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Non-adiabatic gravitational collapse of charged radiating fluid spheres

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Abstract. Dynamical equations governing the non-adiabatic collapse of a shear-free spherical distribution of unisotropic matter in the presence of charge are obtained. A brief outline of constructing a model describing collapse of a charged radiating fluid sphere in the set up developed is given.

Keywords. Non-adiabatic collapse; charged fluid spheres.

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1. Introduction

The state of a stellar structure is governed by the forces of gravitation, thermal processes and the processes of generation and transport of energy. The gravitational attraction is usually counterbalanced by the repulsive hydrostatic pressure of matter or the repulsive forces of electromagnetic interaction due to the presence of charge. When this equilibrium is lost the stellar structure goes into the inevitable state of gravitational collapse. Thermal processes which are ignorable on a short time scale are expected to play a significant role in the life of a star on astronomical scale.

The gravitational collapse of a spherical dust cloud for adiabatic flow under certain simplifying initial conditions was first investigated by Oppenheimer and Snyder (1939). Since then the study of relativistic models describing collapsing bodies has received considerable attention. Vaidya (1951, 1966), Lindquist *et al* (1965) studied outgoing radiation from collapsing spherical bodies. Misner and Sharp (1965) and Misner (1965) related the outgoing radiation in the exterior regions with the collapsing matter in the interior regions. Santos carried a realistic analysis of radiating spherical bodies of non-adiabatic matter in a state of gravitational collapse. His analysis is based on the models for shear-free collapsing fluids with heat flow suggested by Glass (1981). Santos and co-workers (de Oliveira *et al* 1985, 1986; de Oliveira and Santos 1987; Santos 1985) have used this set-up to propose relativistic models for studying collapse of a radiating star.

In this paper we have set up the equations governing the non-adiabatic collapse of shear-free spherical distribution of unisotropic charged matter in the presence of dissipative forces producing heat-flow in the radial directions and obtained the appropriate generalizations of the dynamical equations for spherical collapse obtained by de Oliveira and Santos (1987). This is followed by a brief discussion indicating how this set up provides a relativistic model for studying collapse of charged fluid spheres.

Following Israel (1966), the space-time is considered as separated into distinct region $\mathcal{M}_{(i)}$ and $\mathcal{M}_{(e)}$ by a time-like 3-space $\Sigma_{(b)}$ and this describes the motion of the spherical surface which acts as the boundary separating $\mathcal{M}_{(i)}$ and $\mathcal{M}_{(e)}$. The geometrical and physical quantities belonging to $\mathcal{M}_{(i)}$, $\mathcal{M}_{(e)}$ and $\Sigma_{(b)}$ are respectively indicated by writing bracketed suffixes (i), (e) and (b) with them whenever necessary.

2. The interior space-time

The space-time in the interior of non adiabatically collapsing charged fluid sphere accompanied by a heat-flow in the radial directions, denoted as $\mathcal{M}_{(i)}$, is considered to be represented by the spherically symmetric space-time metric

$$ds^2 = -\exp[\mu(t)][\exp[\lambda(r)]dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] + \exp[\nu(r,t)]dt^2. \tag{1}$$

When $\mu = 0$, the co-ordinates are the usual Schwarzschild co-ordinates used for describing static, equilibrium models of fluid spheres.

The physical content of the space-time region $\mathcal{M}_{(i)}$ is described by the energy-momentum tensor

$$T^\alpha_\beta = (\rho + p)u^\alpha u_\beta - p\delta^\alpha{}_\beta + \pi^\alpha{}_\beta + q^\alpha u_\beta + q_\beta u^\alpha + E^\alpha{}_\beta. \tag{2}$$

In (2), ρ denotes matter density, p isotropic fluid pressure, u^α components of unit time-like flow vector field of matter and q^α components of the space-like radial heat flux vector orthogonal to u^α. Further π^α_β and E^α_β denote respectively the unisotropic pressure tensor and energy-momentum tensor of the electromagnetic field. The metric variables λ, μ and ν are related to the physical variables through the relativistic field equations

$$R^\alpha_\beta - (\tfrac{1}{2})R\delta^\alpha{}_\beta = -8\pi T^\alpha{}_\beta, \tag{3}$$

in the system of units rendering $c = 1$ and $G = 1$.

