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An analysis of quantum chromodynamic structure function beyond leading order

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Abstract. We obtain approximate solutions of Altarelli–Parisi equations beyond leading logarithmic approximation. Our results suggest quantitative utility of higher order terms in the structure function analysis of deep inelastic scattering.

Keywords. Structure functions; Altarelli–Parisi equations.

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1. Introduction

In recent communications (Choudhury and Saikia 1987, 1989, 1990), we presented results on approximate solutions of Altarelli–Parisi (AP) equations (Altarelli and Parisi 1977) in leading order (LLA). In the present paper, we report such approximation solutions beyond leading logarithmic approximation with second order evolution effects. To this end, we use the second order kernels calculated in perturbative QCD (Curci *et al* 1980; Furmanski and Petronzio 1980). The results are then compared with the recent high Q^2 EMC data (Aubert *et al* 1985). Several authors have already dealt with this problem in recent years (Devoto *et al* 1983; Martin *et al* 1988; Diemoz *et al* 1988). Most of them dealt with the problem numerically but we have obtained analytic results which are approximate. We compare our approximate solutions with the solutions obtained by solving the problem exactly but numerically. We also find out as to which region the approximation works and suggest plausible reasons for it.

2. Formalism

The singlet and non-singlet evolution equations with second order evolution effects are

$$Q^2\frac{\partial}{\partial Q^2}F_2^{S}(x,Q^2)=\int_x^1\left\{\left(\frac{\alpha}{2\pi}\right)P_{FF}^{(0)}(\omega)+\left(\frac{\alpha}{2\pi}\right)^2P_{FF}^{(1)}(\omega)\right\}F_2^{S}(x/\omega,Q^2)\,d\omega$$
$$+\int_x^1\left\{\left(\frac{\alpha}{2\pi}\right)P_{GF}^{(0)}(\omega)+\left(\frac{\alpha}{2\pi}\right)^2P_{GF}^{(1)}(\omega)\right\}G(x/\omega,Q^2)\,d\omega \qquad (1)$$

and

$$Q^2\frac{\partial}{\partial Q^2}F_2^{NS}(x,Q^2)=\int_x^1\left\{\left(\frac{\alpha}{2\pi}\right)P_{FF}^{(0)}(\omega)+\left(\frac{\alpha}{2\pi}\right)^2P_{(+)}^{(1)}(\omega)\right\}F_2^{NS}(x/\omega,Q^2)\,d\omega \qquad (2)$$

where $P_{ij}^{(0)}$ are the Altarelli–Parisi splitting kernels and $P_{ij}^{(1)}$ and $P_{(+)}^{(1)}$ are given by

$$
\begin{aligned}
P_{(+)}^{(1)}(\omega) = {} & \left(\frac{1+\omega^2}{1-\omega}\right)_+ \ln\omega\left[-\frac{32}{9}\ln(1-\omega)+2\ln\omega+\frac{50}{9}\right] \\
& + \left(\frac{322}{27}-\frac{2\pi^2}{3}\right)\left(\frac{1+\omega^2}{1-\omega}\right)_+ + \ln\omega\left[-\frac{16}{9}\left(\frac{3}{1-\omega}+2\omega\right)\right. \\
& \left. + \frac{32}{9}(1+\omega)\right] - \frac{8}{9}(1+\omega)\ln^2\omega + \frac{40}{3}(1-\omega) \\
& - \frac{4}{9}\left(\frac{1+\omega^2}{1+\omega}\right)\int_{\omega/(1+\omega)}^{1/(1+\omega)} \frac{dz}{z}\ln\left(\frac{1-z}{z}\right) + \delta(1-\omega)\zeta,
\end{aligned}
\tag{3}
$$

