed that traditional nuclear theories can give a reasonable explanation of the observed binding energy difference between mirror nuclei through the different aspects of CSB (Miller 1990). However, for a better understanding of Nolen-Schiffer (NS) anomaly it is worthwhile to study explicitly the role of quark structure of nucleons in the framework of low energy QCD in CSB processes. In fact since the last decade much work has been reported concerning the role of quark structure of nucleons in elucidating nuclear properties (sec Close 1979; Weise 1984; Richard 1992, for reviews on the subject).

The effect of six quark cluster on the binding energy difference between mirror nuclei has been investigated by Greben and Thomas (1984), Koch and Miller (1985a, b) and Wang *et al* (1988). Based on NRQM (Rujula *et al* 1975) such an approach considers two nucleons to maintain their identities and properties as long as the distance r between them is larger than some cut-off distance r_0. When two nucleons overlap i.e. $r < r_0$, the two-nucleon system is treated as a six-quark cluster (with interaction determined from low energy QCD, Rujula *et al* 1975). Formation and decay of multiquark states in three-nucleon bound system have been dealt with by Dijk and Bakker (1991). Maltman *et al* (1990) have investigated charge symmetry breaking in $A = 3$ nuclear and $A = 4$ hypernuclear isodoublets in terms of quantum-chromodynamically induced corrections to quantum electrodynamics. The present work is an attempt to determine the role of quarks, specially the six-quark cluster formation of overlapping nucleons on the lambda particle binding energy difference of ${}^4_\Lambda$He and ${}^4_\Lambda$H hypernuclei. Let us denote the mass of the six-quark system corresponding to two neutrons as $2\tilde{m}_n$ and that of two protons as $2\tilde{m}_p$. Koch and Miller (1985a) have found that $2\tilde{m}_n - 2\tilde{m}_p$ is less than the corresponding term $2m_n - 2m_p$ for free nucleons. This is caused by the colour magnetic hyperfine interaction between quarks. Koch and Miller (1985a) gave the value of $2\tilde{m}_n - 2\tilde{m}_p$ also in the MIT bag model. Such an effect based on quark model gives a correction of right sign to the NS anomaly of the mirror nuclei ^{3}H, ^{3}He. However, the magnitude of the effect depends upon the probability of formation of six-quark cluster in the nucleus. We have calculated this probability for ${}^4_\Lambda$He and ${}^4_\Lambda$H and have found the contribution of six-quark cluster effects to the corresponding Λ-binding energy difference. The paper is organized as follows. Section 2 contains the theory. Calculations are discussed in §3 and conclusions in §4.

2. Theory

Lambda particle binding energies in the ground states of ${}^4_\Lambda$He and ${}^4_\Lambda$H hypernuclei are given by

$$B_\Lambda({}^4_\Lambda\text{He}) = M({}^3\text{He}) + M_\Lambda - M({}^4_\Lambda\text{He}), \tag{1}$$

$$B_\Lambda({}^4_\Lambda\text{H}) = M({}^3\text{H}) + M_\Lambda - M({}^4_\Lambda\text{H}). \tag{2}$$

Again the binding energies of ^{3}He and ^{3}H are defined as

$$B({}^3\text{He}) = 2m_p + m_n - M({}^3\text{He}), \tag{3}$$

$$B({}^3\text{H}) = 2m_n + m_p - M({}^3\text{H}). \tag{4}$$

The theoretical calculation of the binding energy of a nucleus consists in calculating the total energy which is the sum of kinetic and potential energies of the nucleons in the nucleus minus the kinetic energy of the centre of mass. Binding energy is then the negative of this total energy. When the effect of six-quark cluster formation for internucleon distance $r < r_0$ is considered, an additional contribution $\delta B'$ to the binding energies of $A = 3$ nuclei and $A = 4$ hypernuclei results. We denote by P_{6q} the probability that a pair of neutrons/protons forms a six-quark cluster in $A = 3$ nuclei and by $\bar{P}_{6q}$ for $A = 4$ hypernuclei. Considering ^{3}He, the additional contribution to the binding energy is calculated as follows. With the possibility that six-quark cluster may be formed with a probability P_{6q} (Koch and Miller 1985a),

$$M({}^3\text{He}) = 2m_p(1 - P_{6q}) + 2\tilde{m}_p P_{6q} + m_n - \{B({}^3\text{He}) + \langle V_c \rangle_{\text{He}}\}. \tag{5}$$

