Pramana – J. Phys., Vol. 38, No. 5, May 1992, pp. 477–480. © Printed in India.

Lower bound of m_t from $B_d^0 - \bar{B}_d^0$ mixing and recent $|V_{ub}|/|V_{cb}|$ measurements

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MS received 31 January 1991; revised 9 March 1992

Abstract. A careful analysis of $B_d^0 - \bar{B}_d^0$ mixing data, incorporating the most recent QCD corrections to the parameter x_d, as well as the recent data regarding the matrix elements $|V_{ub}|$ and $|V_{cb}|$ leads to a reliable lower limit on mass of t-quark, for example $m_t \geqslant 115\,\text{GeV}$.

Keywords. Quark mixing matrix; $B_d^0 - \bar{B}_d^0$ mixing.

PACS Nos 12·15; 11·30

Great advances have been made in the measurements of the quark mixing matrix elements (Cabibbo 1963; Kobayashi and Maskawa 1974) and in their implications on the B-decays, CP-violation and related phenomena (Chau 1983; Wolfenstein 1986; Donoghue *et al* 1986, 1987a, b; Paschos and Turke 1989). However, one of the crucial ingredients for more accurate predictions, the t-quark, has defined experimental detection, therefore questions related to the t-quark, in particular its mass (m_t), assume significance.

Several analyses have been carried out to study the implications of CP-violation phenomenon, B meson decays and $B_d^0 - \bar{B}_d^0$ mixing phenomenon on CKM matrix elements and m_t (Paschos and Turke 1989 and references therein). One important conclusion, which emerges from such analyses, is the lower limit on m_t, e.g. $m_t \geqslant 55\,\text{GeV}$. Very recently the analyses of Maalampi and Roos (1990) on the one hand, and Albright (1990) on the other tried to effect an overall fit to some of the parameters, such as m_t, B-decay constant etc., taking into account the data regarding CKM matrix elements, CP-violation in K-sector and $B_d^0 - \bar{B}_d^0$ mixing including certain theoretical developments (Kaufman *et al* 1988; Datta *et al* 1990). Both the analyses indicate a 'heavier' t-quark mass, for example, $m_t \geqslant 100\,\text{GeV}$. This is in good agreement with the latest experimental expectations (Dydak 1990; Malhotra 1990), however, both the analyses are plagued by theoretical as well as experimental uncertainties. The purpose of this study is to find a lower limit on m_t which is largely free from the uncertainties of the data and other inputs as well as by taking into account the latest data, regarding $|V_{cb}|$ and $|V_{ub}|/|V_{cb}|$ (Albright *et al* 1989; Fulton *et al* 1989) and certain recent theoretical improvements due to Buras *et al* (1990) in the expression for $B_d^0 - \bar{B}_d^0$ mixing parameter $x_d (\equiv \Delta m/\Gamma)$.

Recently semileptonic decays of B-mesons have provided an improved estimate on $|V_{ub}|$ particularly the lower limit and similar improvements have been obtained in

$|V_{cb}|$ also (Kramer and Palmer 1990). In the present work, we consider the safe limits for $|V_{cb}|$,

$$0{\cdot}035 \leqslant |V_{cb}| \leqslant 0{\cdot}050, \tag{1}$$

and for $|V_{ub}|/|V_{cb}|$ the following range

$$0{\cdot}08 \leqslant |V_{ub}|/|V_{cb}| \leqslant 0{\cdot}14. \tag{2}$$

To express the data regarding V_{cb} and V_{ub}, in terms of the quark mixing parameters, we consider out of several available versions (Particle data group 1990), the original parametrization (KM) of the mixing matrix as suggested by Kobayashi and Maskawa (1974). The usefulness of this parametrization in the present context becomes obvious when we consider the expression for x_d and its implications on the lower limit of t-quark mass. In the KM parametrization, the mixing matrix is parametrized in terms of three angles $\theta_1, \theta_2, \theta_3$ (with $\sin\theta_i = s_i$, $\cos\theta_i = c_i$) and the phase angle δ, for example,

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & T_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}. \tag{3}$$

