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Relativistic remnants in the reduction of the Bethe-Salpeter equation to the Schrödinger equation

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Abstract. Following Salpeter, the Bethe-Salpeter equation for the bound system of two oppositely charged particles is reduced to a Schrödinger equation for each of the following cases: (a) both particles are spin 1/2 particles, (b) one particle is a spinor while the other is spinless, and (c) both particles are spinless. It is shown that if e is the magnitude of charge carried by each of the particles whose masses are set equal to the electron and proton masses then, strictly speaking, only in case (a) do we obtain the familiar Schrödinger equation for the hydrogen atom. The latter equation is recovered in the other two cases only if relativistic remnants—terms of the order of 10^{-5} and smaller—are neglected in comparison with unity. Attention is drawn to a situation where such remnants may not be negligibly small, viz. the problem of confinement of quarks.

Keywords. Bethe-Salpeter equation; spin; Schrödinger equation.

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1. Introduction

It is well known that spin is neglected in the Schrödinger equation for the hydrogen atom:

$$\left(W-\frac{p^2}{2\mu}\right)\phi(\mathbf{p})=-\frac{e^2}{2\pi^2}\int\frac{d^3\mathbf{p}'\phi(\mathbf{p}')}{(\mathbf{p}-\mathbf{p}')^2},\tag{1}$$

where the symbols have their usual significance. It is not so well known—at least at the level at which elementary texts in quantum mechanics are written—that (1) itself is derivable from a parent Bethe-Salpeter equation (BSE) by a process of reduction, which was first spelled out by Salpeter (1952) (referred to as S hereafter).

The general form of the BSE for the bound states of particles a and b, interacting via the exchange of particle c in the ladder approximation, is:

$$F_aF_b\psi(p_\mu)=(-2\pi i)^{-1}\int d^4p'_\mu\psi(p'_\mu+p_\mu)I^c_{ab}(p'_\mu),\tag{2}$$

where $F_a(F_b)$ is the free propagator for particle $a(b)$, I^c_{ab} is the interaction operator, which has different forms in different theories, and ψ is the Bethe-Salpeter wavefunction. Equation (1) may be obtained from (2) through a suitable limiting procedure ("reduction") provided one chooses

$$F_a = [\gamma^a_\mu \mu_a P_\mu + \gamma^a_\mu p_\mu - m_a + i\varepsilon],$$
$$F_b = [\gamma^b_\mu \mu_b P_\mu - \gamma^b_\mu p_\mu - m_b + i\varepsilon], \tag{3}$$

and

$$I^c_{ab} = \left(\frac{-e^2}{2\pi^2}\right)\gamma^a_4 \gamma^b_4/\mathbf{p}'^2, \tag{4}$$

where centre of mass coordinates have been used such that P_μ is a fixed 4-vector—the energy-momentum of the centre of mass, p_μ is the relative energy-momentum 4-vector, $\gamma^{a,b}_\mu$, are the Dirac matrices for particles a (electron) and b (proton) respectively, $m_a(m_b)$ is the mass of particle $a(b)$, c denotes a photon, and

$$\mu_a = m_a/(m_a + m_b), \quad \mu_b = m_b/(m_a + m_b).$$

The notation used here is that of Bjorken and Drell (1965), with $\hbar = c = 1$.

We note that 'reduction" comprises, among other steps (see below), the neglect of the spins of the electron and the proton, *both of which figure as fermions in the original BSE*. This raises the legitimate question whether (1) would also follow if "reduction" were applied to the BSE for a system whose constituents were a fermion and a scalar, or two scalars. Stated differently, the motivation for this note is the following question: if at all, in what form does the memory of the spins of the constituent particles survive the process of "reduction"? In the next section, we deal with the reduction of the BSE for each of following cases: (a) both particles treated as spin 1/2 particles, (b) one treated as a spinor and the other as a spinless boson, and (c) both treated as spinless bosons. Our treatment is essentially, though not entirely, due to S; case (a) is included here in order to recapitulate the details of reduction, as also for the sake of completeness. The final section is devoted to a brief discussion of our work.

