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Perturbative and non-perturbative evolutions of structure functions at low-x

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Abstract. Approximate solutions of Altarelli–Parisi equations are obtained in low-x limit and have tested them in EMC low-x and low-Q^2 data. The results are compared with the phenomenological non-perturbative evolutions. Data conform to perturbative as well as non-perturbative evolutions but do not conform to the predictions of non-linear evolution.

Keywords. Structure function; Altarelli–Parisi equations; low-x.

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1. Introduction

Study of structure functions at low-x is a topical problem in QCD (Ali and Bartels 1990). Low-x is however attainable both at low-Q^2 and high-Q^2. While low-x and high-Q^2 regimes have been the focus of intense research in view of the ensuing experiments at super colliders (Jarlskog and Rein 1990), the corresponding low-Q^2 regime has not received similar attention. The reason is understandable: the scale where the non-perturbative QCD ends and where the perturbative one begins is not precise. There is no universal criterion to determine Q_0^2—the boundary between the perturbative and non-perturbative domains. In contemporary literature, it ranges from 30·5 GeV^2 (Abbott *et al* 1980) to 1 GeV^2 (Bednyakov 1984).

Because of such a controversy, low-x, low-Q^2 regime has been the topic for non-perturbative evolutions alone (Landshoff 1990) albeit phenomenological. Only recently an attempt has been reported (Glück *et al* 1989) about the quantitative QCD calculations of structure functions in this region. The starting point for the evolution was taken at $\mu \sim 0{\cdot}25$ GeV when valence quarks are assumed to be the only partons. Such a value of μ is quite low at first sight. Nevertheless $\alpha_s(\mu^2)/2\pi$, the quantity appearing in the Q^2 evolution equations is still in the perturbative region since $\alpha_s(\mu^2)/2\pi \sim 0{\cdot}5$. The gluons and the sea partons can then be generated radiatively using QCD. The prediction of such an approach is however found to overshoot the data points (Arneodo *et al* 1989) at small-x and small-Q^2.

The aim of the present paper is to report an alternative description of Q^2 evolution of structure functions at the low-x limit of Altarelli–Parisi (AP) equations (Altarelli and Parisi 1977). Approximate solutions of AP equations have been reported in recent years (Choudhury and Saikia 1987, 1989, 1990) but mostly valid only at intermediate $x(x > 0{\cdot}1)$. The present method on the other hand is valid only for low-x. We then compare our results with EMC data (Arneodo *et al* 1989) for low-x and low-Q^2. As

Q^2 to be used is rather low ($0{\cdot}25\,\mathrm{GeV}^2 < Q^2\,\mathrm{GeV}^2 < 7{\cdot}2\,\mathrm{GeV}^2$) we test our evolutions without the use of phenomenological inputs unlike the conventional practice. Rather we take inputs directly from data. Such a strategy seems safer than to evolve the structure functions down to $Q^2 < Q_0^2$. We also compare the data with phenomenological non-perturbative evolutions (Donnachie and Landshoff 1984; Landshoff 1990). Besides, we also incorporate the effect of gluon evolution as predicted by the non-linear evolution equation (Kim and Ryskin 1990) as an ansatz in our formalism and test its consequences.

2. Theory

(A) *Perturbative evolutions at low-x*

The AP equations have the standard structure

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} - \frac{A_f}{t}\Bigg[\{3+4\ln(1-x)\}F_2^{NS}(x,t) + 2 \int_x^1 \frac{d\omega}{1-\omega}\{(1+\omega^2)F_2^{NS}(x/\omega,t) - 2F_2^{NS}(x,t)\}\Bigg] = 0 \tag{1}$$

$$\frac{\partial F_2^S(x,t)}{\partial t} - \frac{A_f}{t}\Bigg[\{3+4\ln(1-x)\}F_2^S(x,t) + 2\int_x^1 \frac{d\omega}{(1-\omega)}\{(1+\omega^2)F_2^S(x/\omega;t) - 2F_2^S(x,t)\} + \frac{3}{2}N_f\int_x^1 \{\omega^2+(1-\omega)^2\}G(x/\omega,t)d\omega\Bigg] = 0 \tag{2}$$

where $A_f = 4/(33-2N_f)$, N_f being the number of flavour and $t = \ln Q^2/\Lambda^2$.

