Lower bound of $m_t$ from $B^0_d - \bar{B}^0_d$ mixing and recent $|V_{ub}|/|V_{cb}|$ measurements

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MS received 31 January 1991; revised 9 March 1992

Abstract. A careful analysis of $B^0_d - \bar{B}^0_d$ mixing data, incorporating the most recent QCD corrections to the parameter $\Delta A$, as well as the recent data regarding the matrix elements $|V_{ub}|$ and $|V_{cb}|$ leads to a reliable lower limit on mass of $t$-quark, for example $m_t \geq 55 \text{ GeV}$.

Keywords. Quark mixing matrix; $B^0_d - \bar{B}^0_d$ mixing.

PACS Nos 12.15; 11.30

Great advances have been made in the measurements of the quark mixing matrix elements (Cabibbo 1963; Kobayashi and Maskawa 1974) and in their implications on the $B$-decays, CP-violation and related phenomena (Chau 1983; Wolfenstein 1986; Donoghue et al 1986, 1987a, b; Paschos and Turke 1989). However, one of the crucial ingredients for more accurate predictions, the $t$-quark, has defined experimental detection, therefore questions related to the $t$-quark, in particular its mass ($m_t$), assume significance.

Several analyses have been carried out to study the implications of CP-violation phenomenon, $B$ meson decays and $B^0_d - \bar{B}^0_d$ mixing phenomenon on CKM matrix elements and $m_t$ (Paschos and Turke 1989 and references therein). One important conclusion, which emerges from such analyses, is the lower limit on $m_t$, e.g. $m_t \geq 55 \text{ GeV}$. Very recently the analyses of Maalampi and Roos (1990) on the one hand, and Albright (1990) on the other tried to effect an overall fit to some of the parameters, such as $m_t$, $B$-decay constant etc., taking into account the data regarding CKM matrix elements, CP-violation in K-sector and $B^0_d - \bar{B}^0_d$ mixing including certain theoretical developments (Kaufman et al 1988; Datta et al 1990). Both the analyses indicate a 'heavier' $t$-quark mass, for example, $m_t \geq 100 \text{ GeV}$. This is in good agreement with the latest experimental expectations (Dydak 1990; Malhotra 1990), however, both the analyses are plagued by theoretical as well as experimental uncertainties. The purpose of this study is to find a lower limit on $m_t$ which is largely free from the uncertainties of the data and other inputs as well as by taking into account the latest data, regarding $|V_{cb}|$ and $|V_{ub}|/|V_{cb}|$ (Albright et al 1989; Fulton et al 1989) and certain recent theoretical improvements due to Buras et al (1990) in the expression for $B^0_d - \bar{B}^0_d$ mixing parameter $\Delta A (\equiv \Delta m/T)$.

Recently semileptonic decays of $B$-mesons have provided an improved estimate on $|V_{ub}|$ particularly the lower limit and similar improvements have been obtained in
| $V_{cb}$ | also (Kramer and Palmer 1990). In the present work, we consider the safe limits for $|V_{cb}|$.

$$0.035 \leq |V_{cb}| \leq 0.050,$$

and for $|V_{ub}|/|V_{cb}|$ the following range

$$0.08 \leq |V_{ub}|/|V_{cb}| \leq 0.14.$$

To express the data regarding $V_{cb}$ and $V_{ub}$, in terms of the quark mixing parameters, we consider out of several available versions (Particle data group 1990), the original parametrization (KM) of the mixing matrix as suggested by Kobayashi and Maskawa (1974). The usefulness of this parametrization in the present context becomes obvious when we consider the expression for $x_d$ and its implications on the lower limit of $t$-quark mass. In the KM parametrization, the mixing matrix is parametrized in terms of three angles $\theta_1, \theta_2, \theta_3$ (with $\sin \theta_i = s_i, \cos \theta_i = c_i$) and the phase angle $\delta$, for example,

$$V_{CKM} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & T_{tb}
\end{pmatrix} =
\begin{pmatrix}
c_1 & -s_1 c_3 & -s_1 s_3 \\
s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i \delta} & c_1 c_2 s_3 + s_2 c_3 e^{i \delta} \\
s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i \delta} & c_1 s_2 s_3 - c_2 c_3 e^{i \delta}
\end{pmatrix}.
$$