2. Reduction of BSE to Schrödinger equation

2.1 *The spinor-spinor BSE*

It will be seen that in (4) a truncated interaction operator for the photon has been used, even though in the covariant treatment all the four degrees of freedom (the transverse, the longitudinal and the timelike) of the photon contribute to its propagator. The longitudinal and the timelike parts are usually combined into two terms T_1 and T_2 such that T_1 is given by the RHS of (4). If one Fourier transforms T_1 to x-space, it becomes evident that it represents exactly the instantaneous Coulomb interaction in the frame of reference in which (4) was obtained. An essential step in the reduction of (2) to (1), as shown by S, is the neglect of the contributions of the transverse degrees of freedom of the photon to its propagator; the T_2 term is dropped because it finally makes a vanishing contribution when coupled with a conserved vector current.

Let a, b and c in (2) denote an electron, a proton and a photon, respectively.

Substituting (3) and (4) into (2) we have

$$[\gamma_\mu^a \mu_a P_\mu + \gamma_\mu^a p_\mu - m_a + i\varepsilon][\gamma_\mu^b \mu_b P_\mu - \gamma_\mu^b p_\mu - m_b + i\varepsilon]\psi(p_\mu)$$
$$= -(2\pi i)^{-1}\left(\frac{-e^2\gamma_4^a\gamma_4^b}{2\pi^2}\right)\int \frac{d^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p}'-\mathbf{p})^2}, \tag{5}$$

or, choosing $P_\mu = (E, \mathbf{0})$,

$$[\gamma_4^a \mu_a E + \gamma_4^a p_4 - \gamma_4^a \mathbf{p}\cdot\boldsymbol{\alpha}^a - m_a + i\varepsilon][\gamma_4^b \mu_b E - \gamma_4^b p_4 + \gamma_4^b \mathbf{p}\cdot\boldsymbol{\alpha}^b - m_b + i\varepsilon]\psi(p_\mu)$$
$$= -(2\pi i)^{-1}\left(\frac{-e^2\gamma_4^a\gamma_4^b}{2\pi^2}\right)\int \frac{\mathrm{d}^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p}'-\mathbf{p})^2}, \tag{6}$$

where the wavefunction is a 16-component spinor and $\boldsymbol{\alpha}^{a(b)}$ are the Dirac matrices for particle $a(b)$.

Following S we multiply (6) by $\gamma_4^a\,\gamma_4^b$; the resulting equation is then operated on by $\Lambda_+^a(\mathbf{p})\Lambda_+^b(\mathbf{p})$, where $\Lambda_+^{a(b)}(\mathbf{p})$ is the usual positive energy projection operator for particle $a(b)$. We thus obtain

$$[\mu_a E + p_4 - E_a(\mathbf{p}) + i\varepsilon][\mu_b E - p_4 - E_b(\mathbf{p}) + i\varepsilon]\Lambda_+^a(\mathbf{p})\Lambda_+^b(\mathbf{p})\psi(p_\mu)$$
$$= -(2\pi i)^{-1}\left(\frac{-e^2}{2\pi^2}\right)\Lambda_+^a(\mathbf{p})\Lambda_+^b(\mathbf{p})\int \frac{\mathrm{d}^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p}'-\mathbf{p})^2}, \tag{7}$$

where

$$E_{a,b} = (\mathbf{p}^2 + m_{a,b}^2)^{1/2}. \tag{8}$$

Because of the use of the instantaneous approximation, the RHS of (7) is a function of $\mathbf{p}$ alone: on grounds of consistency therefore we may set

$$\text{LHS of (7)} = S(\mathbf{p}), \tag{9}$$

whence (7) becomes

$$S(\mathbf{p}) = \frac{e^2}{4\pi^3 i}\Lambda_+^a(\mathbf{p})\Lambda_+^b(\mathbf{p})\int \frac{\mathrm{d}^4 p'_\mu [\Lambda_+^a(\mathbf{p}')\Lambda_+^b(\mathbf{p}')]^{-1} S(\mathbf{p}')}{(\mathbf{p}'-\mathbf{p})^2\, D(p'_4, \mathbf{p}')}, \tag{10}$$

where

$$D(p'_4, \mathbf{p}') = [\mu_a E + p'_4 - E_a(\mathbf{p}') + i\varepsilon][\mu_b E - p'_4 - E_b(\mathbf{p}') + i\varepsilon]. \tag{11}$$

The procedure for carrying out the integration over p'_4 in (10) is standard: one writes

$$\frac{1}{D(p'_4, \mathbf{p}')} = \frac{A}{[\mu_a E + p'_4 - E_a(\mathbf{p}') + i\varepsilon]} + \frac{B}{[\mu_b E - p'_4 - E_b(\mathbf{p}') + i\varepsilon]}$$

and determines A and B; use is then made of the Dirac identity

$$\frac{1}{x - a \pm i\varepsilon} = \mp i\pi\delta(x-a) + \frac{P}{x-a},$$

and the result (Copson 1970) that the principal value integral

$$P\int\frac{\mathrm{d}x}{(x-u)(x-v)}=0 \tag{12}$$

for real u and v.

