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Finite size nucleonic effects in the nuclear medium

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Abstract. The finite size effects of nucleons inside a nucleus is investigated. This new approach is entirely different from Hagedorn's volume correction method and is more rigorous. The size of the nucleon is varied and the magnitude of the hard-core potential is extracted by minimising the energy with respect to the nuclear radius.

Keywords. Finite size effects; volume correction.

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1. Introduction

It is customary to find in high energy collision theories, correction terms to volume arising due to the finite size of the constituent hadrons (Hagedorn and Rafelski 1981; Hagedorn 1983; Kapusta 1981). The recent investigations on dense hadronic matter (Ritchie *et al* 1991) depend on the various approximations made on the hard-core repulsive interactions. A consistent formalism of excluded volume effect in hadron gas at high temperatures had been given by Cleymans *et al* (1986) and Kuono and Takagi 1989. These correction terms are mostly empirical and lack physical content. In the present scenario of nuclear physics, where demarcation with particle physics had been obliterated, the swelling of nucleon bags inside the nuclear medium is one of the basic assumptions in explaining the EMC data (Aubert *et al* 1983). The thermodynamic implications of finite size nucleonic effects are found to have nontrivial effects in hadronic mass spectrum (Bhadhuri *et al* 1985). We feel that applying a volume correction to the available volume is an ad hoc procedure for the following reasons: (i) the particles do come closer than what their physical boundaries would permit and (ii) the particles would come closer if the energy is increased.

These effects are already known. The purpose of the present note is to incorporate these effects and obtain the height of the hard-core potential as a function of the hard-core radius and the attractive potential well depth, assuming the nuclear radius and the binding energy of the nucleus. We also extract the effective mass of the nucleon as a function of the attractive potential well depth.

2. The method

We start with the following expression for nuclear energy which takes into account the kinetic energy of the individual nucleons, the repulsive hard-core contribution and the attractive potential energy

$$E=\frac{3A(k_f)^2}{10M}+\frac{V_R A(A-1)b^3}{2R^3}\left(1-\frac{9b}{16R}+\frac{b^3}{32R^3}\right)-AV_0. \tag{1}$$

We picture the nucleus as an ideal Fermi gas of A nucleons, each of mass M, in a sphere of radius R, volume V and obeying Pauli exclusion principle. According to the theory of Fermi gas, the number of states of nucleons with momenta between p and $p+\mathrm{d}p$ in a volume V is $4(4\pi p^2\mathrm{d}p)\,V/(2\pi\hbar^3)$, where factor 4 stands for spin and isospin degeneracy. The ground state of the system will correspond to zero temperature with a momentum spread of zero to Fermi momentum $k_f=(3\pi^2A/2V)^{1/3}$, the kinetic energy being given by $T=3Ak_f^2/10M$, which is the first term in our energy equation. On treating this as the total energy of the nucleus, the positive sign of this quantity would clearly indicate the unsatisfactory nature of ideal Fermi gas model which can be improved by considering the nuclear matter as a gas of nucleons moving in a potential well of depth $-V_0$. This model of nuclear matter (a Fermi gas in a potential well) is similar to the free electron model in the theory of metals. It is well known that if the interaction of nucleons corresponds to attractive forces at all distances, then collapse of the nuclei should occur. Evidence for the existence of repulisve force between the nucleons at short distances is furnished by experiments on nucleon–nucleon scattering at high energies. The high energy nucleon–nucleon scattering data agree with the assumption that nucleons experience a strong repulsive force that does not permit them together more closely than a distance equal to the core radius $r_c=0{\cdot}4$ fm. Taking this into account, we obtain the total hard-core energy by summing over all pairs of nucleons and write the net hard-core energy as

$$E_R=\frac{A(A-1)PV_R}{2},$$

where V_R is the magnitude of the positive hard-core potential which acts between ${}^AC_2$ pairs of nucleons, when they approach each other within a hard-core radius b, the macroscopic probability of finding a pair of nucleons closer to b given by

$$P=\frac{1}{(4\pi R^3/3)^2}\int\!\!\int e(b-r_{12})\mathrm{d}^3r_1\mathrm{d}^3r_2 \tag{2}$$

where the function $e(x)$ is defined as

$$e(x)=1,\quad \text{for}\quad x>0$$

$$e(x)=0,\quad \text{for}\quad x<0$$

and r_{12} is the distance between the nucleons. The result of the integrations which runs through the interior of the nucleus gives

$$P=\frac{b^3}{R^3}\left(1-\frac{9b}{16R}+\frac{b^3}{32R^3}\right).$$

Combining all these and writing the total energy of the nucleus as the sum of kinetic energy and attractive and repulsive potential energies (equation 1) we find that the system is unstable since the minimum energy of the system occurs only for very large values of the nuclear radius ($R \gg 1{\cdot}2A^{1/3}$). Though there is a surface term in our equation, its contribution is not appreciable and hence to take care of this we add a confining term BV where B gets fixed by the energy minimization of the nucleus with respect to the radius R i.e.