For an observer following the motion of the fluid i.e., in a co-moving co-ordinate system

$$u^\alpha = (0, 0, 0, \exp[-\nu/2]) \tag{4}$$

and the anisotropic pressure tensor $\pi^\alpha{}_\beta$ has explicit expression

$$\pi^\alpha{}_\beta = \sqrt{3}A(r,t)[C^\alpha C_\beta + \tfrac{1}{3}(\delta^\alpha{}_\beta - u^\alpha u_\beta)] \tag{5}$$

with

$$C^\alpha = (-\exp[(\lambda + \mu)/2], 0, 0, 0). \tag{6}$$

$A(r,t)$ denotes the magnitude of this tensor.

The energy-momentum tensor of electromagnetic field is

$$E^\alpha{}_\beta = \left(\frac{1}{4\pi}\right)\left[F_{\gamma\beta}F^{\alpha\gamma} + \left(\frac{1}{4}\right)\delta^\alpha{}_\beta F^{ab}F_{ab}\right] \tag{7}$$

where $F_{\alpha\beta}$ denotes the components of Maxwell stress tensor which satisfy Maxwell's equations

$$F_{\alpha\beta,\gamma} + F_{\beta\gamma,\alpha} + F_{\gamma\alpha,\beta} = 0, \tag{8}$$

$$F^{\alpha\beta}{}_{;\beta} = 4\pi\sigma u^\alpha, \tag{9}$$

where σ denotes charge density. For electromagnetic fields with spherical symmetry, eqs (8) and (9) imply that $F^{\alpha\beta}$ is expressible in terms of a four potential $\psi_\alpha = (0, 0, 0, \psi)$ as

$$F_{\alpha\beta} = \psi_{\alpha,\beta} - \psi_{\beta,\alpha}$$

and it has the only surviving component $F_{14} = -F_{41} = -\partial\psi/\partial r$.

Further (9) leads to the relations

$$\frac{\partial\psi}{\partial r} = -\frac{\ell(r)}{r^2}\exp[(\lambda + \nu - 3\mu)/2] \tag{10a}$$

$$4\pi\sigma = -\frac{1}{r^2}\frac{\mathrm{d}\ell}{\mathrm{d}r}\exp[-(\lambda + 3\mu)/2], \tag{10b}$$

where $\ell(r)$ is a function of r only. Accordingly it may be noted that the variation of charge density with time is determined by the metric variable $\mu(t)$.

The heat flux vector q_α, for non-adiabatic radial heat flow in $\mathcal{M}_{(i)}$ is the space-like vector

$$q_\alpha = (-\exp[\lambda + \mu]q, 0, 0, 0) \tag{11}$$

with magnitude $q = q(r, t)$. Let p_r and $p_\perp$ defined by the relations

$$p_r = p + (2A/\surd 3),\ p_\perp = p - (A/\surd 3) \tag{12}$$

represent the fluid pressure in the radial and the transverse directions respectively.

