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Screening of a $U(1) \times U(1)$ dyon by a domain wall in a double axion model

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Abstract. The screening of the electric-like charge degrees of freedom of a $U(1) \times U(1)$ dyon by an axion-like domain wall is discussed. Depending on the combination of degenerate minima at the interior and exterior boundary of the spherical domain wall a multiplicity of "screened generated electric-like $U(1) \times U(1)$ charges" are generated in the region exterior to the domain wall. Such a result demonstrates how a $U(1) \times U(1)$ dyon-domain wall configuration may generate multiplicate phenomena in the early universe from a single $U(1) \times U(1)$ source.

Keywords. Dyon; charge screening; domain wall.

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1. Introduction

The discovery in particle physics that particle interactions can be described by a gauge theory with masses generated through spontaneous symmetry breaking has led to a host of new phenomena that have deep rooted cosmological implications with regard to interactions in the early universe since the agent that breaks the gauge symmetry is a scalar multiplet whose field configurations about a vacuum state have well defined topological significance (Aitchison and Hey 1982). Monopoles (Albrecht and Turok 1985), cosmic strings (Kawano 1989), domain walls and texture (Vilenkin 1983) may all represent the generic seeds for large scale structure with both the topological properties of the defect and its geometrical location relative to surrounding matter being of importance in determining the dynamics of structure growth. One of the major accomplishments of inflationary cosmology was that it diluted the density of primitive monopoles generated around the time of GUT breaking to a level commensurate with present observational limits (Guth 1981). However monopoles might be formed long after the period of GUT breaking around the time of nucleosynthesis and might survive until the present epoch distributed in an inhomogeneous fashion throughout the universe (Hill 1983). If such monopoles or dyons do exist it would be of interest to ask how such configurations might be affected or screened by another defect such as a domain wall. In fact recent speculations concerning galaxy formation have suggested that domain walls created around the time of decoupling may be instrumental in generating density perturbations that in turn could give rise to the primitive origin of large scale structure (Goetz and Notzold 1991). If monopoles, dyons and domain walls are generated simultaneously following GUT breaking or decoupling the simultaneous presence of these defects and their effect on particle interactions would leave observable traces in the latter universe (a dyon is a monopole with electric charge degree of freedom). In a previous note we have discussed the screening of the electric charge degree of freedom of an abelian dyon by an axion cloud with the result that the axion cloud screens, anti-screens, or reverses the sign of the electric charge depending on the boundary conditions of the axion cloud (Wolf 1991). In the present note we replace the axion cloud by a domain wall with the domain wall generated by two axion-like pseudo-scalar fields. From a theoretical point of view such a model has a foundation rooted in string theory (Kim 1987). Our analysis uses a phenomenological lagrangian of two axion-like pseudo-scalars coupled to two $U(1)$ gauge fields. The main result of the analysis illustrates the electric charge screening phenomena of each $U(1)$ electric-like charge by the domain wall with multiple values of the screened charges resulting from the different combinations of degenerate minimum in the pseudo-scalar potential. Since different screened charges result for each combination of the degenerate minimum at the exterior and interior of the spherical domain wall, reaction rates of fundamental processes affected by external electric fields may be affected in an inhomogeneous fashion throughout the universe which might affect the local baryon asymmetry as well as the density of matter thus giving rise to dyon-domain wall induced density fluctuations. This result is unique to a double-axion-like-model coupled to two $U(1)$ gauge fields. Screening of electric charge by vacuum polarized positive charge is a feature of QED (Euler and Heisenberg 1936) and anti-screening in QCD is a feature generated by the non-abelian structure of the gauge fields which leads to the confining potential of QCD (Creutz 1980). The screening effect that we illustrate here is of a topological nature and may well have significance in early universe phenomena as mentioned above. It is in this spirit that the investigation should be taken, namely as a simple illustration of how one topological defect may affect another with the altering of particle phenomena exterior to the domain wall through the charge screening mechanism being the primary result.