Using the above parametrization of $V_{CKM}$, eqs (1), (2) and $s_1 = 0.22 \pm 0.003$ (Leutwyler and Roos 1984; Donoghue et al 1987), we obtain

$$0 \leq s_2 \leq 0.08,$$

$$0.018 \leq s_3 \leq 0.030.$$

For the effect of $B_d^0 - \bar{B}_d^0$ mixing on $t$-quark mass, we consider the expression for $x_d$ which takes into account the QCD corrections in the $m_t > M_W$ limit also (Buras et al 1990), for example,

$$x_d = \frac{G_F^2}{6 \pi^2 M_B B_{B_d} f_B^2 M_W^2} \lambda_i^2 F(x_i),$$

where $\lambda_i = (V_{td}^* V_{tb})$ and $x_i = m_t^2/M_W^2$. The function $F(x_i)$ includes the effect of Inami-Lim function (Inami and Lim 1981). In the present parametrization, it is easy to check that

$$V_{tb} \approx 1, |V_{td}|^2 = s_1^2 s_3^2.$$

The usefulness of the KM parametrization becomes obvious as $x_d$ becomes directly proportional to $s_2$. A major uncertainty with the standard model estimates for $x_d$ is introduced by the 'hadronic factor' $(B_{B_d} f_B^2)$. There are no reliable estimates for this, however, we would just require its maximum value for which there are fairly reliable estimates, e.g. $(B_{B_d} f_B^2) \leq (0.150 \text{ GeV})^2$ (Schubert 1987). It is clear that a lower estimate for $m_t$ can be obtained from the expression for $x_d$ by considering the lowest values for $x_d$, as well as the largest values of $s_2$ and the hadronic factor. The present best value of combined ARGUS and CLEO of $x_d$ is given as (PDG 90)

$$x_d = 0.69 \pm 0.13.$$
Before we can proceed with our lower bound of $m_t$, we have to further refine our estimate of the maximum value of $s_2$. It is not difficult to check from the analysis of Kim et al (1990) that, in order to have a simultaneous fit of $\epsilon_\mu$, $\epsilon'/\epsilon$ along with $x_d$, it is not possible for $s_2$ to exceed the value 0.065, i.e.,

$$s_2 \lesssim 0.065.$$  \hspace{1cm} (9)

From (6) and the following values of the parameters $	au_B = (11.8 \pm 1.1) \times 10^{-13}$ s, $G_F = 1.166 \times 10^{-5}$ GeV$^{-2}$, $M_B = 5.0$ GeV

$$M_W = 80 \text{ GeV}, \quad (B_{B_s}\mathcal{J}^2) \leq (0.150 \text{ GeV})^2,$$

we obtain the following lower estimate for $m_t$, e.g.,

$$m_t \geq 115 \text{ GeV}.$$  \hspace{1cm} (11)

It is clear that such a higher estimate is in agreement with the present experimental estimates (LEP, UA2, CDF)

$$m_t = 127 \pm 37 \pm 3 \pm 30 \text{ GeV (Dydak 1990)}$$

$$= 134^{+36}_{-41} \text{ GeV (Malhotra 1990)}.$$  \hspace{1cm} (12)

A few comments are in order. While using $F(t)$ to evaluate $m_t$, we have used the most recent analysis of Buras et al regarding the QCD corrections to $B_d^0 - \bar{B}_d^0$ mixing in the presence of heavy top quark mass, however, if we use the QCD corrections of Kaufman et al (1987) and Datta et al (1990) we find a rather higher-lower limit on $m_t$, e.g.,

$$m_t \geq 150 \text{ GeV},$$  \hspace{1cm} (13)

even with very conservative values of the parameters. Furthermore, in our estimate of the lower bound on $m_t$ we have used the lowest value of $x_d$, in case we use the central value for $x_d$, we will obtain

$$m_t \geq 140 \text{ GeV}.$$  \hspace{1cm} (14)

It needs to mention that a possible source of uncertainty can be the upper estimate for the hadronic factor $(B_{B_s}\mathcal{J}^2)$. In case its value registers a higher-upper limit, then our estimate for the lower bound of $m_t$ will get correspondingly depressed compared with the values given by eqs (11) and (13).

To conclude our findings, we have found a reliable lower estimate of $m_t$, for example $m_t \geq 115 \text{ GeV}$, which relies on the conservative range for $|V_{ub}|$ and $|V_{cb}|$ as well as is free from uncertainties associated with the CKM matrix elements and hadronic factor.

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