Thus, in (10)

$$\int\frac{\mathrm{d}p_4'}{D(p_4',\mathbf{p}')}=\frac{-2i\pi}{E-E_a(\mathbf{p}')-E_b(\mathbf{p}')}, \tag{13}$$

so that we obtain

$$S(\mathbf{p})=-\frac{e^2}{2\pi^2}\Lambda_+^a(\mathbf{p})\Lambda_+^b(\mathbf{p})\int\frac{\mathrm{d}^3\mathbf{p}'[\Lambda_+^a(\mathbf{p}')\Lambda_+^b(\mathbf{p}')]^{-1}S(\mathbf{p}')}{(\mathbf{p}-\mathbf{p}')^2[E-E_a(\mathbf{p}')-E_b(\mathbf{p}')]}. \tag{14}$$

With the definition

$$\frac{[\Lambda_+^a(\mathbf{p})\Lambda_+^b(\mathbf{p})]^{-1}S(\mathbf{p})}{[E-E_a(\mathbf{p})-E_b(\mathbf{p})]}=\phi(\mathbf{p}) \tag{15}$$

and the use of nonrelativistic kinematics

$$|\mathbf{p}|\ll m_a, m_b, \tag{16}$$

$$[E-E_a(\mathbf{p})-E_b(\mathbf{p})]\simeq W-\frac{p^2}{2\mu}, \tag{17}$$

where

$$W=E-m_a-m_b \tag{18}$$

and

$$\frac{1}{\mu}=\frac{1}{m_a}+\frac{1}{m_b}, \tag{19}$$

(14) becomes identical with (1).

Before we go on to consider the BSE for the spinor-scalar case, or the scalar-scalar case, let us enumerate the various steps which comprise reduction. These are: (a) the use of the ladder approximation, (b) the use of the instantaneous approximation, (c) the neglect of the negative energy components, (d) the neglect of the spins of the electron and the proton, and (e) the use of nonrelativistic kinematics. We note that while the photon has been treated as a spin-one particle in the foregoing, it could equally well have been treated as spinless—without affecting the process of reduction. This remark is important for the remainder of our considerations.

2.2 *The spinor-scalar BSE*

We identify a and b in (2) as in § 2·1 above, except that we regard b (the proton) to be spinless. As for c (the photon), let us note that if we were to treat it as a spin-one particle, the interaction operator in (2) will have the form (e.g., Mainland 1986).

$$I_{ab}^c\sim\frac{\gamma_4^a(2p_4-p_4')}{\mathbf{p}'^2}; \tag{20}$$

whence, despite the use of the instantaneous approximation, the kernel of the BSE will continue to be a function of **p** as well as p_4. Reduction of the equation to a 3-dimensional form via the use of an equation analogous to (9) would then be no longer possible. In view of this difficulty and the fact (noted above) that the reduction of the spinor-spinor BSE is not affected by the spin of the photon, we assume that the photon is spinless. Let $g_a(f_b)$ denote the strength of interaction between the photon and the electron (proton). From (2) we then have

$$[\gamma^a_\mu \mu_a P_\mu + \gamma^a_\mu p_\mu - m_a + i\varepsilon][\mu_b^2 P_\mu^2 + p_\mu^2 - 2\mu_b p_\mu P_\mu - m_b^2 + i\varepsilon]\psi(p_\mu)$$
$$= (-2\pi i)^{-1}\left(\frac{-g_a f_b}{2\pi^2}\right)\int \frac{d^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p}' - \mathbf{p})^2}, \tag{21}$$

where P_μ, p_μ, m_a, etc. have the same significance as earlier. However, $\psi(p_\mu)$ is now a 4-component spinor.

From dimensional considerations it follows that the product $(g_a f_b)$ must have the dimensions of mass; its magnitude shall be fixed later.

