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Effect of meson exchange current and three-body force on the charge form factor of ^{3}He

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Abstract. We report a calculation of meson exchange current contribution to the charge form factor of ^{3}He by the hyper spherical harmonic expansion method with the inclusion of two-pion exchange three nucleon force. Results indicate that the charge form factor is changed appreciably in the right direction at high momentum transfer, due to the inclusion of the meson exchange current.

Keywords. Few-nucleon systems; hyper spherical harmonics.

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Several theoretical attempts were made to obtain the charge form factor (CFF) of trinucleon systems from three-body calculations using standard two-body force. But the theoretical results do not agree well with the experimental numbers. Experimentally (McCarthy *et al* 1977) the first diffraction minimum for ^{3}He occurs at $q^2 \simeq 11{\cdot}8\,\mathrm{fm}^{-2}$, where q is the momentum transfer and a secondary maximum of magnitude ($F_{\max}$) about 6×10^{-3} at $q^2 \simeq 15{\cdot}65\,\mathrm{fm}^{-2}$. Calculations of CFF for trinucleon system using standard two-body force have been performed by many workers using Faddeev equations (Faddeev 1960; Harper *et al* 1972; Sasakawa and Sawada 1979) or the hyper spherical harmonic (HH) approach (Ballot and Fabre 1980; Bruinsma and Wageningen 1973; Erens *et al* 1971). But in all the cases the position of the dip is at a value of q^2 which is too large and the magnitude of secondary maximum is considerably too low and occurs at too large value of q^2. Several recent calculations include a three body force (3BF) in addition to the two-body force (Das and Coelho 1982; Das *et al* 1982a; Coelho *et al* 1982; Friar *et al* 1986; Sauer 1981; Sasakawa *et al* 1986; Schiavilla *et al* 1990; Strueve *et al* 1987). The binding energy improves appreciably with inclusion of 3BF, but no tangible improvement in CFF is found.

Kloet and Tjon (Kloet and Tjon 1974) suggested that the discrepancy between the experimental result and theoretical calculation of CFF arose due to the neglect of the exchange effects in which the photon interacts with a pion, which is exchanged between two nucleons. They calculated the effect of meson exchange current (MEC), while the trinucleon wave function was obtained by a momentum space Faddeev calculation using a standard two-nucleon force. They found that the position of the dip moves to a lower value of $q^2 (\simeq 12\,\mathrm{fm}^{-2})$ and the magnitude of the secondary maximum increases by about a factor of two. This demonstrated that at high momentum transfer, exchange of charged mesons plays an important role.

In this work we calculate the contribution of MEC to the CFF by the hyper spherical harmonic expansion (HHE) method. In addition to the two nucleon force (2BF), three nucleon force (3BF) of Fujita and Miyazawa (Fujita and Miyazawa 1957) with a suitable cut off parameter has been employed in the three body calculation. The HHE method (Ballot and Fabre 1980) deals directly with wave function in configuration space and inclusion of long-range, hardcore or many body interactions is straight forward in this method. Also, the structure of the equations remain unaltered due to the inclusion of such interactions.

In the HHE (Ballot and Fabre 1980) method the wave function is expanded in the complete basis of hyper spherical harmonics (HH) spanning the hyper angular space

$$\Psi(\mathbf{x}, \mathbf{y}) = r^{-5/2} \sum_{K\alpha} \Phi_{K\alpha}(r) \mathscr{P}_{K\alpha}(\Omega). \tag{1}$$