$\langle V_c \rangle_{\mathrm{He}}$ measures how much Coulomb repulsion is lost by cutting the two-proton integral off at distances $r < r_0$. Due to this the binding energy in ^{3}He increases by $\langle V_c \rangle_{\mathrm{He}}$. Therefore

$$M(^3\mathrm{He}) = 2m_p + m_n - B(^3\mathrm{He}) - \delta B(^3\mathrm{He}), \tag{6}$$

where

$$\delta B(^3\mathrm{He}) = (2m_p - 2\tilde{m}_p)P_{6q} + \langle V_c \rangle_{\mathrm{He}} \tag{7}$$

is the change in the binding energy coming from quark structure contribution.

For ^{3}H nucleus we have similarly the relation,

$$M(^3\mathrm{H}) = 2m_n + m_p - \{B(^3\mathrm{H}) + \delta B(^3\mathrm{H})\}, \tag{8}$$

where

$$\delta B(^3\mathrm{H}) = (2m_n - 2\tilde{m}_n)P_{6q} + \langle V_c \rangle_{\mathrm{H}}. \tag{9}$$

For $A = 4$ hypernuclear systems, the additional six-quark cluster contributions to binding energies are expressed as

$$\delta B(^4_\Lambda\mathrm{He}) = (2m_p - 2\tilde{m}_p)\bar{P}_{6q} + \langle \bar{V}_c \rangle_{\mathrm{He}}, \tag{10}$$

and

$$\delta B(^4_\Lambda\mathrm{H}) = (2m_n - 2\tilde{m}_n)\bar{P}_{6q} + \langle \bar{V}_c \rangle_{\mathrm{H}}. \tag{11}$$

Like the $\langle V_c \rangle_{\mathrm{He,H}}$ term, $\langle \bar{V}_c \rangle_{\mathrm{He,H}}$ refers to the corresponding Coulomb energy terms for $^4_\Lambda$He and $^4_\Lambda$H hypernuclei. $\langle V_c \rangle_{\mathrm{H}}$, $\langle \bar{V}_c \rangle_{\mathrm{H}}$ terms are normally zero, but they may give rise to very small spurious contributions (1 keV) due to Coulomb interaction between two neutrons (Koch and Miller 1985a).

The Λ-binding energy difference of $^4_\Lambda$He and $^4_\Lambda$H hypernuclei according to (1) and (2) is,

$$B_\Lambda(^4_\Lambda\mathrm{He}) - B_\Lambda(^4_\Lambda\mathrm{H}) = \{M(^3\mathrm{He}) - M(^3\mathrm{H})\} - \{M(^4_\Lambda\mathrm{He}) - M(^4_\Lambda\mathrm{H})\}. \tag{12}$$

Concentrating on the effect of six-quark cluster formation only on the binding energy difference (denoted by ΔB_{6q}), we get from (12),

$$\begin{aligned}\Delta B_{6q} &= [B_\Lambda(^4_\Lambda\mathrm{He}) - B_\Lambda(^4_\Lambda\mathrm{H})]_{6q} \\ &= [\delta B(^3\mathrm{H}) - \delta B(^3\mathrm{He})] - [\delta B(^4_\Lambda\mathrm{H}) - \delta B(^4_\Lambda\mathrm{He})].\end{aligned} \tag{13}$$

The first and second terms on the RHS of (13) are derived from the first and second terms on the RHS of (12), respectively. Using (7), (9), (10) and (11), we obtain

$$\begin{aligned}\Delta B_{6q} = &\{(2m_n - 2m_p) - (2\tilde{m}_n - 2\tilde{m}_p)(P_{6q} - \bar{P}_{6q})\} \\ &+ \{\langle V_c \rangle_{\mathrm{H}} - \langle \bar{V}_c \rangle_{\mathrm{H}}\} + \{\langle \bar{V}_c \rangle_{\mathrm{He}} - \langle V_c \rangle_{\mathrm{He}}\}.\end{aligned} \tag{14}$$

Our calculations are based on (14).