We choose $P_\mu = (E, \mathbf{0})$ and multiply (21) by γ^a_4 to obtain

$$[\mu_a E + p_4 - H_a(\mathbf{p}) + i\varepsilon][(p_4 - \mu_b E)^2 - E_b^2(\mathbf{p}) + i\varepsilon]\psi(p_\mu)$$
$$= (-2\pi i)^{-1}\left(\frac{-g_a f_b}{2\pi^2}\right)\gamma^a_4 \int \frac{d^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p} - \mathbf{p}')^2}, \tag{22}$$

where $H_a(\mathbf{p})$ is the Dirac hamiltonian

$$H_a(\mathbf{p}) = \mathbf{p}\cdot\boldsymbol{\alpha}^a - m_a \beta^a. \tag{23}$$

and $E_{a,b}(\mathbf{p})$ are given by (8).

Operating on (22) with the positive energy projection operator $\Lambda^a_+(\mathbf{p})$, we obtain

$$[\mu_a E + p_4 - E_a(\mathbf{p}) + i\varepsilon][(p_4 - \mu_b E)^2 - E_b^2(\mathbf{p}) + i\varepsilon]\Lambda^a_+ \psi(p_\mu)$$
$$= (-2\pi i)^{-1}\left(\frac{-g_a f_b}{2\pi^2}\right)\Lambda^a_+(\mathbf{p})\gamma^a_4 \int \frac{d^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p} - \mathbf{p}')^2}. \tag{24}$$

With the definition

$$[\mu_a E + p_4 - E_a(\mathbf{p}) + i\varepsilon][(p_4 - \mu_b E)^2 - E_b^2(\mathbf{p}) + i\varepsilon]\Lambda^a_+ \psi(p_\mu) = S(\mathbf{p}), \tag{25}$$

(24) becomes

$$S(\mathbf{p}) = (-2\pi i)^{-1}\left(\frac{-g_a f_b}{2\pi^2}\right)\Lambda^a_+(\mathbf{p})\gamma^a_4 \int \frac{d^4 p'_\mu [\Lambda^a_+(\mathbf{p}')]^{-1} S(\mathbf{p}')}{(\mathbf{p} - \mathbf{p}')^2 D(p'_4, \mathbf{p}')}, \tag{26}$$

where

$$D(p'_4, \mathbf{p}') = [\mu_a E + p'_4 - E_a(\mathbf{p}') + i\varepsilon][(p'_4 - \mu_b E)^2 - E_b^2(\mathbf{p}') + i\varepsilon]. \tag{27}$$

Introducing

$$x = p'_4 + \mu_a E, \tag{28}$$

the integral over p'_4 in (26) may be written as

$$\int \frac{dp'_4}{D(p'_4, \mathbf{p}')} = \int \frac{dx}{[x - E_a(\mathbf{p}') + i\varepsilon][x - E + E_b(\mathbf{p}') - i\varepsilon][x - E - E_b(\mathbf{p}') + i\varepsilon]}. \tag{29}$$

Using partial fractions, the Dirac identity, etc., one can show that

$$\int \frac{\mathrm{d}p_4'}{D(p_4',\mathbf{p}')} = \frac{-i\pi}{E_b(\mathbf{p}')[E - E_a(\mathbf{p}') - E_b(\mathbf{p}')]}. \tag{30}$$

Substituting (30) in (26) and defining

$$\frac{\gamma_4^a[\Lambda_+^a(\mathbf{p})]^{-1}S(\mathbf{p})}{E_b(\mathbf{p})[E - E_a(\mathbf{p}) - E_b(\mathbf{p})]} = \phi(\mathbf{p}),$$

we obtain

$$E_b(\mathbf{p})[E - E_a(\mathbf{p}) - E_b(\mathbf{p})]\phi(\mathbf{p}) = \left(\frac{-g_a f_b}{4\pi^2}\right)\int \frac{\mathrm{d}^3\mathbf{p}'\phi(\mathbf{p}')}{(\mathbf{p}'-\mathbf{p})^2}. \tag{31}$$

Upon using (16), we have

$$E_b(\mathbf{p}) \simeq m_b + \frac{\mathbf{p}^2}{2m_b}, \tag{32}$$

and

$$E - E_a(\mathbf{p}) - E_b(\mathbf{p}) \simeq W - \frac{p^2}{2\mu},$$

where W and μ have been defined in (18) and (19), respectively. In the spirit in which terms of the order of p^4 and higher have been neglected so far, we may write

$$E_b(\mathbf{p})[E - E_a(\mathbf{p}) - E_b(\mathbf{p})] \simeq m_b\left[W - \frac{p^2}{2\mu}\left(1 - \frac{W\mu}{m_b^2}\right)\right], \tag{33}$$

so that (31) yields

$$\left[W - \frac{p^2}{2\mu}\left(1 - \frac{W\mu}{m_b^2}\right)\right]\phi(\mathbf{p}) = \left(\frac{-g_a f_b}{4\pi^2 m_b}\right)\int \frac{\mathrm{d}^3\mathbf{p}'\phi(\mathbf{p}')}{(\mathbf{p}'-\mathbf{p})^2}. \tag{34}$$