2. Charge screening in a double axion-like model by a domain wall

We begin our analysis by writing down the following lagrangian of gravity, two $U(1)$ gauge fields and two axion-like pseudo-scalars

$$
\begin{aligned}
L = {} & \frac{c^4}{16\pi G} R(-g)^{1/2} + \\
& \left[\frac{\partial_\mu \phi_1 \partial^\mu \phi_1}{2} + \frac{\partial_\mu \phi_2 \partial^\mu \phi_2}{2} - \frac{A_2}{\bar{A}}\left(\phi_1^2 - \frac{A_1}{A_2}\right)^2 \right. \\
& - \frac{\bar{A}_2}{4}\left(\phi_2^2 - \frac{\bar{A}_1}{\bar{A}_2}\right)^2 + \alpha_1\left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{1\alpha\beta} F_{1\mu\nu}}{\sqrt{-g}}\right)\phi_1 \\
& + \alpha_2\left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{2\alpha\beta} F_{2\mu\nu}}{\sqrt{-g}}\right)\phi_2 + \alpha_3\left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{1\alpha\beta} F_{2\mu\nu}}{\sqrt{-g}}\right)\phi_1 \\
& \left. + \alpha_4\left(\frac{\varepsilon^{\mu\nu\alpha\beta} F_{1\alpha\beta} F_{2\mu\nu}}{\sqrt{-g}}\right)\phi_2 - \frac{1}{16\pi} F_{1\mu\nu} F_1^{\mu\nu} - \frac{1}{16\pi} F_{2\mu\nu} F_2^{\mu\nu} \right] \sqrt{-g} \\
= {} & L_G + L_M (\alpha_1, \alpha_2, \alpha_3, \alpha_4 \text{ are coupling constants}).
\end{aligned} \tag{1}
$$

The above lagrangian is a phenomenological generalization of that used in single axion models (Weinberg 1978) (Pecci and Quinn 1977). The kinetic terms are of the usual form for ϕ_1, ϕ_2. The two potentials have degenerative minimum at

$$\phi_1 = \pm\left(\frac{A_1}{A_2}\right)^{1/2}, \quad \phi_2 = \pm\left(\frac{\bar{A}_1}{\bar{A}_2}\right)^{1/2}$$

and the terms of the form

$$\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{1.2\alpha\beta}F_{1.2\mu\nu}}{\sqrt{-g}}\right)\phi_{1.2}$$

are effective terms which represent coupling between each pseudo-scalar and its corresponding $U(1)$ field, the mixed terms of the form

$$\frac{\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}F_{2\mu\nu}\phi_{1.2}}{\sqrt{-g}}$$

represents a pseudo-scalar coupled to both of the gauge fields. Also

$$F_{1\mu\nu} = \frac{\partial A_{1\mu}}{\partial x^\nu} - \frac{\partial A_{1\nu}}{\partial x^\mu}, \quad F_{2\mu\nu} = \frac{\partial A_{2\mu}}{\partial x^\nu} - \frac{\partial A_{2\nu}}{\partial x^\mu},$$

$A_{1\mu}$, $A_{2\mu}$ are the four vector potentials for the two $U(1)$ groups.

Varying (1) with respect to ϕ_1, ϕ_2 gives

$$-\Box\phi_1 - A_2\phi_1\left(\phi_1^2 - \frac{A_1}{A_2}\right) + \alpha_1\frac{\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}F_{1\mu\nu}}{\sqrt{-g}} + \alpha_3\frac{\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}F_{2\mu\nu}}{\sqrt{-g}} = 0 \quad (2)$$

$$-\Box\phi_2 - \bar{A}_2\phi_2\left(\phi_2^2 - \frac{\bar{A}_1}{\bar{A}_2}\right) + \alpha_2\frac{\varepsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}F_{2\mu\nu}}{\sqrt{-g}} + \alpha_4\frac{\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}F_{2\mu\nu}}{\sqrt{-g}} = 0. \quad (3)$$