The relativistic field equation (3) subsequently provides the system of four equations as explicitly stated below:

$$8\pi T^1{}_1 = -8\pi p_r + \frac{\ell^2}{r^4}\exp[-2\mu] = -\exp[-(\lambda+\mu)]\left[\frac{\nu'}{r} + \frac{1}{r^2}\right] + \frac{\exp[-2\mu]}{r^2} + \exp[-\nu]\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{1}{2}\dot{\mu}\dot{\nu}\right], \tag{13}$$

$$8\pi T^2{}_2 = -8\pi p_\perp - \frac{\ell^2}{r^4}\exp[-2\mu] = -\exp[-(\lambda+\mu)] - \left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda' + \nu'}{4} - \frac{\lambda'}{2r} + \frac{\nu'}{2r}\right] + \exp[-\nu]\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{1}{2}\dot{\mu}\dot{\nu}\right], \tag{14}$$

$$8\pi T^3{}_3 = 8\pi T^2{}_2$$

$$8\pi T^4{}_4 = 8\pi\rho + \frac{\ell^2}{r^4}\exp[-2\mu] = \exp[-(\lambda+\mu)]\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{\exp[-\mu]}{r^2} + \frac{3}{4}\exp[-\nu]\dot{\mu}^2, \tag{15}$$

$$8\pi T^1{}_4 = 8\pi q\exp[\nu/2] = -\tfrac{1}{2}\exp[-(\lambda+\nu)]\dot{\mu}\nu'. \tag{16}$$

Here and in what follows an overhead prime and an overhead dot denote differentiations with respect to r and t respectively. Equation (14) when subtracted from (13) leads to

$$8\pi(p_r - p_\perp) = -\exp[-(\lambda+\mu)]\left[\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} - \frac{\lambda'}{2r} - \frac{\nu'}{2r} - \frac{1}{r^2}\right] - \frac{\exp[-\mu]}{r^2}\left[1 - \frac{2\ell^2}{r^2}\right]. \tag{17}$$

This equation determines the measure of anisotropy of pressure $(p_r - p_\perp)$ at all points of the interior space-time region $\mathscr{M}_{(i)}$.

3. The exterior space-time

The space time in the exterior region, denoted as $\mathscr{M}_{(e)}$, of the non-adiabatically collapsing charged fluid sphere is expected to be filled with radiation and is appropriately described by Reissner-Nordström-Vaidya metric

$$ds^2_{(e)} = -\mathscr{Y}^2 d\theta^2 - \mathscr{Y}^2 \sin^2\theta d\phi^2 + \left(1 - \frac{2m}{\mathscr{Y}} + \frac{Q^2}{y^2}\right)dv^2 + 2dv d\mathscr{Y}, \tag{18}$$

where $M = M(v)$ denotes the total mass enclosed within the spherical region of radius $\mathscr{Y}$, and Q is the constant representing total charge associated with the collapsing body. The energy-momentum tensor in $\mathscr{M}_{(e)}$ is denotes by

$$T^\alpha_\beta = \varepsilon\zeta^\alpha\zeta_\beta + \frac{1}{4\pi}\left[F_{\gamma\beta}F^{\alpha\gamma} + \frac{1}{4}\delta^\alpha{}_\beta F_{\mu\nu}F^{\mu\nu}\right], \tag{19}$$

where $\zeta_\alpha = (0, 0, 0, 1)$. The non-zero components of T^α_β are

$$T^1{}_1 = -T^2{}_2 = -T^3{}_3 = T^4{}_4 = \frac{Q^2}{8\pi\mathscr{Y}^2}, \quad T^1{}_4 = \varepsilon - \frac{1}{\mathscr{Y}^2}\frac{dM}{dv} \tag{20}$$

where ε represents the density of radiation energy.

4. Boundary conditions

The space-time regions $\mathscr{M}_{(i)}$ and $\mathscr{M}_{(e)}$ are joined on their boundary of separation by stipulating the junction conditions

$$ds^2_{(i)} = ds^2_{(e)} = ds^2_{(b)} \tag{21}$$

$$\mathscr{K}_{\mu\nu(i)} = \mathscr{K}_{\mu\nu(e)}, \tag{22}$$

given by Israel, on the boundary $\Sigma_{(b)}$. The intrinsic metric $ds^2_{(b)}$ on the boundary surface $\Sigma_{(b)}$ is expressible as

$$ds^2_{(b)} = -\mathscr{R}^2(\tau)(d\theta^2 + \sin^2\theta d\phi^2) + d\tau^2. \tag{23}$$