If we fix (recall that the product $g_a f_b$ has the dimensions of mass)

$$\frac{g_a f_b}{2m_b} = e^2 \tag{35}$$

(34) is identical with (1), except for the occurrence of the term $W\mu/m_b^2$. If one assumes that, as a first approximation, W changes insignificantly because of this term, one finds that

$$\frac{W\mu}{m_b^2} \simeq 7{\cdot}9 \times 10^{-12},$$

which indeed may be neglected in comparison with unity and (34) then reduces to (1).

2.3 *The scalar-scalar BSE*

We identify a, b and c in (2) as in §2·2, except that we also treat a (electron) to be spinless. Let $f_a(f_b)$ denote the strength of interaction between the photon and the electron (proton). With the same restrictions on I^c_{ab} in (2) as in § 2·2, we have, with $P_\mu = (E, \mathbf{0})$,

$$[(p_4 + \mu_a E)^2 - E_a(\mathbf{p}) + i\varepsilon][(p_4 - \mu_b E)^2 - E_b(\mathbf{p}) + i\varepsilon]\psi(p_\mu) = -(2\pi i)^{-1}\left(\frac{-f_a f_b}{2\pi^2}\right)\int \frac{d^4 p'_\mu \psi(p'_\mu)}{(\mathbf{p}' - \mathbf{p})^2}. \tag{36}$$

where $\mu_a, E_a(\mathbf{p})$, etc. have the same significance as earlier; $\psi(p_\mu)$ however is now a scalar. Note that $(f_a f_b)$ has the dimensions of mass squared.

Equation (36) may be written as

$$S(\mathbf{p}) = -(2\pi i)^{-1}\left(\frac{-f_a f_b}{2\pi^2}\right)\int \frac{d^4 p'_\mu S(\mathbf{p}')}{(\mathbf{p}' - \mathbf{p})^2 D(p'_4, \mathbf{p}')}, \tag{37}$$

where

$$S(\mathbf{p}) = D(p_4, \mathbf{p})\psi(p_\mu) \tag{38}$$

and

$$D(p_4, \mathbf{p}) = [(p_4 + \mu_a E)^2 - E_a(\mathbf{p}) + i\varepsilon][(p_4 - \mu_b E)^2 - E_b(\mathbf{p}) + i\varepsilon]. \tag{39}$$

The algebra involved in carrying out the integration over p'_4 in (37) is straightforward, though somewhat more tedious than in the earlier cases. The result is:

$$\int \frac{dp'_4}{D(p'_4, \mathbf{p}')} = \frac{-i\pi[E_a(\mathbf{p}') + E_b(\mathbf{p}')]}{E_a(\mathbf{p}')E_b(\mathbf{p}')[E - E_a(\mathbf{p}') - E_b(\mathbf{p}')][E + E_a(\mathbf{p}') + E_b(\mathbf{p}')]}. \tag{40}$$

Substituting (40) in (37) and defining

$$\phi(\mathbf{p}) = \frac{[E_a(\mathbf{p}) + E_b(\mathbf{p})]S(\mathbf{p})}{E_a(\mathbf{p})E_b(\mathbf{p})[E - E_a(\mathbf{p}) - E_b(\mathbf{p})][E + E_a(\mathbf{p}) + E_b(\mathbf{p})]}, \tag{41}$$

we obtain

$$\frac{E_a(\mathbf{p})E_b(\mathbf{p})[E - E_a(\mathbf{p}) - E_b(\mathbf{p})][E + E_a(\mathbf{p}) + E_b(\mathbf{p})]}{[E_a(\mathbf{p}) + E_b(\mathbf{p})]}\phi(\mathbf{p}) = \left(\frac{-f_a f_b}{4\pi^2}\right)\int \frac{d^3\mathbf{p}'\phi(\mathbf{p}')}{(\mathbf{p}' - \mathbf{p})^2}. \tag{42}$$