Here r is the hyper radial variable which is the invariant global length in a six-dimensional space and Ω represents a set of five hyper angles (Ballot and Fabre 1980). These are defined in terms of the particle co-ordinates $\mathbf{r}_i$ $(i = 1, 2, 3)$ through

$$\mathbf{x} = \mathbf{r}_2 - \mathbf{r}_1,$$

$$\mathbf{y} = \sqrt{\tfrac{2}{3}}(\mathbf{r}_3 - (\mathbf{r}_1 + \mathbf{r}_2)/2)$$

and

$$r = (x^2 + y^2)^{1/2}, \quad \Omega = \{x, y, \Phi\}$$

where

$$\Phi = \tan^{-1}(x/y). \tag{2}$$

The label $K\alpha$ stands for the five quantum numbers related to the five degrees of freedom in Ω. The complete orthonormal HH $(\mathscr{P}_{K\alpha}(\Omega))$ is the angular part of the homogeneous harmonic polynomial of degree K $(K = 0, 1, 2, \ldots, \infty)$ in the six dimensional space and is given by

$$\mathscr{P}_{K\alpha}(\Omega) = \sum_{m_x m_y} \langle \ell_x \ell_y m_x m_y | LM \rangle \, Y_{\ell_x m_x}(x) \, Y_{\ell_y m_y}(y) {}^{(2)}P_{2K}^{\ell_y, \ell_x}(\Phi) \tag{3}$$

where

$${}^{(2)}P_{2K}^{\ell_y, \ell_x}(\Phi) = N_{2K}^{\ell_y, \ell_x} (\sin \Phi)^{\ell_x} (\cos \Phi)^{\ell_y} P_n^{\ell_x + 1/2, \ell_y + 1/2}(\cos^2 \Phi) \tag{4}$$

$P_n^{\alpha\beta}(x)$ is a Jacobi polynomial and

$$N_{2K}^{\ell_y, \ell_x} = \left[\frac{4(K+1) n! (n + \ell_x + \ell_y + 1)!}{(n + \ell_x + \frac{3}{2})(n + \ell_y + \frac{3}{2})} \right]^{1/2} \tag{5}$$

Here $n = (2K - \ell_x - \ell_y)/2$ is an integer.

Substitution of (1) into the non-relativistic Schrödinger equation for three nucleons and projection onto a particular HH lead to a system of coupled differential equations (Ballot and Fabre 1980). Adiabatic approximation (Das *et al* 1982b; Levinger and Fabre 1981) has been used in order to decouple the system of differential equation. The CFF of ^3He is then calculated using the wave function thus obtained (Das *et al* 1982a). The exchange contribution of ^3He is given by (Kloet and Tjon 1974)

$$F_{\text{ch}}^{\text{ex}}(q^2) = \frac{-g^2}{2(4\pi\sqrt{m})^3} [F_v(q^2) + G_v(q^2) + 3F_s(q)^2 + 3G_s(q)^2] K(q^2). \tag{6}$$

The function $K(q^2)$ is defined as

$$K(q^2)=\int \mathrm{d}\mathbf{p}_1\,\mathrm{d}\mathbf{p}'_1\,\mathrm{d}\mathbf{q}_1\,\Phi_0(p_1,q_1)\Phi_0\left(p'_1,\left|\mathbf{q}_1-\frac{\mathbf{q}}{2\sqrt{3M}}\right|\right)$$
$$\times\frac{q^2-2(\mathbf{p}_1-\mathbf{p}'_1)\cdot\mathbf{q}\sqrt{M}}{[(\mathbf{p}_1-\mathbf{p}'_1)\sqrt{M}-\frac{1}{2}\mathbf{q}]^2+\mu^2} \tag{7}$$

where $\Phi_0(p,q)$ is the momentum space wave function for the S-state of trinucleon and $\mathbf{p}_1$ and $\mathbf{q}_1$ are relative momenta related to the particle momenta $\mathbf{K}_i(i=1,2,3)$ by

$$\mathbf{p}_1=\frac{1}{2\sqrt{M}}(\mathbf{K}_2-\mathbf{K}_3);\quad \mathbf{q}_1=\frac{1}{2\sqrt{3M}}(\mathbf{K}_2+\mathbf{K}_3-2\mathbf{K}_1),$$

M and μ are the nucleon (939 MeV) and pion (139·6 MeV) masses. The πNN coupling g has been taken equal to 13·6. F_v and F_s are respectively the isovector and isoscalar charge form factor of the nucleons, while G_v and G_s are its isovector and isoscalar magnetic form factors and are given in Janssens *et al* (1966).