$$
\begin{aligned}
P_{FF}^{(1)}(\omega) = {} & \frac{8}{3}+\frac{56}{3}\omega-\frac{896}{27}\omega^2+\frac{320}{27}\omega^{-1}+\ln\omega\left(\frac{56}{9}+24\omega+\frac{128}{9}\omega^2\right) \\
& - \frac{56}{9}\ln^2\omega(1+\omega)+\ln\omega\left(\frac{1+\omega^2}{1-\omega}\right)_+\left[\frac{26}{9}-\frac{32}{9}\ln(1-\omega)+2\ln\omega\right] \\
& + \left(\frac{1+\omega^2}{1-\omega}\right)_+\left(\frac{322}{27}-\frac{2\pi^2}{3}\right)-\frac{4}{9}\left(\frac{1+\omega^2}{1+\omega}\right)\int_{\omega/(1+\omega)}^{1/(1+\omega)}\frac{dZ}{Z}\ln\left(\frac{1-Z}{Z}\right) \\
& + \delta(1-\omega)\zeta_F^{(1)},
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
P_{GF}^{(1)}(\omega) = {} & 132-\frac{44}{3}\omega+\left(-\frac{236}{3}+\frac{848}{3}\omega\right)\ln\omega+\frac{80}{3}\omega^{-1} \\
& - \left(\frac{44}{3}+\frac{128}{3}\omega\right)\ln^2\omega-\frac{40}{3}\ln(1-\omega)+\ln\omega(\omega^2+(1-\omega)^2) \\
& \times\left[-\frac{2}{3}\ln\omega+\frac{296}{3}-\frac{32}{3}\ln(1-\omega)\right] \\
& + 2\ln(1-\omega)(\omega^2+(1-\omega)^2)\left[\frac{20}{3}-\frac{10}{3}\ln(1-\omega)\right] \\
& + (\omega^2+(1-\omega)^2)\left(-\frac{356}{3}+\frac{2\pi^2}{9}\right)+12(\omega^2+(1+\omega)^2) \\
& \times\int_{\omega/(1+\omega)}^{1/(1+\omega)}\frac{dz}{z}\ln\left(\frac{1-z}{z}\right)
\end{aligned}
\tag{5}
$$

where $\zeta = -0{\cdot}1598$ and $\zeta_F^{(1)}$ is defined as

$$
P_{FF}^{(1)}(\omega) = \hat{P}_{FF}^{(1)}(\omega) + \delta(1-\omega)\zeta_F^{(1)}. \tag{6}
$$

Here the diagonal entries of $P_{FF}^{(1)}(\omega)$ (Eq. (4)) are rewritten by separating the part which is proportional to $\delta(1-\omega)$ with co-efficient $\zeta_F^{(1)}$. The co-efficient $\zeta_F^{(1)}$ is then related to the corresponding probabilities $P_{FF}^{(1,T)}$ and $P_{GF}^{(1,T)}$ for the time-like processes via the dispersion relations (Curci *et al* 1980). It is given by

$$
\zeta_F^{(1)} = -\int_0^1 d\omega\,\omega\,[\hat{P}_{FF}^{(1,T)}(\omega) + \hat{P}_{GF}^{(1,T)}(\omega)]. \tag{7}
$$

Explicit forms of $P_{FF}^{(1,T)}$ and $P_{GF}^{(1,T)}$ are given by Furmanski and Petronzio (1980). We also note that the running coupling constant in higher order has the form

$$\frac{\alpha(Q^2)}{2\pi}=\frac{2}{\beta_0\ln(Q^2/\Lambda^2)}\left[1-\frac{\beta_1}{\beta_0^2}\frac{\ln\ln(Q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2)}\right] \tag{8}$$

where $\beta_0=25/3$, $\beta_1=154/3$ in four flavour case.