Using the above parametrization of V_{CKM}, eqs (1), (2) and $s_1 = 0{\cdot}22 \pm 0{\cdot}003$ (Leutwyler and Roos 1984; Donoghue *et al* 1987), we obtain

$$0 \leqslant s_2 \leqslant 0{\cdot}08, \tag{4}$$

$$0{\cdot}018 \leqslant s_3 \leqslant 0{\cdot}030. \tag{5}$$

For the effect of $B_d^0 - \bar{B}_d^0$ mixing on t-quark mass, we consider the expression for x_d which takes into account the QCD corrections in the $m_t \geqslant M_W$ limit also (Buras *et al* 1990), for example,

$$x_d = \tau_B \frac{G_F^2}{6\pi^2} M_B B_{B_d} f_B^2 M_W^2 \lambda_t^2 F(x_t), \tag{6}$$

where $\lambda_t = (V_{td}^* V_{tb})$ and $x_t = m_t^2/M_W^2$. The function $F(x_t)$ includes the effect of Inami-Lim function (Inami and Lim 1981). In the present parametrization, it is easy to check that

$$V_{tb} \cong 1, |V_{td}|^2 = s_1^2 s_2^2. \tag{7}$$

The usefulness of the KM parametrization becomes obvious as x_d becomes directly proportional to s_2^2. A major uncertainty with the standard model estimates for x_d is introduced by the 'hadronic factor' $(B_{B_d} f_B^2)$. There are no reliable estimates for this, however, we would just require its maximum value for which there are fairly reliable estimates, e.g. $(B_{B_d} f_B^2) \leqslant (0{\cdot}150\,\mathrm{GeV})^2$ (Schubert 1987). It is clear that a lower estimate for m_t can be obtained from the expression for x_d by considering the lowest values for x_d, as well as the largest values of s_2 and the hadronic factor. The present best value of combined ARGUS and CLEO of x_d is given as (PDG 90)

$$x_d = 0{\cdot}69 \pm 0{\cdot}13. \tag{8}$$

Before we can proceed with our lower bound of m_t, we have to further refine our estimate of the maximum value of s_2. It is not difficult to check from the analysis of Kim *et al* (1990) that, in order to have a simultaneous fit of ε_k, ε'/ε alongwith x_d, it is not possible for s_2 to exceed the value 0·065, i.e.,

$$s_2 \lesssim 0{\cdot}065. \tag{9}$$

From (6) and the following values of the parameters $\tau_B = (11{\cdot}8 \pm 1{\cdot}1) \times 10^{-13}\,\mathrm{s}$, $G_F = 1{\cdot}166 \times 10^{-5}\,\mathrm{GeV}^{-2}$, $M_B = 5{\cdot}0\,\mathrm{GeV}$

$$M_W = 80\,\mathrm{GeV}, \quad (B_{B_d} f_B^2) \leqslant (0{\cdot}150\,\mathrm{GeV})^2, \tag{10}$$

we obtain the following lower estimate for m_t, e.g.,

$$m_t \geqslant 115\,\mathrm{GeV}. \tag{11}$$

It is clear that such a higher estimate is in agreement with the present experimental estimates (LEP, UA2, CDF)

$$m_t = 127 \pm 37 \pm 3 \pm 30\,\mathrm{GeV}\ (\text{Dydak 1990}) \tag{12}$$
$$= 134^{+36}_{-41}\,\mathrm{GeV}\ (\text{Malhotra 1990}).$$

A few comments are in order. While using $F(t)$ to evaluate m_t, we have used the most recent analysis of Buras *et al* regarding the QCD corrections to $B_d^0 - \bar{B}_d^0$ mixing in the presence of heavy top quark mass, however, if we use the QCD corrections of Kaufman *et al* (1987) and Datta *et al* (1990) we find a rather higher-lower limit on m_t, e.g.,

$$m_t \geqslant 150\,\mathrm{GeV}, \tag{13}$$

even with very conservative values of the parameters. Furthermore, in our estimate of the lower bound on m_t we have used the lowest value of x_d, in case we use the central value for x_d, we will obtain

$$m_t \geqslant 140\,\mathrm{GeV}. \tag{14}$$

It needs to mention that a possible source of uncertainty can be the upper estimate for the hadronic factor $(B_{B_d} f_B^2)$. In case its value registers a higher-upper limit, then our estimate for the lower bound of m_t will get correspondingly depressed compared with the values given by eqs (11) and (13).

To conclude our findings, we have found a reliable lower estimate of m_t, for example $m_t \geqslant 115\,\mathrm{GeV}$, which relies on the conservative range for $|V_{ub}|$ and $|V_{cb}|$ as well as is free from uncertainties associated with the CKM matrix elements and hadronic factor.

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