Upon using (16) and retaining terms up to order p^2 only, we have

$$\left[W - \frac{p^2}{2\mu}\left(1 - \frac{W}{W_p} - \frac{\mu W}{\rho^2} + \frac{W}{M}\right)\right]\phi(\mathbf{p}) = \left(\frac{-f_a f_b}{4\pi^2 \mu W_p}\right)\int \frac{d^3\mathbf{p}', \phi(p')}{(\mathbf{p}' - \mathbf{p})^2}, \tag{43}$$

where

$$W_p = W + 2m_a + 2m_b, \tag{44}$$

$$\frac{1}{\rho^2} = \frac{1}{m_a^2} + \frac{1}{m_b^2}, \tag{45}$$

and

$$M = m_a + m_b. \tag{46}$$

If we fix (recall that $f_a f_b$ has the dimensions of mass squared)

$$\frac{f_a f_b}{2\mu W_p} = e^2, \tag{47}$$

(43) is identical with (1) except for the terms W/W_p, $\mu W/p^2$ and W/M. These terms are of order 10^{-9}, 10^{-5} and 10^{-8} respectively, if (as a first approximation) one assumes that their occurrence changes W insignificantly. One may then neglect them in comparison with unity, whence (43) reduces to (1).

3. Concluding remarks

(a) Following S, we have shown in this paper that strictly speaking, the familiar equation for hydrogen atom may be obtained—through a suitable limiting procedure—from a parent BSE for an electron and a proton in which the spins of both these particles are duly respected. Subjecting the BSE wherein either/both of these particles is/are considered as spinless to the same procedure yields an equation which contains terms additional to the usual terms in the equation for hydrogen atom. However, as has been shown, these additional terms are negligibly small. We thus conclude that the BSE for an electron and a proton, irrespective of the spins assigned to them (whether $\frac{1}{2}$ or 0), reduces to the familiar equation for hydrogen atom.

(b) The above remark has the following significance for $Z \neq 1$ atoms: one need not worry about whether or not the nuclei of such atoms are in states of integer or half-integer angular momentum and continue to use for them the equation for the hydrogen atom with appropriate replacements for m_b and e^2. This may seem like a statement without significant content because, in practice, when one uses a Schrödinger equation one does not bother about the parent BSE from which it could be obtained. There are situations however, where one acually begins with a BSE, but wishes to solve the corresponding Schrödinger equation. An example of such a situation is the finite-temperature Schrödinger equation (FTSE) for an electron-proton system (Malik *et al* 1989), the importance of which in the context of solar emission lines has recently been pointed out (Malik *et al* 1991). Suppose now that one wishes to deal with the He^+ system in a medium at non-zero temperature. Should one use the said FTSE with appropriate replacements for m_b and e^2, or use a fresh FTSE obtained via the long route in which one begins with the BSE incorporating the appropriate spins, affects temperaturization (Malik *et al* 1989) and finally implements reduction as discussed above? As a matter of fact it is this question that led us to the exercise undertaken here; our finding is that it suffices to use the first alternative, which is also much the simpler of the two.

(c) Dealing with the problem of confinement of quarks provides an example of a situation where the additional terms referred to in remark (a) may not be negligible. An approach to this problem consists of stipulating a confining potential and solving the relevant Schrödinger equation (e.g., Quigg and Rosner 1976); another approach consists of starting with a BSE with a suitable confining kernel and studying the corresponding Schrödinger equation in different limits. We note that for the BSE which reduces to a Schrödinger equation with a harmonic oscillator potential, as dealt with by Alabiso and Schierholz (1977), Henley (1979), and Biswas *et al* (1982), quarks have been considered to be spinless. Since the total energy of such a system is now not restricted by the condition $E \leqslant m_a + m_b$, terms like $W\mu/m_b^2$ and W/M, etc., are no longer expected to be insignificantly small. Let us note however that the reduction *sans the neglect of spin* of the spinor-spinor BSE leads to a much more complicated equation than the equations encountered in this paper. To elaborate, the BS amplitude for such a case is a 16-component bispinor, whose various constituents are coupled. The extraction of the angular dependence of the equation then becomes a nontrivial problem. Such a program has, in fact, been carried out and we refer the reader to Henley (1980) for further details. We conclude by pointing out that, as also noted by Henley (1980) (except that he finds it surprising), the addition of spin leads to new structure in the spectrum for even the nonrelativistic domain.

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