$$\frac{\partial E}{\partial R} = 0. \tag{3}$$

On including the confining term, we modify our energy equation as

$$E = \frac{3A(k_f)^2}{10M} + \frac{V_R A(A-1)P}{2} - AV_0 + BV. \tag{4}$$

The physical significance of this confining term (BV) is to ensure the stability of the nucleus with its time evolution. One can refer to the quark shell model of nucleus (Hofestadt and Petry 1987), where this concept is dealt in detail.

3. Results and discussion

Using (3) and (4) and assuming the nuclear radius law, $R = 1{\cdot}2A^{1/3}$ and the binding energy per nucleon = 8 MeV, we solve for the two unknowns V_R and B. In figure 1, for some typical values of $V_0 = 45$, 50 and 55 MeV, we have plotted the hard-core potential V_R against the hard core radius b, where b is varied from 0·1 to 1 fm. We find that V_R decreases as b increases and this correct behaviour of the hard-core

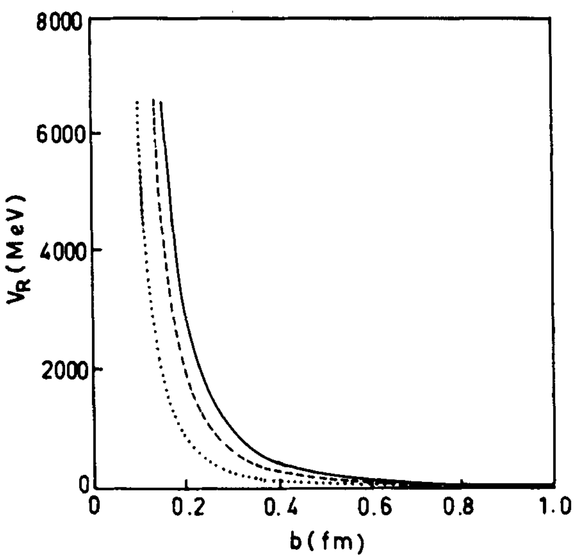

Figure 1. Hard-core potential (V_R) as a function of hard-core radius (b) for $V_0 = 45$ MeV (dotted line), 50 MeV (dashed line) and 55 MeV (continuous line).

potential justifies the way in which the finite size effects are incorporated. One should note that for b values of the order of 0·4 fm, $V_R \sim 200$ MeV, which is large when compared with the Fermi energy of the nucleus, and for $b < 0{\cdot}4$ fm, there is a steep rise in V_R, which indeed is the dimension of the little bag model for the individual nucleon. We also find that in the nuclear medium, the repulsive potential V_R increases with the well depth V_0. This could be anticipated as V_0 increases the kinetic energy of the nucleus should also increase to keep the equilibrium size in tact. These observations prove that applying of volume correction terms has no meaning as the particles are not billiard balls but fuzzy objects consisting of a quark core surrounded by a pion cloud (Myhrer and Wroldsen 1984a, b) their interactions being different in different energy regions. This idea had indeed been evinced by the little bag and cloudy bag models.

Now combining the first two terms of the energy equation, we write the net kinetic energy as

$$T_M^* = \frac{3A(k_f)^2}{10M} + \frac{V_R A(A-1)P}{2}, \tag{5}$$

which can be thought of as the kinetic energy arising from the nucleons of effective mass M^*. For values of V_0 and E mentioned earlier we get $M^* = 0{\cdot}82$ M, which any way would be a function of the potential well depth and for V_0 varying between 45 and 60 MeV, M^* ranges from 0·9 M to 0·7 M (Roy and Nigam 1972).

To conclude, we reiterate that the introduction of hard-core energy and the confining potential which give considerable energy contributions, had been necessitated to yield the required value of nuclear energy. Our model is quite simple but there are a few shortcomings in our theory. For instance, we have not included the quantal effects such as antisymmetrising the two body wavefunctions with respect to spin and isospin components, which would alter the probability distribution that we have used (equation 2). Also for the attractive potential energy, our mean potential well could be very well replaced by summing over the two body interactions. One can also make certain refinements to account for the variation in nuclear energy with mass number A. However our motivation is only to see the finite size nucleonic effects in the nucleus, and all these would not alter the physics involved in our theory.

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