Varying (1) with respect to $A_{1\mu}$, $A_{2\mu}$ gives

$$\frac{\partial}{\partial x^\nu}\left(\frac{\sqrt{-g}}{4\pi}F_1^{\mu\nu}\right) - \frac{\partial}{\partial x^\nu}(4\alpha_1\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}\phi_1) - \frac{\partial}{\partial x^\nu}(2\alpha_3\varepsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}\phi_1)$$
$$-\frac{\partial}{\partial x^\nu}(2\alpha_4\varepsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}\phi_2) = 0 \quad (4)$$

$$\frac{\partial}{\partial x^\nu}\left(\frac{\sqrt{-g}}{4\pi}F_2^{\mu\nu}\right) - \frac{\partial}{\partial x^\nu}(4\alpha_2\varepsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}\phi_2)$$
$$-\frac{\partial}{\partial x^\nu}(2\alpha_3\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}\phi_1) - \frac{\partial}{\partial x^\nu}(2\alpha_4\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}\phi_2) = 0. \quad (5)$$

We now station a $U(1) \times U(1)$ dyon with center ar $r = 0$, we also allow the effective radius of the dyon to be R_D, the inner radius of the domain wall is at R_1 and the outer radius is at R_2. For the region $R_D \leqslant r \leqslant R_1$ we neglect the variation of the pseudo-scalar fields (ϕ_1, ϕ_2), this will be a good approximation if the spatial separation

$(R_1 - R_D)$ is small $(\Delta R_{R_1} = R_1 - R_D)$, from eq. (2) we use the approximation $e^\lambda \simeq e^\nu \simeq 1$ in the spherically symmetric metric

$$(dS)^2 = e^\nu (dx^4)^2 - e^\lambda (dr)^2 - r^2 (d\theta)^2 - r^2 \sin^2\theta (d\phi)^2,$$

$$\frac{d}{dr}(r^2 \phi_{1,r}) = r^2 \left[A_2 \phi_1 \left(\phi_1^2 - \frac{A_1}{A_2} \right) - \alpha_1 8 E_1 B_1 - 4\alpha_3 (E_1 B_2 + E_2 B_1) \right]$$

or

$$r\phi_{1,r} - R_D^2 (\phi_{1,r})_{R_D} = \int_{R_D}^{r} r^2 \left[A_2 \phi_1 \left(\phi_1^2 - \frac{A_1}{A_2} \right) - \alpha_1 8 E_1 B_1 - 4\alpha_3 (E_1 B_2 + E_2 B_1) \right] dr$$

for $R_D < r < R_1$, we see that if $(\phi_{1,r})_{R_D} = 0$, then $(\phi_{1,r})_r$ $(R_D < r < R_1)$ will differ from $(\phi_{1,r})_{R_D} = 0$ by terms of order $\Delta R_r = (r - R_D)$, $(r < R_1)$ and higher, thus the approximation

$$\frac{d\phi_1}{dr} \equiv 0$$

is good in this region if ΔR_r is small, and consequently from a power series expansion about $r = R_D$ using (2) we have a negligible variation of ϕ_1 in the region $R_D \leqslant r \leqslant R_1$. This argument applies also to ϕ_2 from (3). Thus we set

$$\phi_1 = \sqrt{\frac{A_1}{A_2}}, \quad \phi_2 = \sqrt{\frac{\bar{A}_1}{\bar{A}_2}}$$

for $R_D \leqslant r \leqslant R_1$. For the region $R_2 \leqslant r$ we set

$$\phi_1 = -\sqrt{\frac{A_1}{A_2}}, \quad \phi_2 = -\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}$$

which are the second degenerate minimum of the pseudo-scalar fields with potential terms expressed in (1). We also neglect variation of the fields ϕ_1, ϕ_2 for $R_2 \leqslant r$ which will be a good approximation if R_2 is large compared to R_D since the electric-like and magnetic-like fields will fall off as $1/r^2$ and thus give little contribution to the spatial variation of ϕ_1 and ϕ_2 in (2) and (3) in this region. Also as stated the neglect of the spatial variation of ϕ_1, ϕ_2 for $R_D \leqslant r \leqslant R_1$ is a good approximation if R_1 and R_D are close to one another. For all $r > R_D$ we have no magnetic charge associated with each group and thus

$$\frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta} F_{1\alpha\beta}) = 0, \quad \frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta} F_{2\alpha\beta}) = 0 \tag{6}$$