Defining,

$$I_1^{NS}(x,t) = 2\int_x^1 \frac{d\omega}{(1-\omega)}\{(1+\omega^2)F_2^{NS}(x/\omega,t) - 2F_2^{NS}(x,t)\} \tag{3}$$

$$I_1^S(x,t) = 2\int_x^1 \frac{d\omega}{(1-\omega)}\{(1+\omega^2)F_2^S(x/\omega,t) - 2F_2^S(x,t)\} \tag{4}$$

$$I_2^S(x,t) = \frac{3}{2}N_f\int_x^1 [\omega^2+(1-\omega)^2]G(x/\omega,t)d\omega \tag{5}$$

one can recast (1) and (2) as

$$\frac{\partial F_2^{NS}(x,t)}{\partial t} - \frac{A_f}{t}[\{3+4\ln(1-x)\}F_2^{NS}(x,t) + I^{NS}(x,t)] = 0 \tag{6}$$

and

$$\frac{\partial F_2^S(x,t)}{\partial t} - \frac{A_f}{t}[\{3+4\ln(1-x)\}F_2^S(x,t) + I_1^S(x,t) + I_2^S(x,t)] = 0. \tag{7}$$

Let us introduce the variable $u = 1-\omega$, and note that

$$x/(1-u) \approx x + ux, \tag{8}$$

$$F_2^{NS}(x/\omega,t)\approx F_2^{NS}(x,t)+ux\frac{\partial F_2^{NS}(x,t)}{\partial x}, \tag{9}$$

and

$$F_2^{S}(x/\omega,t)\approx F_2^{S}(x,t)+ux\frac{\partial F_2^{S}(x,t)}{\partial x} \tag{10}$$

which are good approximations for small-x; $x\sim 0$.

Putting (8)–(10) in (3)–(5), and performing the u integrations,

$$I^{NS}(x,t)=\{-(1-x)(3+x)\}F_2^{NS}(x,t)+ \\ 2/3x(1-x)(x^2-x+4)\frac{\partial F_2^{NS}(x,t)}{\partial x}, \tag{11}$$

$$I_1^{S}(x,t)=\{-(1-x)(3+x)\}F_2^{S}(x,t)+ \\ 2/3x(1-x)(x^2-x+4)\frac{\partial F_2^{S}(x,t)}{\partial x}, \tag{12}$$

$$I_2^{S}(x,t)=3/2N_f\left\{2/3(1-x)(2-x+2x^2)G(x,t)\right. \\ \left.+\frac{1}{6}x(1-x)^2(10-10x+3x^2)\frac{\partial G(x,t)}{\partial x}\right\}. \tag{13}$$

Using (11)–(13) in (6) and (7),

$$\frac{2F_2^{NS}(x,t)}{\partial t}-\frac{A_f}{t}\left[A(x)F_2^{NS}(x,t)+B(x)\frac{\partial F_2^{NS}(x,t)}{\partial t}\right]=0 \tag{14}$$

and

$$\frac{\partial F_2^{S}(x,t)}{\partial t}-\frac{A_f}{t}\left[A(x)F_2^{S}(x,t)+B(x)\frac{\partial F_2^{S}(x,t)}{\partial x}\right. \\ \left.+(C(x)G(x,t))+D(x)\frac{\partial G(x,t)}{\partial x}\right], \tag{15}$$

where,

$$A(x)=3+4\ln(1-x)-(1-x)(3+x) \tag{16}$$

$$B(x)=2/3x(1-x)(x^2-x+4) \tag{17}$$

$$C(x)=1/2N_f(1-x)(2-x+2x^2) \tag{18}$$

$$D(x)=1/4N_f x(1-x)^2(10-10x+3x^2). \tag{19}$$

Let us now find the solutions of (14) and (15). To that end, (14) can be recast in standard form (Sneddon 1957)