The function $K(q^2)$ in (7) involves a nine-dimensional integral in momentum space. It can be reduced to more manageable form by using appropriate Fourier transforms in the following way. Introducing angle θ between the vectors $\boldsymbol{\Delta}$ and $\mathbf{q}$, where $\boldsymbol{\Delta}=(\mathbf{p}_1-\mathbf{p}'_1)\sqrt{M}$, we may write

$$K(q^2)=\int \mathrm{d}\mathbf{p}_1\,\mathrm{d}\mathbf{p}'_1\,\mathrm{d}\mathbf{q}_1\,\Phi_0(p_1,q_1)\Phi_0\left(p'_1,\left|\mathbf{q}_1-\frac{\mathbf{q}}{2\sqrt{3M}}\right|\right)$$
$$\times\frac{q^2(1-(2\Delta/q)\cos\theta)}{(\Delta^2+\frac{1}{4}q^2+\mu^2)-\Delta q\cos\theta}. \tag{8}$$

In the HHE approach, the trinucleon wave function (in co-ordinate space) is given by;

$$\Psi(\mathbf{x},\mathbf{y})=\sum_K\sum_{m_xm_y}\sum_{\ell_x\ell_y}\langle \ell_x m_x \ell_y m_y|LM\rangle\, Y_{\ell_x m_x}(\theta_x,\phi_x)$$
$$\times Y_{\ell_y m_y}(\theta_y,\Phi_y)\mathcal{N}_K u_K(r) r^{-5/2}{}^{(2)}P_{2K}^{\ell_y,\ell_x}(\Phi) \tag{9}$$

$\mathcal{N}_K={}^0N_{2K}{}^0F_{2K}^{\ell_y\ell_x}(\pi/2)$ is taken from the work of Ballot and Fabre (1980).

Now appropriate Fourier transforms are given by

$$\Phi_0(p_1,q_1)=\left[\frac{\sqrt{M}}{2\pi}\right]^3\int\Psi(\mathbf{x},\mathbf{y})\exp(i\mathbf{x}\cdot\mathbf{p}_1\sqrt{M})\cdot\exp(i\mathbf{y}\cdot\mathbf{q}_1\sqrt{M})\mathrm{d}^3x\,\mathrm{d}^3y \tag{10}$$

$$\frac{1-(2\Delta/q)\cos\theta}{(\mu^2+\frac{1}{4}q^2+\Delta^2)-\Delta q\cos\theta}=\frac{1}{(2\pi)^3}\int Y(\mathbf{x}'',\mathbf{q})\exp(-i\boldsymbol{\Delta}\cdot\mathbf{x})\mathrm{d}^3x. \tag{11}$$

Substituting these in (8), the integral reduces to

$$K(q^2)=\left(\frac{1}{\sqrt{M}}\right)^3\int\Psi(\mathbf{x},\mathbf{y})\Psi^*(\mathbf{x},\mathbf{y})\exp(i\mathbf{q}\cdot\mathbf{y}/2\sqrt{3})\,Y(\mathbf{x},\mathbf{q})\mathrm{d}^3x\,\mathrm{d}^3y. \tag{12}$$

In actual calculation we have considered for simplicity that the bound state is a pure S state (for which $L = M = 0$, $\ell_x = \ell_y = \ell = 0$)

$$\Psi(\mathbf{x}, \mathbf{y}) = \frac{1}{4\pi} \sum_K {}^0N_{2K}\, {}^0F^{00}_{2K}(\pi/2) r^{-5/2} u_K(r) {}^{(2)}P^{00}_{2K}(\Phi). \tag{13}$$

Substituting (13) in (12) and doing the angular integration one has

$$K(q^2) = \frac{4\pi^2 q}{M^{3/2}} \int \sum_{K,K'} \mathcal{N}_K \mathcal{N}_{K'} u_K(r) u_{K'}(r) r^{-5} {}^{(2)}P^{00}_{2K}(\Phi) {}^{(2)}P^{00}_{2K'}(\Phi)$$
$$\times j_0\left(\frac{qy}{2\sqrt{3}}\right) \frac{\exp(-\mu x)}{x} j_1(\tfrac{1}{2}xq) x^2 \mathrm{d}x y^2 \mathrm{d}y. \tag{14}$$

Where $j_n(x)$ is the spherical Bessel function of order n. Equation (14) expresses $K(q^2)$ as a two-dimensional integral, which is then evaluated numerically using the hyperradial wave functions $u_K(r)$, obtained by the HHE method.