giving

$$B_1 = \frac{q_1}{r^2}, \quad B_2 = \frac{q_2}{r^2} \tag{7}$$

for $r > R_D (q_1, q_2 =$ magnetic charges of each $U(1)$ group). Here we have used

$$F_{1_{14}} = E_1(r), \; F_{2_{14}} = E_2(r), \; F_{1_{23}} = r^2 \sin\theta B_1, \; F_{2_{23}} = r^2 \sin\theta B_2$$

where E_1, E_2, B_1, B_2 are the radial electric and magnetic fields for each $U(1)$ group. For the energy momentum tensor we have from (1)

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial L_M}{\partial g^{\mu\nu}} = \partial_\mu\phi_1\partial_\nu\phi_1 - \frac{g_{\mu\nu}}{2}(\partial_\alpha\phi_1\partial^\alpha\phi_1) + \partial_\mu\phi_2\partial_\nu\phi_2$$
$$-\frac{g_{\mu\nu}}{2}(\partial_\alpha\phi_2\partial^\alpha\phi_2) + g_{\mu\nu}\frac{A_2}{4}\left(\phi_1^2 - \frac{A_1}{A_2}\right)^2 + g_{\mu\nu}\frac{\bar{A}_2}{4}\left(\phi_2^2 - \frac{\bar{A}_1}{\bar{A}_2}\right)^2$$
$$+\frac{g_{\mu\nu}}{16\pi}F_{1\alpha\beta}F_1^{\alpha\beta} + \frac{g_{\mu\nu}}{16\pi}F_{2\alpha\beta}F_2^{\alpha\beta} - \frac{1}{4\pi}F_{1\mu\alpha}F_{1\nu}{}^{\alpha} - \frac{1}{4\pi}F_{2\mu\alpha}F_{2\nu}{}^{\alpha}. \qquad (8)$$

We now employ the spherically symmetric metric

$$(\mathrm{d}S)^2 = e^\nu(\mathrm{d}x^4)^2 - e^\lambda(\mathrm{d}r)^2 - r^2(\mathrm{d}\theta)^2 - r^2\sin^2\theta(\mathrm{d}\phi)^2,$$

for $R_D < r < R_1$ we have for the energy momentum tensor in this region $T_1^1 = T_4^4$ which in combination with the Einstein equations yields $\lambda + \nu = 0$ for $R_D < r < R_1$. Using this result we have for (4) and (5) for $R_D < r < R_1$

$$\frac{\partial}{\partial r}\left(\frac{r^2 E_1}{4\pi}\right) = 0,$$
$$\frac{\partial}{\partial r}\left(\frac{r^2 E_2}{4\pi}\right) = 0, \qquad (9)$$

giving

$$E_1 = \frac{e_1}{r^2}, \quad E_2 = \frac{e_2}{r^2}$$

for $R_D < r < R_1$. The other terms in (4) and (5) vanish for $R_D < r < R_1$ since ϕ_1, ϕ_2 are constant and B_1 and B_2 are given by (7) implying

$$\frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta}F_{1\alpha\beta}) = 0; \quad \frac{\partial}{\partial x^\nu}(\varepsilon^{\mu\nu\alpha\beta}F_{2\alpha\beta}) = 0$$

For $R_D < r < R_1$. Note e_1 and e_2 are dyon electric-like charges observed for $R_D < r < R_1$. For $R_D < r < R_1$, eq. (8) gives

$$T_4^4 = \frac{e_1^2}{8\pi r^4} + \frac{e_2^2}{8\pi r^4} + \frac{q_1^2}{8\pi r^4} + \frac{q_2^2}{8\pi r^4}. \qquad (10)$$