$$P_1(x,t,F_2^{NS})\frac{\partial F_2^{NS}}{\partial x}+Q_1(x,t,F_2^{NS})\frac{\partial F_2^{NS}}{\partial t}=R_1(x,t,F_2^{NS}) \tag{20}$$

where,

$$P_1(x,t,F_2^{NS})=A_fB(x), \\ Q_1(x,t,F_2^{NS})=-t, \tag{21}$$

and

$$R_1(x,t,F_2^{NS}) = -A_f A(x) F_2^{NS}(x,t).$$

The general solution of (20) is

$$F(u_1, v_1) = 0 \tag{22}$$

where F is an arbitrary function and

$$u_1(x,t,F_2^{NS}) = C_1$$

$$v_1(x,t,F_2^{NS}) = C_2 \tag{23}$$

form a solution of the equations

$$\frac{\mathrm{d}x}{P_1(x,t,F_2^{NS})} = \frac{\mathrm{d}t}{Q_1(x,t,F_2^{NS})} = \frac{\mathrm{d}F_2^{NS}}{R_1(x,t,F_2^{NS})}. \tag{24}$$

Solving (24) one obtains

$$u_1(x,t,F_2^{NS}) = tX^{NS}(x) \tag{25}$$

$$v_1(x,t,F_2^{NS}) = F_2^{NS}(x,t)\; Y^{NS}(x) \tag{26}$$

where

$$X^{NS}(x) = \exp\left[\frac{1}{A_f}\int \frac{\mathrm{d}x}{B(x)}\right] \tag{27}$$

$$Y^{NS}(x) = \exp\left[\int \frac{A(x)\mathrm{d}x}{B(x)}\right]. \tag{28}$$

Thus the structure function $F_2^{NS}(x,t)$ has to satisfy (22) with u_1 and v_1 given by (25) and (26). It thus have no unique solution. The simplest possibility is that a linear combination of u_1 and v_1 is to satisfy (22) so that

$$A_{NS}u_1 + B_{NS}v_1 = 0. \tag{29}$$

Putting the values of u_1 and v_1 in (29) we obtain

$$F_2^{NS}(x,t) = -\frac{A_{NS}}{B_{NS}}t\left[\frac{X^{NS}(x)}{Y^{NS}(x)}\right]. \tag{30}$$

Defining

$$F_2^{NS}(x,t_0) = -\frac{A_{NS}}{B_{NS}}t_0\left[\frac{X^{NS}(x)}{Y^{NS}(x)}\right] \tag{31}$$

one then has

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0)\left(\frac{t}{t_0}\right). \tag{32}$$

In order to solve (15) we need to relate singlet $F_2^S(s,t)$ with gluon distribution $G(x,t)$. We assume that their t-evolution is identical (Choudhury and Saikia 1989).

$$G^{(x,t)} = g(x) F_2^S(x,t). \tag{33}$$

This results in,

$$\frac{\partial F_2^S(x,t)}{\partial t} - \frac{A_f}{t}\left[L(x)F_2^S(x,t) + M(x)\frac{\partial F_2^S(x,t)}{\partial x}\right] = 0 \tag{34}$$

where,

$$L(x) = A(x) + 3/2N_f C(x)g(x) + 3/2N_f D(x)\frac{\mathrm{d}g(x)}{\mathrm{d}x} \tag{35}$$

and

$$M(x) = B(x) + 3/2N_f D(x)g(x). \tag{36}$$

The general solution of (34) can now be obtained by recasting it in a form similar to (20) with replacements

$$\begin{aligned} &P_1 \to P_2 = P_f M(x) \\ &Q_1 \to Q_2 = -t \\ &R_1 \to R_2 = -P_f\, L(x)F_2^S(x,t). \end{aligned} \tag{37}$$