For the nucleon form factors we take the analytic forms given in Das *et al* (1982a). For 2BF we have taken the semi-realistic S-projected Afnan and Tang S-3 potential (Afnan and Tang 1968) (which is appropriate, in view of the fact that we consider only the space totally symmetric S state of the trinucleon) and for 3BF the Fujita-Miyazawa (FM-3BF) form of two pion exchange three-body force (Fujita and Miyazawa 1957). Since the FM-3BF has a strong singularity (going as r^{-6} for $r \to 0$) and is attractive for the equilateral triangle configuration of the trinucleon, a phenomenological cut off parameter (x_0) is used (Das *et al* 1982a) to regularise the very short range behaviour of 3BF. The value of x_0 is chosen to be 0·277 fm as in reference (Das *et al* 1982a). So that no nodes appear in the hyperradial wave function $U_0(r)$, while $F_{\max}$ has the largest value.

In figures 1 and 2 $|F_{\text{ch}}(q^2)|$ has been plotted as a function of q^2. In figure 1 curve a represents the CFF for 2BF only and curve b represents CFF for 2BF including exchange correction. In figure 2, we plot the CFF of ^{3}He for 2BF plus FM-3BF without MEC contribution [curve a] and finally for 2BF plus FM-3BF together with MEC contribution [curve b].

Our calculation shows that the position of the first diffraction minimum ($q^2_{\min}$) moves slightly to the left when 3BF is included. The quantity $q^2_{\min}$ changes from 15·91 fm^{-2} with 2BF alone to 15·54 fm^{-2} when 2BF and 3BF (with coupling constant $C_p = 0{\cdot}46$ MeV) are included (calculated BE = 6·485 MeV). This behaviour is opposite to what has been observed in other calculations (Friar *et al* 1986; Sauer 1981). But, for $C_p = 0{\cdot}9$ MeV (calculated BE = 6·922 MeV), the first diffraction minimum moves slightly to the right ($q^2_{\min} = 16{\cdot}39$ fm^{-2}) (Das *et al* 1982a), in agreement with other calculations. This may be due to the fact that a larger cut off radius ($x_0 = 0{\cdot}340$ fm) is necessary for the stronger 3BF. This arises from the singular (and therefore unphysical) nature of the FM-3BF for very short separations. In any case, the change in the value of $q^2_{\min}$ is quite small.

Although the position of first diffraction minimum ($\simeq 15{\cdot}9$ fm^{-2}) does not change appreciably with the inclusion of 3BF only, the inclusion of MEC contribution has a dramatic effect. In the case of 2BF together with MEC [figure 1, curve (b)] the dip moves appreciably to the left to $q^2_{\min} \simeq 13{\cdot}4$ fm^{-2} and the magnitude of the secondary maximum increases to about 2×10^{-3} at $q^2 \simeq 18$ fm^{-2}. In the case of 2BF plus 3BF