When eq. (10) is inserted in the $\binom{4}{4}$ Einstein equation we find after integration

$$e^{-\lambda} = e^\nu = 1 - \frac{2GM}{rc^2} + \frac{Ge_1^2}{r^2c^4} + \frac{Ge_2^2}{r^2c^4} + \frac{Gq_1^2}{r^2c^4} + \frac{Gq_2^2}{r^2c^4} \qquad (11)$$

for $R_D < r < R_1$, here $M =$ mass of central dyon. By a similar argument we have

$$B_1 = \frac{q_1}{r^2}, \; B_2 = \frac{q_2}{r^2}, \; E_1 = \frac{\bar{e}_1}{r^2}, \; E_2 = \frac{\bar{e}_2}{r^2}, \qquad \text{for } r > R_2 \qquad (12)$$

giving

$$e^{-\lambda} = e^{\nu} = 1 - \frac{2G(M + M_D)}{rc^2} + \frac{G\bar{e}_1^2}{r^2c^4} + \frac{G\bar{e}_2^2}{r^2c^4} + \frac{Gq_1^2}{r^2c^4} + \frac{Gq_2^2}{r^2c^4} \tag{13}$$

for $r > R_2$. (Here again we use $\lambda + \nu = 0$ for $r > R_2$ which follows from $T_1^1 = T_4^4$ for $r > R_2$.) We note that q_1, q_2 are the magnetic charges carried by the $U(1) \times U(1)$ dyon as observed for all $r > R_D$ since no magnetic charge is present for $r > R_D$; $\bar{e}_1, \bar{e}_2$ represents the effective electric charges of the dyon-domain wall configuration as observed for $r > R_2$(M_D = mass of the domain wall).

We next calculate the screened electric-like charges $\bar{e}_1$, $\bar{e}_2$ in terms of the dyon charges (e_1, e_2), q_1, q_2. Equations (4) and (5) can be integrated between R_1 and R_2 to give, using $\lambda + \nu = 0$ at $r = R_1$ and $r = R_2$,

$$\left[\frac{r^2E_1}{4\pi}\right]_{R_1}^{R_2} = [8\alpha_1 r^2 B_1 \phi_1]_{R_1}^{R_2} + [4\alpha_3 r^2 B_2 \phi_1]_{R_1}^{R_2} + [4\alpha_4 r^2 B_2 \phi_2]_{R_1}^{R_2} \tag{14}$$

$$\left[\frac{r^2E_2}{4\pi}\right]_{R_1}^{R_2} = [8\alpha_2 r^2 B_2 \phi_2]_{R_1}^{R_2} + [4\alpha_3 r^2 B_1 \phi_1]_{R_1}^{R_2} + [4\alpha_4 r^2 B_1 \phi_2]_{R_1}^{R_2}. \tag{15}$$

Using (7), (9) at $r = R_1$ and (12) at $r = R_2$ we have from (14) and (15), using the continuity of the electric fields at $r = R_1$, $r = R_2$ and the boundary values of ϕ_1, ϕ_2

$$\bar{e}_1 - e_1 = -64\pi\alpha_1 q_1\sqrt{\frac{A_1}{A_2}} - 32\pi\alpha_3 q_2\sqrt{\frac{A_1}{A_2}} - 32\pi\alpha_4 q_2\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}$$

$$\bar{e}_2 - e_2 = -64\pi\alpha_2 q_2\sqrt{\frac{\bar{A}_1}{\bar{A}_2}} - 32\pi\alpha_3 q_1\sqrt{\frac{A_1}{A_2}} - 32\pi\alpha_4 q_1\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}. \tag{16}$$

If we had chosen

$$\phi_1 = +\sqrt{\frac{A_1}{A_2}} \text{ at } r = R_1, \quad \phi_1 = -\sqrt{\frac{A_1}{A_2}} \text{ at } r = R_2$$

$$\phi_2 = -\sqrt{\frac{\bar{A}_1}{\bar{A}_2}} \text{ at } r = R_1, \quad \phi_2 = +\sqrt{\frac{\bar{A}_1}{\bar{A}_2}} \text{ at } r = R_2 \tag{17}$$

we would obtain a different set of values for the screened charges of $\bar{e}_1$, $\bar{e}_2$ in (16). There are four possibilities depending on the choice of ϕ_1 and ϕ_2 at the inner and outer boundaries of the domain wall.