$\mathcal{K}_{\mu\nu(i)}$ and $\mathcal{K}_{\mu\nu(e)}$ respectively denote the extrinsic curvatures of $\Sigma_{(b)}$ expressed in terms of respective unit space-like normal vectors to $\Sigma_{(b)}$, $n_{\alpha(i)}$ in $\mathcal{M}_{(i)}$ and $n_{\alpha(e)}$ in $\mathcal{M}_{(e)}$ by the defining equations

$$\mathcal{K}_{\mu\nu} = -n_\alpha \frac{\partial^2 x^\alpha}{\partial\zeta^\mu \partial\zeta^\nu} - n_\alpha \Gamma^\alpha_{jk} \frac{\partial x^j}{\partial\zeta^\mu}\frac{\partial x^k}{\partial\zeta^\nu}. \tag{24}$$

In (24) ζ^μ are the co-ordinates θ, ϕ, τ on $\Sigma_{(b)}$ and x^α are the co-ordinates appropriate to $\mathcal{M}_{(i)}$ and $\mathcal{M}_{(e)}$. The unit space-like normals $n_{\alpha(i)}$ and $n_{\alpha(e)}$ to $\Sigma_{(b)}$ in $\mathcal{M}_{(i)}$ and $\mathcal{M}_{(e)}$ are given by

$$n_{\alpha(i)} = (\exp[(\lambda + \mu)/2], 0, 0, 0) \tag{25}$$

$$n_{\alpha(e)} = \left(\frac{\partial v}{\partial\tau}, 0, 0, -\frac{\partial y}{\partial\tau}\right). \tag{26}$$

The extrinsic curvatures $\mathcal{K}_{\mu\nu(i)}$ and $\mathcal{K}_{\mu\nu(e)}$ have the expressions

$$\mathcal{K}_{\tau\tau(i)} = -\tfrac{1}{2}[\exp[-(\lambda+\mu)/2]\nu']_{(b)} \tag{27}$$

$$\mathcal{K}_{\theta\theta(i)} = \mathcal{K}_{\phi\phi(i)}/\sin^2\theta = [r\exp[(\mu-\lambda)/2]]_{(b)} \tag{28}$$

$$\mathcal{K}_{\tau\tau(e)} = \left[\frac{1}{v}\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} - \left(\frac{M}{y^2} - \frac{Q^2}{y^3}\right)\frac{\mathrm{d}y}{\mathrm{d}\tau}\right]_{(b)}, \tag{29}$$

$$\mathcal{K}_{\theta\theta(e)} = \mathcal{K}_{\phi\phi(e)}/\sin^2\theta = \left[y\frac{\mathrm{d}y}{\mathrm{d}\tau} + y\frac{\mathrm{d}v}{\mathrm{d}\tau}\left\{1 - \frac{2M}{y} + \frac{Q^2}{y^2}\right\}\right]_{(b)}, \tag{30}$$

$$\mathcal{K}_{\mu\nu(i)} = \mathcal{K}_{\mu\nu(e)} = 0 \text{ for } \mu \neq \nu. \tag{31}$$

The junction conditions (21) imply the following relations:

$$r_{(b)}\exp[\mu/2] = \mathcal{R}(\tau),\ \frac{\mathrm{d}t}{\mathrm{d}\tau} = \exp[-\nu/2],\ y_{(b)} = \mathcal{R}(\tau),$$

$$\left(\frac{\mathrm{d}y}{\mathrm{d}\tau}\right)^2_{(b)} = \left[2\frac{\mathrm{d}y}{\mathrm{d}v} + 1 - \frac{2M}{y} + \frac{Q^2}{y^2}\right]^{-1}_{(b)}. \tag{32}$$