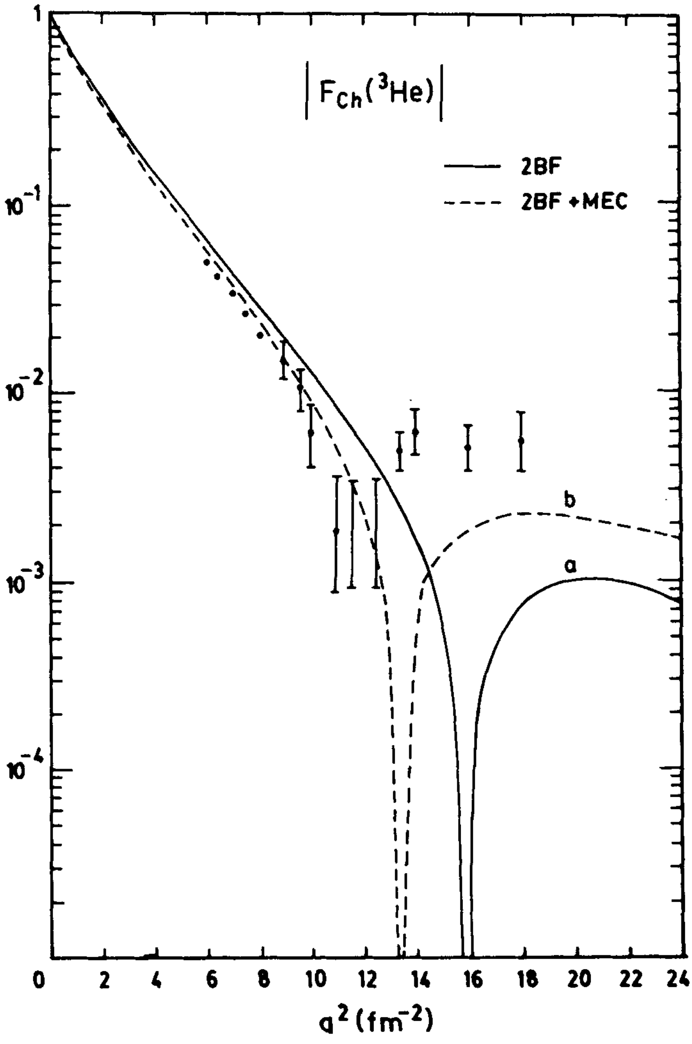

Figure 1. Absolute value of charge form factor of ^{3}He for 2BF respectively (a) without exchange correction and (b) with exchange correction. Experimental points are taken from Kloet and Tjon (1974) and McCarthy *et al* (1977).

including MEC contribution [figure 2, curve (b)] the position of the dip occurs at $q^2_{min} \simeq 13{\cdot}2\,fm^{-2}$. Furthermore F_{max} also increases to about 3×10^{-3} at $q^2 \simeq 18\,fm^{-2}$.

Thus even though the effect of 3BF alone is insignificant, the combined effect of 3BF and MEC changes CFF in the right direction by an appreciable amount. Although these still fall short of the experimental values, we hope that the numbers will be closer to experimental values when a realistic 2BF (which has sufficiently repulsive short range behaviour) is used. Furthermore one should use all other wave function components.

Except for the fact that the first diffraction minimum moves slight to the left with the inclusion of FM-3BF with $C_p = 0{\cdot}46$ MeV (which is probably due to rather unphysical nature of the FM-3BF), our results agree qualitatively with previous