We next develop an approximate solution for the scalar fields within the domain wall. We first approximate $e^{\nu} \approx e^{\lambda} \approx 1$ within the wall for $R_1 \leqslant r \leqslant R_2$, eqs (4) and (5) integrate to be for r between R_1 and R_2 where we use the boundary conditions

$$\phi_1 = \sqrt{\frac{A_1}{A_2}}, \quad \phi_2 = \sqrt{\frac{\bar{A}_1}{\bar{A}_2}} \text{ at } r = R_1, \quad \phi_1 = -\sqrt{\frac{A_1}{A_2}},$$

$$\phi_2 = -\sqrt{\frac{\bar{A}_1}{\bar{A}_2}} \text{ at } r = R_2,$$

$$\frac{r^2E_1}{4\pi}-\frac{e_1}{4\pi}=8\alpha_1q_1\phi_1-8\alpha_1q_1\sqrt{\frac{A_1}{A_2}}+4\alpha_3q_2\phi_1-4\alpha_3q_2\sqrt{\frac{A_1}{A_2}}$$
$$+4\alpha_4q_2\phi_2-4\alpha_4q_2\sqrt{\frac{\bar{A}_1}{\bar{A}_2}} \tag{18}$$

$$\frac{r^2E_2}{4\pi}-\frac{e_2}{4\pi}=8\alpha_2q_2\phi_2-8\alpha_2q_2\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}+4\alpha_3q_1\phi_1-4\alpha_3q_1\sqrt{\frac{A_1}{A_2}}$$
$$+4\alpha_4q_1\phi_2-4\alpha_4q_1\sqrt{\frac{\bar{A}_1}{\bar{A}_2}} \tag{19}$$

giving

$$E_1=\frac{e_1}{r^2}+\left[32\pi\alpha_1q_1\phi_1-32\pi\alpha_1q_1\sqrt{\frac{A_1}{A_2}}+16\pi\alpha_3q_2\phi_1-16\pi\alpha_3q_2\sqrt{\frac{A_1}{A_2}}+16\pi\alpha_4q_2\phi_2-16\pi\alpha_4q_2\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}\right]\frac{1}{r^2} \tag{20}$$

$$E_2=\frac{e_2}{r^2}+\left[32\pi\alpha_2q_2\phi_2-32\pi\alpha_2q_2\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}+16\pi\alpha_3q_1\phi_1-16\pi\alpha_3q_1\sqrt{\frac{A_1}{A_2}}+16\pi\alpha_4q_1\phi_2-16\pi\alpha_4q_1\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}\right]\frac{1}{r^2} \tag{21}$$

Equations (2) and (3) read for $R_1 \leqslant r \leqslant R_2$

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\phi_{1,r})-A_2\phi_1\left(\phi_1^2-\frac{A_1}{A_2}\right)-8\alpha_1E_1B_1-4\alpha_3(E_1B_2+E_2B_1)=0 \tag{22}$$

$$\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}(r^2\phi_{2,r})-\bar{A}_2\phi_2\left(\phi_2^2-\frac{\bar{A}_1}{\bar{A}_2}\right)-8\alpha_2E_2B_2-4\alpha_4(E_1B_2+E_2B_1)=0. \tag{23}$$

To develop a power series solution about $r=R_1$, we set

$$\phi_1(R_1)=\sqrt{\frac{A_1}{A_2}},\quad \left(\frac{\mathrm{d}\phi_1}{\mathrm{d}r}\right)_{R_1}=\bar{K}_1,\quad \phi_2(R_1)=\sqrt{\frac{\bar{A}_1}{\bar{A}_2}},\quad \left(\frac{\mathrm{d}\phi_2}{\mathrm{d}r}\right)_{R_1}=\bar{K}_2,$$

here we note that the derivatives of ϕ_1, ϕ_2 at $r=R_1$ as we approach the point $r=R_1$ from the right are not 0, but are given by $\bar{K}_1$ and $\bar{K}_2$ respectively (constants to be determined). The fact that the derivatives of ϕ_1, ϕ_2 are not continuous across the boundary $r=R_1$ and $r=R_2$ should not cause alarm since there is no physical restriction insisting that they should be. The values of the fields ϕ_1, ϕ_2 however are continuous across the boundaries (R_1, R_2).