The condition (22) establishing continuity of the extrinsic curvatures of the boundary $\Sigma_{(b)}$, implies

$$[-\tfrac{1}{2}\exp[-(\lambda+\mu)/2]\nu']_{(b)} = \left[\frac{1}{v}\frac{\mathrm{d}^2 y}{\mathrm{d}\tau^2} - \left\{\frac{M}{y^2} - \frac{Q^2}{y^2}\right\}\frac{\mathrm{d}y}{\mathrm{d}\tau}\right] \tag{33}$$

$$[r\exp((\mu-\lambda)/2)]_{(b)} = \left[y\frac{\mathrm{d}y}{\mathrm{d}\tau} + y\frac{\mathrm{d}v}{\mathrm{d}\tau}\left(1 - \frac{2M}{y} + \frac{Q^2}{y^2}\right)\right]. \tag{34}$$

5. Implications of boundary conditions

The total mass $M(v)$ of the collapsing sphere containing non-adiabatic charged matter, enclosed within the boundary $\Sigma_{(b)}$ can be obtained by eliminating $\mathrm{d}y/\mathrm{d}\tau$ from (34) using the relations (32). It is found that

$$M(v) = \tfrac{1}{2} r_{(b)} \exp[\mu/2] \left[1 - \exp[-\lambda] + \frac{1}{4} r^2 \exp[\mu - v] \dot{\mu}^2 + \frac{Q^2}{r^2} \exp[-\mu] \right]_{(b)} \tag{35}$$

Let

$$m(r,t) = \tfrac{1}{2} r \exp[\mu/2] \left[1 - \exp[-\lambda] + \frac{1}{4} r^2 \exp[\mu - v] \dot{\mu}^2 + \frac{Q^2}{r^2} \exp[-\mu] \right] \tag{36}$$

be considered as total mass of matter enclosed up to radius $r \leqslant r_{(b)}$. Using the field equations (13) and (16) the gradient $\partial m/\partial r$ is expressible in the form

$$\frac{\partial m}{\partial r} = 4\pi \rho r^2 \exp[3\mu/2] + 2\pi q r^3 \dot{\mu} \exp[(5\mu + 2\lambda - v)/2] + \tfrac{1}{2} \ell \ell' \exp[-\mu/2]. \tag{37}$$

The total mass enclosed within $\Sigma_{(b)}$ is observed to be

$$\begin{aligned} M(v) &= \int_0^{r_{(b)}} \frac{\partial m}{\partial r} \mathrm{d}r \\ &= m_0(t) + \int_0^{r_{(b)}} [4\pi r^2 \exp[3\mu/2] + 2\pi \dot{\mu} q r^3 \exp[(5\mu + 2\lambda - v)/2] \\ &\quad + (\ell^2/2r^2) \exp[-\mu/2]] \mathrm{d}r + \frac{1}{2} \frac{Q^2}{r_{(b)}} \exp[-\mu/2] \end{aligned} \tag{38}$$

where $m_0(t)$ is arbitrary function of time which is zero for the distributions without singularities within. We set $m_0(t) = 0$.

Eliminating $\mathrm{d}\mathscr{y}/\mathrm{d}\tau$ and $\mathrm{d}^2\mathscr{y}/\mathrm{d}\tau^2$ from the junction condition (33), using (32) and the field equations (13) and (16) it is found that

$$8\pi p_{r_{(b)}} = 8\pi (q \exp[(\lambda + \mu)/2])_{(b)}. \tag{39}$$

This relation implies that the fluid pressure at the boundary in the radial direction of the shear-free collapsing charged fluid undergoing dissipation in the form of radial heat flow is directly related with the heat flux across the boundary. If there is no heat flow across the boundary radial pressure on the boundary must vanish.