The general solution of (34) is then

$$U(u_2, v_2) = 0 \tag{38}$$

where U is an arbitrary function with

$$u_2 = tX^s(x), \tag{39}$$

and

$$v_2 = F_2^S(x,t)\; Y^S(x), \tag{40}$$

while $X^S(x)$ and $Y^S(x)$ are

$$X^S(x) = \exp\left[\frac{1}{A_f}\int \frac{\mathrm{d}x}{M(x)}\right], \tag{41}$$

$$Y^S(x) = \exp\left[\int \frac{L(x)}{M(x)}\mathrm{d}x\right]. \tag{42}$$

A simple possibility similar to (29) then yields,

$$F_2^S(x,t) = F_2(x,t_0)\left(\frac{t}{t_0}\right) \tag{43}$$

with

$$F_2^S(x,t_0) = -\frac{A_s}{B_s}(t_0)\frac{X^S(x)}{Y^S(x)}. \tag{44}$$

(32) and (43) are our main results to yield the structure functions

$$F_2(x,t) = \frac{5}{18}F_2^S(x,t) + \frac{3}{18}F_2^{NS}(x,t). \tag{45}$$

(B) *Non-perturbative evolutions*

Some time back, Donnachie and Landshoff (1984) parametrized the small-Q^2 SLAC data (Stein *et al* 1984) as

$$V \equiv F_2^{\text{valence}} = 1{\cdot}33x^{0{\cdot}56}(1-x)^3\left(\frac{Q^2}{Q^2+0{\cdot}85}\right)^{0{\cdot}44} \tag{46}$$

and

$$S \equiv F_2^{\text{Non-valence}} = 0{\cdot}17x^{-0{\cdot}08}(1-x)^7\left(\frac{Q^2}{Q^2+0{\cdot}36}\right)^{1{\cdot}08} \tag{47}$$

so that they also agree with real photon data (Caldwell *et al* 1978). Recently it was compared with the new small-Q^2 data of EMC (Arneodo *et al* 1989). In our present work, we study how well the non-perturbative evolutions (46) and (47) compare with the perturbative ones (33) and (43). We use the following expressions for isoscalar structure functions with (u,d), (u,d,s) and (u,d,s,c) sea components respectively.

$$F_2(x,Q^2) = \frac{5}{6}V + \left\{\frac{10}{9}S, \frac{15}{9}S, \frac{20}{9}S\right\}. \tag{48}$$

(C) *Non-linear evolution*

Kim and Ryskin (1991) have shown that the non-linear evolution equation (Gribov *et al* 1983) predicts that at small-x, gluon evolves faster than suggested in (33) Specifically,

$$F_2^{\text{sea}}(x,Q^2) \approx \frac{7}{5}\Sigma e_f^2 xG(x,Q^2)\frac{\alpha s}{2\pi}. \tag{49}$$

Identifying $F_2^{\text{sea}}(x,Q^2)$ as the singlet structure functions at low-x,

$$xG(x,Q^2) \approx C_g t F_2^S(x,Q^2) \tag{50}$$

where

$$C_g = \frac{20}{21}\frac{1}{A_f \Sigma e_f^2}. \tag{51}$$

Solution of singlet evolution with (51) yields for very small-x,

$$F_2^S(x,Q^2) = F_2^S(x,Q_0^2))\left(\frac{Q^2}{Q_0}\right)^{C_g A_f N_f}, \tag{52}$$

which for $N_f = 4$ yields a $(Q^2/Q_0)^{24/7}$ evolution.

3. Results and discussion

We present our results (solid lines) for isoscalar target in figure 1(a–d) for representative values of x; $x = 0{\cdot}0025, 0{\cdot}0035, 0{\cdot}0240$ and $0{\cdot}0400$. Data at $Q^2 = 0{\cdot}25\,\text{GeV}^2, 0{\cdot}35\,\text{GeV}^2$, $0{\cdot}35\,\text{GeV}^2$ and $0{\cdot}70\,\text{GeV}^2$ are respectively taken as inputs to test the evolutions (43). The agreement is found to be excellent. In the same figures, we also plot the non-perturbative evolutions (dashed lines) obtained from (46) and (47) and using (48). Using an SU(4) flavour symmetric sea, we find that the non-perturbative results are slightly below the data without strange, and charm components of the sea. If the onsets of strange and sea components are taken around $Q^2 \sim 2{\cdot}8\,\mathrm{GeV}^2$ and $Q^2 \sim 5{\cdot}6\,\mathrm{GeV}^2$ respectively, the non-perturbative evolutions can also explain the data equally well.