Equations (22) and (23) give for the second derivative of ϕ_1, ϕ_2 at $r=R_1$

$$(\phi_{1,rr})_{R_1}=\frac{2}{R_1}\phi_1'(R_1)+(8\alpha_1E_1B_1)_{R_1}+4\alpha_3(E_1B_2+E_2B_1)_{R_1} \tag{24}$$

$$(\phi_{2,rr})_{R_1} = \frac{2}{R_1}(\phi_2'(R_1)) + (8\alpha_2 E_2 B_2)_{R_1} + 4\alpha_4(E_1 B_2 + E_2 B_1)_{R_1}. \tag{25}$$

In eqs (24) and (25), $\phi_1'(R_1) = \bar{K}_1$, $\phi_2'(R_1) = \bar{K}_2$; and E_1, B_1, E_2, B_2 are given by (7) and (9) at $r = R_1$. The power series solution for ϕ_1, ϕ_2 about $r = R_1$ is

$$\phi_1(r) = \phi_1(R_1) + \left(\frac{d\phi_1}{dr}\right)_{R_1}(r - R_1) + \frac{1}{2}\left(\frac{d^2\phi_1}{dr^2}\right)_{R_1}(r - R_1)^2 \ldots \tag{26}$$

$$\phi_2(r) = \phi_2(R_1) + \left(\frac{d\phi_2}{dr}\right)_{R_1}(r - R_1) + \frac{1}{2}\left(\frac{d^2\phi_2}{dr^2}\right)_{R_1}(r - R_1)^2 \ldots \tag{27}$$

we may calculate any derivative of ϕ_1, ϕ_2 from (22) and (23) by differentiating and thus obtain a power series solution as in (26), (27) to any order. To calculate $\bar{K}_1$, $\bar{K}_2$ approximately we take a finite number of terms in (26) and (27) and set

$$\phi_1 = -\sqrt{\frac{A_1}{A_2}}$$

at $r = R_2$,

$$\phi_2 = -\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}$$

at $r = R_2$ and solve for $\bar{K}_1$, $\bar{K}_2$.

To obtain the mass of the domain wall we first calculate the T_4^4 component of the energy momentum tensor for $R_1 \leqslant r \leqslant R_2$, in (8) we have approximately $e^{\lambda} \approx e^{\nu} \approx 1$ for $R_1 \leqslant r \leqslant R_2$, (8) becomes in this region

$$T_4^4 = \frac{1}{2}(\phi_{1,r})^2 + \frac{1}{2}(\phi_{2,r})^2 + \frac{A_2}{4}\left(\phi_1^2 - \frac{A_1}{A_2}\right)^2$$
$$+ \frac{\bar{A}_2}{4}\left(\phi_2^2 - \frac{\bar{A}_1}{\bar{A}_2}\right)^2 + \frac{E_1^2}{8\pi} + \frac{E_2^2}{8\pi} + \frac{B_1^2}{8\pi} + \frac{B_2^2}{8\pi}. \tag{28}$$

If we substitute (26) and (27) for ϕ_1, ϕ_2 into (28), and B_1, B_2 from (7) and E_1, E_2 from (20) and (21) into (28) we have an approximate expression for T_4^4 for $R_1 \leqslant r \leqslant R_2$. The $(^4_4)$ component of the Einstein equations reads

$$\frac{d}{dr}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} T_4^4 r^2. \tag{29}$$

Integrating (29) between R_1 and R_2 gives (boundaries of domain wall)

$$(re^{-\lambda})_{R_2} - (re^{-\lambda})_{R_1} = R_2 - R_1 - \frac{8\pi G}{c^4}\int_{R_1}^{R_2} r^2 T_4^4 \, dr \tag{30}$$

where we substitute (28) for T_4^4. In eq. (30) we substitute $(re^{-\lambda})_{R_1}$ from (11) and $(re^{-\lambda})_{R_2}$ from (13), this equation will then allow us to solve for M_D (mass of domain wall).