6. Equations governing collapse

In this section the dynamical equations governing the collapse of charged, non-adiabatic unisotropic spherical distribution have been obtained. We introduce the proper-time derivative operator in $\mathscr{M}_{(i)}$ as

$$\mathscr{D} \equiv \exp[-v/2] \frac{\partial}{\partial t} \tag{40}$$

and the proper velocity

$$U \equiv r\mathscr{D}\exp[\mu/2] = r\dot{\mu}\exp[(\mu - \nu)/2] \tag{41}$$

following the approach due to de Oliveira and Santos. The total charge $\ell(r)$ enclosed within the spherical region of radius $r_{(b)}$ is obtained from (10) as

$$\ell(r) = -\exp[3\mu/2]\int_0^r 4\pi\sigma r^2\exp[\lambda/2]\,\mathrm{d}r \tag{42}$$

which implies that

$$\mathscr{D}\ell(r) = 0. \tag{43}$$

Subsequently the expression (36) for $m(r,t)$ on using the field equations (13) and (16) leads to

$$\mathscr{D}m = -4\pi r^2\exp[\mu]\left[p_r U + q\frac{\mathrm{d}}{\mathrm{d}r}(r\exp[-\mu/2])\right]. \tag{44}$$

Equation (36) can be rewritten as

$$\exp[\lambda] = \left[1 + U^2 - \frac{2m\exp[-\mu/2]}{r} + \frac{\ell^2\exp[-\mu]}{r^2}\right]^{-1}. \tag{45}$$

The Bianchi identity $T^{\alpha}_{1;\alpha} = 0$ in the interior region gives the following expression for the pressure gradient

$$\frac{\partial}{\partial r}(p_r) = -(\rho + p_r)\frac{\nu'}{2} - \frac{2}{r}(p_r - p_\perp) + \frac{\ell\ell'}{4\pi r^4}\exp(-2\mu) - $$
$$-\left(\frac{5}{2}\dot{\mu}q + \dot{q}\right)\exp[\lambda + \mu - (\nu/2)]. \tag{46}$$

This relation shows the dependence of the gradient of pressure in the radial direction on the matter density, radial pressure P_r, the measure of unisotropy, variation of charge, heat flux and its variation with time. The relations (45), (46) and (13) lead to

$$\mathscr{D}U = -\frac{\exp[-\mu/2]}{(\rho + p_r)}\left[\frac{\partial}{\partial r}(p_r) + \frac{2}{r}(p_r - p_\perp) - \frac{\ell\ell'\exp[-2\mu]}{4\pi r^4}\right]$$
$$\times\left[1 + U^2 - \frac{2m\exp[-\mu/2]}{r} + \frac{\ell^2\exp[-\mu]}{r^2}\right]\cdot$$
$$-\frac{\exp[-\mu/2]}{r}\left[4\pi p_r r^2\exp(\mu) + \frac{m\exp[-\mu/2]}{r} - \frac{\ell^2\exp[-\mu]}{r^2}\right] -$$
$$-\frac{(r\exp[\mu/2]\mathscr{D}q + 5Uq)}{r(\rho + p_r)}. \tag{47}$$

Equations (36), (41), (45), (46) and (47) are the dynamical equations governing the collapse of non-adiabatic collapse of charged spherical distributions of unisotropic matter which represent in the generalized form the dynamical equations due to Misner and Sharp (1964), Berkenstein (1971) and de Oliveira and Santos (1987).

7. Discussion

A brief outline of a relativistic model for studying the collapse of a radiating sphere with charged non-adiabatic fluid in the interior is discussed below in the set-up under consideration.

The matter content in the region $\mathscr{M}_{(i)}$ be assumed initially to be the spherically symmetric unisotropic fluid in equilibrium described by the static metric

$$ds_{(i)}^2 = -\exp[\lambda(r)]dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) + \exp[\nu(r)]dt^2. \tag{48}$$

On setting $\mu(t) = 0$ and $\nu(r,t) = \nu(r)$, the metric (1) reduces to this form. The space-time in the exterior region $\mathscr{M}_{(e)}$ is described by Reissner–Nordstrom metric

$$ds_{(e)}^2 = -y^2(d\theta^2 + \sin^2\theta d\phi^2) + \left(1 - \frac{2M}{y} + \frac{Q^2}{y^2}\right)dv^2 + 2dv\,dy, \tag{49}$$

where M and Q respectively denote the total mass and the total charge of the fluid sphere.