3. Calculations

Calculation of P_{6q} for $A = 3$ nuclei has been reported by Koch and Miller (1985a,b). To calculate $\bar{P}_{6q}$ for $A = 4$ hypernuclei, we have used the wave-function of Dalitz *et al*

(1972) whose spatial part is of the form

$$\psi = \prod_{i>j}^{A-1} g(r_{ij}) \prod_{i=1}^{A-1} f(r_{i\Lambda}), \tag{15}$$

where

$$g(r) = u_g(r)/r, \quad r < d_g,$$
$$= A_g r^{n_g}[\exp(-\alpha_g r) + B_g \exp(-\beta_g r)], \quad r > d_g \tag{16}$$

u_g is a solution of the equation,

$$-\frac{\hbar^2}{2\mu_g}\frac{d^2}{dr^2}u_g(r) + [V_g(r) - e_g]u_g(r) = 0 \tag{17}$$

μ_g is the reduced mass of two nucleons.

The values of the constants, n_g, α_g, β_g have been given by Dalitz *et al* (1972). $f(r_{i\Lambda})$ satisfies an equation of form similar to (16). In the present work the differential equation, (17) has been solved numerically by Numerov's method. $\bar{P}_{6q}$ is then given by

$$\bar{P}_{6q} = 4\pi \int_{d_{NN}}^{r_0} r^2 g^2(r)\mathrm{d}r, \tag{18}$$

after normalizing $g(r)$. The value of the parameter d_{NN} of the NN potential is taken as 0·45 fm, from Dalitz *et al* (1972). The value of $r_0 = 1$ fm has been taken from Koch and Miller (1985a). Calculation of $\langle V_c \rangle_{\mathrm{He}}$ has been discussed in detail by Koch and Miller (1985a). For $r_0 = 1$ fm, $\langle V_c \rangle_{\mathrm{He}}$ is 93 keV. Calculation of $\langle V_c \rangle_{\mathrm{He}}$ for ${}^4_\Lambda$He is carried out with the normalized $g(r)$. $\langle \bar{V}_c \rangle_{\mathrm{He}}$ in our calculation is found to be 57·3 keV. Therefore, $\langle \bar{V}_c \rangle_{\mathrm{He}} - \langle V_c \rangle_{\mathrm{He}}$ comes to $-35{\cdot}7$ keV. The $\langle V_c \rangle_{\mathrm{H}} - \langle \bar{V} \rangle_{\mathrm{H}}$ term will be insignificant and is neglected.

Finally we consider the mass difference of six-quark clusters for two neutrons $(2\tilde{m}_n)$ and two protons $(2\tilde{m})_p$ which is $-0{\cdot}58$ MeV according to more realistic model of Koch and Miller (1985a). Following the MIT bag model (Thomas 1983), Koch and Miller (1985a) gave a value of 0·432 MeV for $2\tilde{m}_n - 2\tilde{m}_p$. Using these two values of $2\tilde{m}_n - 2\tilde{m}_p$, the calculated value of $\langle \bar{V}_c \rangle_{\mathrm{He}} - \langle V_c \rangle_{\mathrm{He}}$ and of $\bar{P}_{6q}$ (equation 18), the Λ-binding energy difference between ${}^4_\Lambda$He and ${}^4_\Lambda$H has been obtained by us for each case (see table 1).

Table 1. Six quark cluster effects on the Λ-binding energy difference between mirror hypernuclei ${}^4_\Lambda$He and ${}^4_\Lambda$H: The values of ΔB_{6q} obtained by us are shown in the last column of the table. Values of other quantities mentioned in the text are shown in other columns.

Model	$2\tilde{m}_n - 2\tilde{m}_p$ (MeV)	$2m_n - 2m_p$ (MeV)	P_{6q}	$\bar{P}_{6q}$ (Present calculation)	$\langle \bar{V}_c \rangle_{\mathrm{He}} - \langle V_c \rangle_{\mathrm{He}}$ (keV)	ΔB_{6q} (keV)
NRQM	−0·58*					59·06 (NRQM)
		2·6	0·064**	0·0342	−35·7	
MIT Bag	0·432*					29·20 (MIT Bag)

*Koch and Miller (1985a); **Koch and Miller (1985b).

4. Conclusions

It has been possible to account for a part (59 keV in the NRQM and 29 keV in the bag model) of the Λ-binding energy difference between mass 4 mirror hypernuclei, ${}^4_\Lambda\mathrm{He}$ and ${}^4_\Lambda\mathrm{H}$, by incorporating the quark structure of nucleons. The correction, as expected, is small and in the right direction. Here we have made a simple-minded estimate of the small six quark cluster effect. To improve upon the result a more realistic wavefunction of mass 4 hypernuclei than that used in the present work is required and we propose to study this later.

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