3. Conclusion

The above calculation has suggested a general approach to the problem involving multiple axion-like fields coupled to gauge fields in the vicinity of a dyon-like core. The fact that four different screened effective charges result depending on the choice of minima for the pseudo-scalar fields at the inner and outer boundary of the domain wall suggests that phenomena sensitive to such a domain wall-dyon core would experience multiplicate effects depending on the screened charges that result from the choice of minima for the two pseudo-scalar fields. Such phenomena might include dyon-induced plasma oscillations, Zeeman-like splittings of atomic hydrogen by the dyon-domain wall if the atomic hydrogen is present in the vicinity, and any particle decays that are sensitive to background electric and magnetic fields. Multiple signatures in particle decays might signal the multiple screened charges of the domain wall dyon core background that in turn would serve to identify double axion-like models surrounding a dyon core.

We also note from (16) that the differences $\bar{e}_1 - e_1, \bar{e}_2 - e_2$ depend on the strengths of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ which have dimensions of $(\mathrm{cm/erg})^{1/2}$ in $1/m$ ($c = \hbar = 1$) (Pecci and Quinn, 1977), or in an explicit model on the inverse of the vacuum expectation value at a high scale that gives masses to the heavy fields in the theory where these heavy fields have been integrated out of the theory. Or said differently, the vacuum expectation value is proportional to the heavy (integraged out masses) and thus we expect $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ to be small and the effective charges $\bar{e}_1, \bar{e}_2$ will differ little from e_1, e_2 from (16). We would thus expect that the above phenomena (dyon induced plasm oscillations, Zeeman-like splittings of the atomic hydrogen and various particle decays) would experience multiplicative signatures which differ little from the primary signatures and should be searched for through a very narrow window about the primary signatures. Thus the detection of the above multiplicative signatures would be a difficult task.

In addition to the production of monopoles around the time of nucleosynthesis as pointed out by Hill (1983), the production of CHAMPS (Dimopoulous *et al* 1990) (DeRujula *et al* 1990) (charged massive particles between 20 and 1000 TeV predicted by particle theory) might provide us with a viable component for dark matter in galatic halos, if such CHAMPS accumulated in the vicinity of the monopoles discussed in Hill (1983) we would have the components of a macroscopic dyon that may serve as the core of the above model. Since topological defects have received a lot of attention in the theoretical community it is important to pursue the above model of a domain wall-dyon core along with the multiplicative phenomena it might generate in the surrounding space such as multiple Zeeman splittings and multiple particle signatures from elementary processes in the dyon-domain wall background. This is the hope embodied in this investigation, namely to encourage work on more improved models that might more closely simulate the effective fields generated near a dyon core-domain wall configuration.

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References

Aitchison I J R and Hey A J G 1982 *Gauge theories in particle physics* (Adam Higler Ltd., Bristol)
Albrecht A and Turok N 1985 *Phys. Rev. Lett.* **54** 1868
Creutz M 1980 *Phys. Rev.* **D21** 2308
Dimopoulous S, Eicher B, Esmailzadah R and Starkman G D 1990 *Phys. Rev.* **D41** 2388
DeRujula A, Glashow S L and Sarid U 1990 *Nucl. Phys.* **B133** 173
Euler A and Heisenberg W 1936 *Z. Phys.* **98** 714
Goetz G and Notzold D 1991 *Nucl. Phys.* **B351** 645
Guth A 1981 *Phys. Rev.* **D23** 347
Hill C T 1983 *Proc. of NATO Advanced Study Inst. on Magnetic Monopoles*, Oct. 14–17, 1982 Wingspread, Wisconsin (New York: Plenum Press) p. 159
Kawano L 1989 *Ph.D. Thesis*, The University of Chicago
Kim J E 1987 *Proc. of 23rd Int. Conf. on High Energy Phys.*, July 16–23, 1986 Berkeley, CA (Singapore: World Scientific) p. 322
Pecci R D and Quinn H R 1977 *Phys. Rev. Lett.* **38** 1440
Vilenkin A 1983 *Phys. Lett.* **B133** 177
Weinberg S 1978 *Phys. Rev. Lett.* **40** 223
Wolf C 1991 *Acta Phys. Slovaca* **41** 109