The corresponding results of a faster gluon evolution (50) as predicted by the non-linear evolution equation (Gribov *et al* 1983) and incorporated in the present formalism through (52) are also shown in figure 1 (a–d) (dot dashed lines). Data however do not accommodate such fast growth of gluons, presumably indicating the region of x under study is far below the onset of non-linear effects.

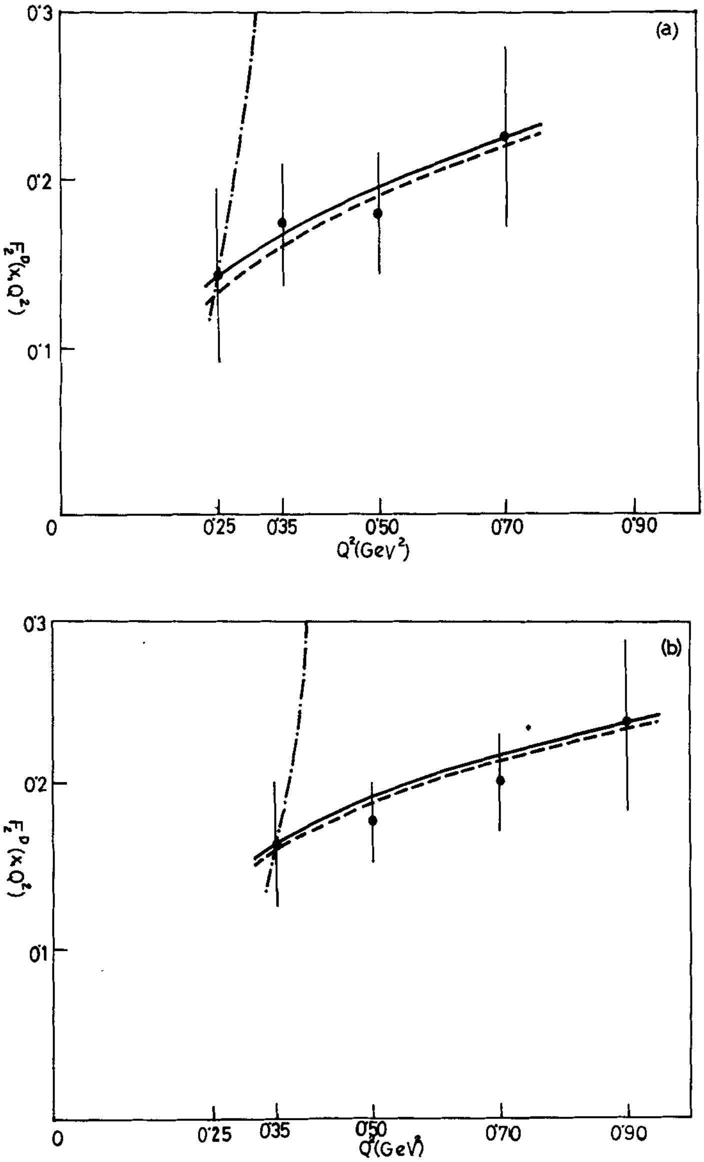

Figure 1 (a–b).