To proceed further, let us use the variable (Furmanski and Petronzio 1981)

$$t=\frac{2}{\beta_0}\ln[\alpha(Q_0^2)/\alpha(Q^2)] \tag{9}$$

so that

$$\frac{\alpha(t)}{2\pi}=\frac{\alpha(t_0)}{2\pi}\exp\left(-\frac{\beta_0}{2}t\right). \tag{10}$$

With variable t defined in (9), eq. (2) becomes

$$\frac{dF_2^{NS}}{dt}(x,t)=\int_x^1\left[P_{FF}^{(0)}(\omega)+\frac{\alpha(t)}{2\pi}\left\{P_{(+)}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{FF}^{(0)}(\omega)\right\}\right]F_2^{NS}\left(\frac{x}{\omega}t\right)d\omega. \tag{11}$$

Let us now assume that x and t dependence of the structure functions are factorizable (Choudhury and Saikia 1989):

$$F_2^{NS}(x,t)=f^{NS}(x)h^{NS}(t), \tag{12}$$

with the condition

$$f^{NS}(x)=F_2^{NS}(x,t_0), \tag{13}$$

so that

$$f^{NS}(x)\frac{dh^{NS}}{dt}(t)=h^{NS}(t)\int_x^1 P_{FF}^{(0)}(\omega)f^{NS}(x/\omega)d\omega$$
$$+\frac{\alpha(t)}{2\pi}h^{NS}(t)\int_x^1\left[P_{(+)}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{FF}^{(0)}(\omega)\right]f^{NS}(x/\omega)d\omega. \tag{14}$$

Dividing by $f^{NS}(x)$ and using (10), (14) can be written as

$$\frac{dh^{NS}}{dt}(t)=\left[I_1^{NS}(x)+I_2^{NS}(x)\frac{\alpha(t)}{2\pi}\exp\left(-\frac{\beta_0}{2}t\right)\right]h^{NS}(t) \tag{15}$$

where

$$I_1^{NS}(x)=\int_x^1 P_{FF}^{(0)}(\omega)\frac{f^{NS}\left(\frac{x}{\omega}\right)}{f^{NS}(x)}d\omega \tag{16}$$

and

$$I_2^{NS}(x)=\int_x^1\left\{P_{(+)}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{FF}^{(0)}(\omega)\right\}\frac{f^{NS}\left(\frac{x}{\omega}\right)}{f^{NS}(x)}d\omega. \tag{17}$$

Solution of (15) is

$$h^{\rm NS}(t)=h^{\rm NS}(t_0)\exp\left\{I_1^{\rm NS}(x)\cdot t-\frac{2}{\beta_0}\frac{\alpha(t)}{2\pi}I_2^{\rm NS}(x)\left(1-\exp\left(\frac{\beta_0 t}{2}\right)\right)\right\}. \tag{18}$$

Let us now find similar solution of (1). Using t variable defined in (9), eq. (1) can be recast as

$$\frac{{\rm d}F_2^{\rm S}}{{\rm d}t}(x,t)=\int_x^1\left[P_{\rm FF}^{(0)}(\omega)+\frac{\alpha(t)}{2\pi}\left\{P_{\rm FF}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{\rm FF}^{(0)}(\omega)\right\}\right]F_2^{\rm S}\left(\frac{x}{\omega},t\right){\rm d}\omega$$
$$+\int_x^1\left[P_{\rm GF}^{(0)}(\omega)+\frac{\alpha(t)}{2\pi}\left\{P_{\rm GF}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{\rm GF}^{(0)}(\omega)\right\}\right]G\left(\frac{x}{\omega},t\right){\rm d}\omega. \tag{19}$$

We now assume the factorizability of singlet and gluon structure functions:

$$F_2^{\rm S}(x,t)=f^{\rm S}(x)h^{\rm S}(t), \tag{20}$$

$$G(x,t)=f^{g}(x)h^{g}(t) \tag{21}$$

with the conditions

$$f^{\rm S}(x)=F_2^{\rm S}(x,t_0)$$
$$f^{g}(x)=G(x,t_0) \tag{22}$$

so that (19) is expressible as

$$\frac{{\rm d}h^{\rm S}}{{\rm d}t}(t)=h^{\rm S}(t)\int_x^1\left[P_{\rm FF}^{(0)}(\omega)+\frac{\alpha(t)}{2\pi}\left\{P_{\rm FF}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{\rm FF}^{(0)}(\omega)\right\}\right]\frac{f^{\rm S}(x/\omega)}{f^{\rm S}(x)}{\rm d}\omega$$
$$+h^{g}(t)\int_x^1\left[P_{\rm GF}^{(0)}(\omega)+\frac{\alpha(t)}{2\pi}\left\{P_{\rm GF}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{\rm GF}^{(0)}(\omega)\right\}\right]\frac{f^{g}(x/\omega)}{f^{\rm S}(x)}{\rm d}\omega. \tag{23}$$

Let us now assume that the t evolution of gluon is similar to that of singlet (Choudhury and Saikia 1989)

$$h^{g}(t)\approx h^{\rm S}(t) \tag{24}$$

so that (23) can be recast as

$$\frac{{\rm d}h^{\rm S}(t)}{h^{\rm S}(t)}=\left[I_1^{\rm S}(x)+I_2^{\rm S}(x)\frac{\alpha(t_0)}{2\pi}\exp\left(-\frac{\beta_0}{2}t\right)\right]{\rm d}t \tag{25}$$

where

$$I_1^{\rm S}(x)=\int_x^1\left[P_{\rm FF}^{(0)}(\omega)f^{\rm S}(x/\omega)+P_{\rm GF}^{(0)}(\omega)f^{g}(x/\omega)\right]\frac{{\rm d}\omega}{f^{\rm S}(x)}, \tag{26}$$

$$I_2^{\rm S}(x)=\int_x^1\left[\left\{P_{\rm FF}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{\rm FF}^{(0)}(\omega)\right\}f^{\rm S}(x/\omega)\right.$$
$$\left.+\left\{P_{\rm GF}^{(1)}(\omega)-\frac{\beta_1}{2\beta_0}P_{\rm GF}^{(0)}(\omega)\right\}f^{g}(x/\omega)\right]\frac{{\rm d}\omega}{f^{\rm S}(x)}. \tag{27}$$

The solution of (25) is

$$h^{S}(t)=h^{S}(t_0)\exp\left\{I_1^{S}(x)\cdot t-\frac{2}{\beta_0}\frac{\alpha(t)}{2\pi}I_2^{S}(x)\left(1-\exp\left(\frac{\beta_0}{2}t\right)\right)\right\}. \tag{28}$$

Equations (18) and (28) can be used to obtain

$$F_2^{NS}(x,Q^2)=F_2^{NS}(x,Q_0^2)\left[\frac{\alpha(Q_0^2)}{\alpha(Q^2)}\right]^{(2/\beta_0)I_1^{NS}(x)}$$
$$\times\exp\left[I_2^{NS}(x)\frac{2}{\beta_0}\frac{\alpha(Q^2)}{2\pi}\left\{\frac{\alpha(Q_0^2)}{\alpha(Q^2)}-1\right\}\right], \tag{29}$$

$$F_2^{S}(x,Q^2)=F_2^{S}(x,Q_0^2)\left[\frac{\alpha(Q_0^2)}{\alpha(Q^2)}\right]^{(2/\beta_0)I_1^{S}(x)}$$
$$\times\exp\left[I_2^{S}(x)\frac{2}{\beta_0}\frac{\alpha(Q^2)}{2\pi}\left\{\frac{\alpha(Q_0^2)}{\alpha(Q^2)}-1\right\}\right] \tag{30}$$

which are our main results in evaluating the structure function

$$F_2^{\mu P}(x,Q^2)=\frac{3}{18}F_2^{NS}(x,Q^2)+\frac{5}{18}F_2^{S}(x,Q^2). \tag{31}$$

3. Results

We discuss here how our formalism conforms to experiment. To use the results, first we evaluate $\zeta_F^{(1)}$ defined in (6) and (7). We find the value to be 24·5157 which differs from that of Floratos *et al* (1981) and Baulieu and Kounnas (1982) by 9·6%. Such difference may arise because we have used the four-point Gaussian quadrature in the evaluation of the integrals occurred in $\zeta_F^{(1)}$.