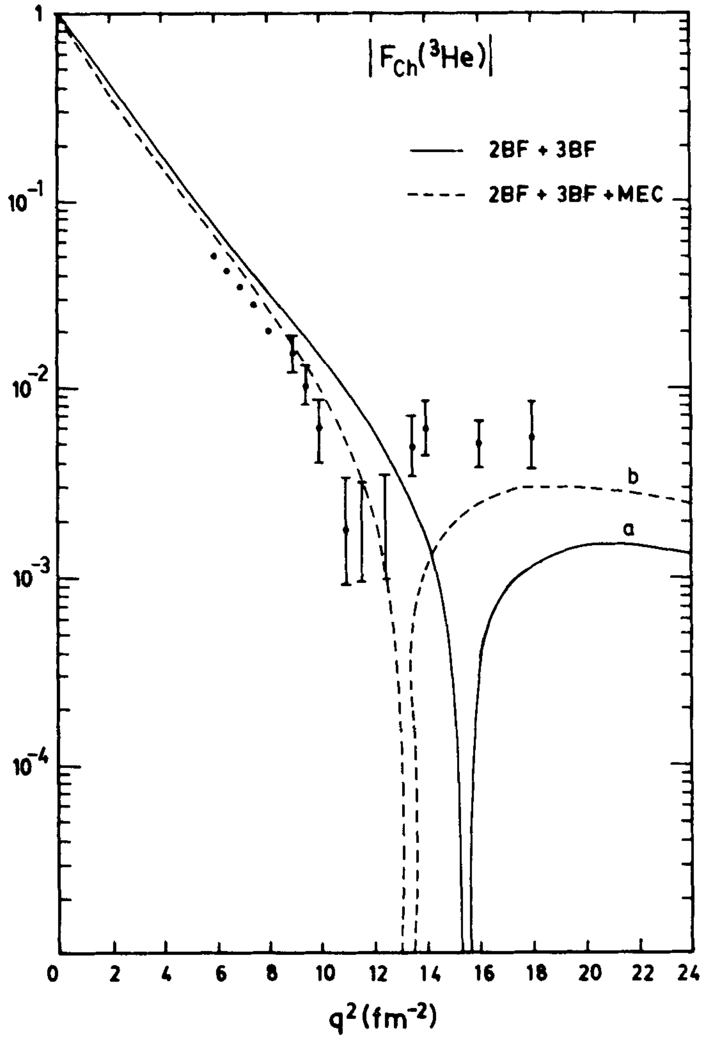

Figure 2. Absolute value of charge form factor of ^{3}He including 3BF ($C_p = 0{\cdot}46$ MeV, $x_0 = 0{\cdot}277$ fm) respectively (**a**) without exchange correction and (**b**) with exchange correction. Experimental points are taken from Kloet and Tjon (1974) and McCarthy *et al* (1977).

calculations (Kloet and Tjon 1974; Friar *et al* 1986; Sauer 1981; Sasakawa *et al* 1986; Schiavilla *et al* 1990).

However, all the previous calculations have used the Faddeev method. To our knowledge, this is the first calculation using the HHE method to study the effect of MEC and 3BF on ^{3}He charge form factor, and provides independent confirmation of the results.

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References

Afnan J R and Tang Y C 1968 *Phys. Rev.* **175** 1337
Ballot J L and Fabre R M 1980 *Ann. Phys.* **127** 62
Bruinsma Raud V and Wageningen R 1973 *Phys. Lett.* **B44** 221
Coelho H T, Das T K and Fabre R M 1982 *Phys. Lett.* **B109** 255
Das T K and Coelho H T 1982 *Phys. Rev.* **C26** 697
Das T K, Coelho H T and Fabre R M 1982a *Phys. Rev.* **C26** 2288
Das T K, Coelho H T and Fabre R M 1982b *Phys. Rev.* **C26** 2281
Erens G, Visschers J L and Van Wageningen R 1971 *Ann. Phys.* **67** 461
Faddeev L D 1960 *Zh. Eksp. Theor. Fis.* **39** 1459 (English Translation 1961 *Sov. Phys. JETP* **12** 1014)
Friar J L, Gibson B F, Payne G L and Chen C R 1986 *Phys. Rev.* **C34** 1463
Fujita J and Miyazawa H 1957 *Prog. Theor. Phys.* **17** 360
Harper E P, Kim Y E and Tubis A 1972 *Phys. Rev. Lett.* **28** 1533
Janssens T, Hofstadter R, Hughes E B and Yearian M R 1966 *Phys. Rev.* **B142** 922
Kloet W M and Tjon J A 1974 *Phys. Lett.* **B49** 419
Levinger J S and Fabre R M 1981 *Bull. Am. Phys. Soc.* **26** 34
McCarthy J S, Sick I and Whitney R R 1977 *Phys. Rev.* **C15** 1396
Sasakawa T *et al* 1986 *Czech. J. Phys.* **B36** 313
Sasakawa and Sawada T 1979 *Phys. Rev.* **C19** 2035
Sauer P U 1981 *Nucl. Phys.* **A463** 273C
Schiavilla R *et al* 1990 *Phys. Rev.* **C41** 309
Strueve W, Hajduk Ch, Sauer P U and Theis W 1987 *Nucl. Phys.* **A465** 651