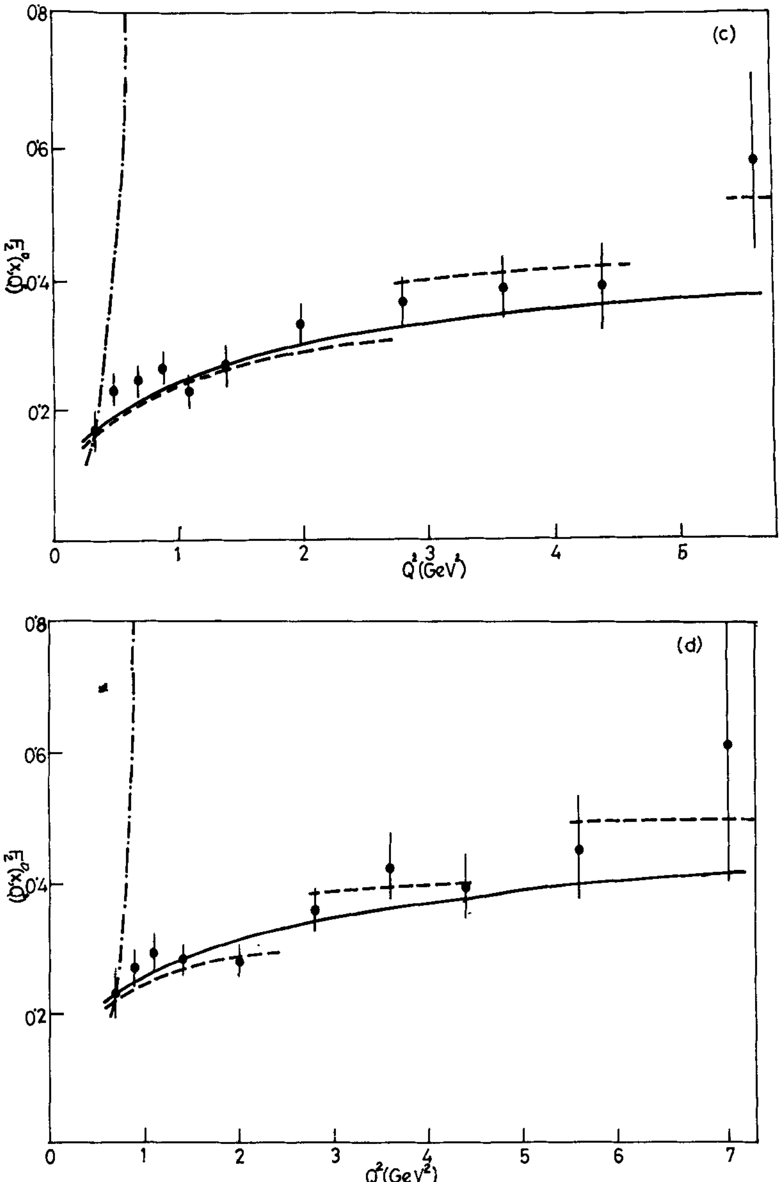

Figure 1(c–d).

Figure 1(a–d). $F_2^p(x, Q^2)$ *vs* Q^2 **for** $x = 0{\cdot}0025$, **0·0035, 0·0240 and 0·0400 respectively. Solid** lines represent the prediction of the perturbative evolution while the dashed ones represent those of non-perturbative evolution. Dot-dashed one are predictions of non-linear evolution. Strange and charm seas are assumed to contribute at $Q^2 \sim 2{\cdot}8\,\mathrm{GeV}^2$ and $\sim 5{\cdot}6\,\mathrm{GeV}^2$ respectively. Data are taken from Arneodo *et al* (1989).

To conclude, we have studied the low-x and low-Q^2 region of deep inelastic scattering using the Altarelli–Parisi equations in Taylor approximation. As the partial differential equations become of two variables (x, t), they do not yield unique solutions. We then suggest a simple possibility (29) which nearly conforms to experiment (Arneodo *et al* 1989). Other possibilities may also be considered as equally valid solutions of the Taylor approximated Altarelli–Parisi equations. We have not reported them here as they will lose the present simplicity. We also find that the phenomenological non-perturbative evolutions can explain the data equally well if proper account of strange and charm components of the sea are taken into account as Q^2 increases. Data however seems to rule out non-linear evolution at least in the range of x under study.

The present method also yields x evolution of structure functions at low-x. They are however not unique and needs the information of relative x evolution of singlet to gluon distribution. Therefore, it is not discussed here.

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