The matter density ρ_0, the fluid pressures along the radial and the transverse directions $(p_r)_0$ and $(p_\perp)_0$ of the distribution are found to have the explicit expressions

$$8\pi\rho_0 = -\exp[-\lambda]\left(-\frac{\lambda'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} - \frac{\ell^2}{r^4}, \tag{50}$$

$$8\pi(p_r)_0 = \exp[-\lambda]\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} + \frac{\ell^2}{r^4}, \tag{51}$$

$$8\pi(p_r - p_\perp)_0 = -\exp[-\lambda]\left(\frac{\nu''}{2} + \frac{\nu'^2}{4} - \frac{\lambda'\nu'}{4} - \frac{\lambda' + \nu'}{2r} - \frac{1}{r^2}\right) - \frac{1}{r^2} + \frac{\ell^2}{r^4}. \tag{52}$$

The junction condition (39) at the static boundary reduces to

$$(p_r)_{0(b)} = 0. \tag{53}$$

When the equilibrium of this fluid is lost it will start collapsing with its boundary $\Sigma_{(b)}$ shrinking continuously. The interior space-time $\mathscr{M}_{(i)}$ and the exterior space-time $\mathscr{M}_{(e)}$ be subsequently described by the metrics (1) and (18) respectively.

The matter density and the fluid pressure of the collapsing fluid are related with the matter density and the fluid pressure of the static model from which collapse starts by the relations

$$8\pi\rho = 8\pi\rho_0\exp[-\mu] + \frac{3}{4}\exp[-\mu]\dot{\mu}^2 + \frac{\ell^2\exp[-\mu]}{r^4} \times (1 - \exp[-\mu]), \tag{54}$$

$$8\pi p_r = 8\pi (p_r)_0 \exp[-\mu] - \exp[-\nu]\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{1}{2}\dot{\mu}\dot{\nu}\right] + \\ + \frac{\ell^2 \exp[-\mu]}{r^4}(\exp[-\mu] - 1), \tag{55}$$

$$8\pi(p_r - p_\perp) = 8\pi(p_r - p_\perp)_0 \exp[-\mu] + \frac{2\ell^2}{r^4}\exp[-\mu](\exp[-\mu] - 1). \tag{56}$$

The junction condition (39) at the boundary $\Sigma_{(b)}$ of the collapsing fluid sphere accordingly becomes

$$\exp[\mu]\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2 - \frac{1}{2}\dot{\mu}\dot{\nu}\right]_{(b)} - \left[\frac{1}{2}\exp[(\nu - \lambda)/2]\nu'\right]_{(b)} \exp[\mu/2]\dot{\mu} - \\ - \left[\frac{\ell^2 \exp[\nu]}{r^4}\right]_{(b)}(\exp[-\mu] - 1) = 0, \tag{57}$$

the differential equation which determines $\mu(t)$.

With $\lambda = \lambda(r)$ and $\nu = \nu(r)$, chosen appropriate to an equilibrium model and $\mu(t)$ determined by

$$\exp[\mu]\left[\ddot{\mu} + \frac{3}{4}\dot{\mu}^2\right]_{(b)} - \frac{1}{2}[\exp[(\nu - \lambda)/2]\nu']_{(b)}\exp[\mu/2]\dot{\mu} - \left[\frac{\ell^2 \exp[\nu]}{r^4}\right]_{(b)} \\ \times (\exp[-\mu] - 1) = 0, \tag{58}$$

the space-time metric (1) provides a relativistic model for studying collapse of charged fluid sphere accompanied by a radial heat flow.

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