The integrals I_1^{NS}, I_2^{NS}, I_1^{S} and I_2^{S} defined in eqs (16), (17), (26) and (27) are evaluated numerically. The input functions $F_2^{NS}(x,t_0)$, $F_2^{S}(x,t_0)$ and $G(x,t_0)$ are taken from Harriman *et al* (Harriman *et al* 1990). These inputs are obtained by making fits to recent DIS data and using the evolution equations beyond the LLA. Hence it will be possible to compare our approximate solutions with the exact but numerical ones starting from the same input densities.

In figure 1, we present $F_2(x,Q^2)$ as a function of Q^2 at x values $x=0{\cdot}03$, 0·05, 0·08, 0·125, 0·175, 0·25, 0·35, 0·45, 0·55, 0·65 and 0·75 covered by EMC data (Aubert *et al* 1985). The dotted lines represent the results of Harriman *et al* (1990) while the solid ones correspond to our results. We observe the following pattern in our results:
At $x=0{\cdot}03$, the difference with exact result is nearly 30%. At $x=0{\cdot}05$, 0·08, 0·125, and 0·175 it becomes 25%, 18%, 15% and 12% respectively. At $x=0{\cdot}75$ and at low Q^2, the difference is 8·6% which decreases to zero at higher Q^2. The pattern indicates that at small x, the present approximation seems to differ from exact numerical results. This may be due to higher evolution of gluons than suggested in (24).

To conclude, our results suggest the quantitative utility of the approximate solutions of Altarelli–Parisi equations beyond leading order in the structure function analysis of deep inelastic scattering.

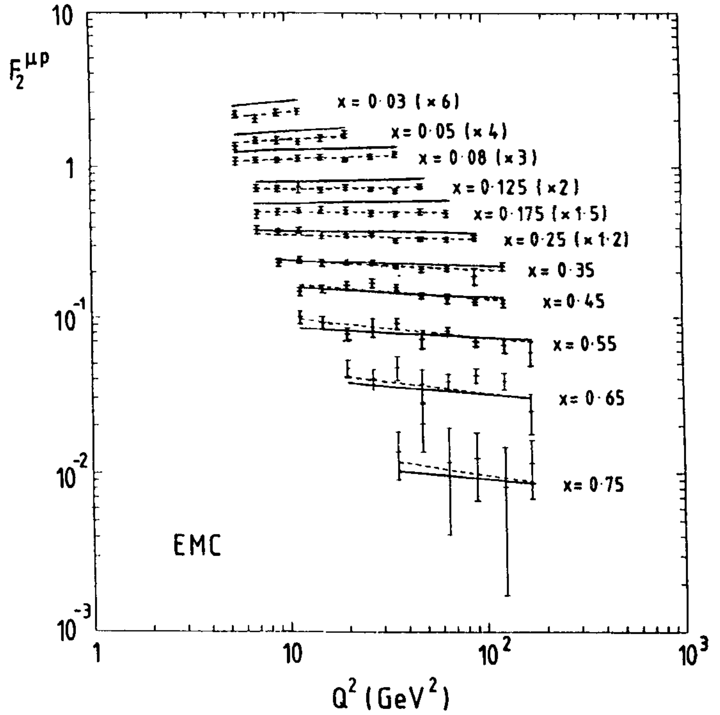

Figure 1. We present $F_2(x, Q^2)$ as function of Q^2 at representative x values, viz $x = 0{\cdot}03$, 0·05, 0·08, 0·125, 0·175, 0·25, 0·35, 0·45, 0·55, 0·65 and 0·75 covered by the EMC data (Aubert *et al* 1985). The dotted lines represent the exact numerical results of Harriman *et al* (1990) while the solid ones correspond to our approximate but analytical results.

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