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Slowly rotating white holes

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Abstract. A model of gravitationally anticollapsing objects, white holes, is constructed on the basis of the Kerr metric in the limit of small rotation with a corresponding interior metric. The extended space-time manifold is considered and the spectral shift of radiation from the point of view of a remote observer is calculated for different parameters of such white holes.

Keywords. Kerr metric; white hole; extended manifold.

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1. Introduction

White holes (WH) are known to be gravitationally anticollapsing objects obtained by means of time inversion in solutions of general relativity (GR) for gravitationally collapsing objects, black holes (BH) (Penrose 1979). The model of a WH on the basis of extended space-time manifolds (STM) with horizons (see, Hawking and Ellis 1973), the so called otonic model* (Trofimenko 1978, 1986, 1988; Trofimenko and Gurin 1986), includes the symmetrical consideration of WH and BH (anticollapsar and collapsar) in complete STMs. Within the framework of this model there is the possibility of avoiding the main problems of WHs (Trofimenko and Gurin 1989), which were orginally discovered in the model of 'retarded cores'. This model is based on the assumption that a part of matter is retarded in its cosmological expansion as compared with the metagalaxy's matter (Novikov 1964; Ne'eman 1965). The main cause of problems of the WH model consists in the effect of infinite blue shift at the event horizon that leads to the catastrophic quantum particle creation and accretion transforming a WH into BH (Eardley 1974; Zeldovich *et al* 1974; Lohiya and Panchapakesan 1978; Wald and Ramaswamy 1980; Redmount 1985; Novikov and Frolov 1987).

The otonic model of WH has a high heuristic potential for explanation of different cosmic phenomena. The specific features of variety of regions in STM of WH give possibility of the stable existence of anticollapsing bodies and their manifestations as ejection of matter from WHs (Trofimenko and Gurin 1989).

*By the term 'oton' one means an object which is described within the framework of GR with event horizons. This term has been firstly suggested by Zeldovich and Novikov (1971) and generalized further in the work of Trofimenko (1978, 1986) and Trofimenko and Gurin (1986). The typical representatives of otons are BHs and WHs.

However, there are only a few papers devoted to the analysis of rotating WHs (Dadhich 1977; Trofimenko 1989; Trofimenko and Gurin 1989). In the Dadhich's work (1977) the spectral shift of radiation emitted from the WH's surface was calculated in the two-dimensional case for the symmetry axis and in the equatorial plane, when the Kerr–Newman metric is reduced to the Reissner–Nordström-like form. Such simplification gives the possibility to perform analytical calculations. According to the features of the spectral shift, anticollapsars are divided into four classes depending on the relative location of WH boundaries and horizons (Trofimenko 1989; Trofimenko and Gurin 1989). However, there is a certain risk in the similar method due to the usage of the reduced two-dimensional metric for description of the rotating body field with axial symmetry, and the Dadhich's model reflects the situation only qualitatively. The present analysis shows the role of causal structure of STM in the determination of the anticollapsar's fate and features of its radiation. The spectral shift is one of the important characteristics from the point of view of the observational identification of WHs.

2. The model of slowly rotating white hole

The complete model of the rotating WH analogous to that for a rotating BH must include the description of the rotating anticollapsar's dynamics on the background of the extended Kerr (Kerr–Newman) manifold. Such a procedure is rather complicated in connection with the necessity to solve non-stationary Einstein equations and cannot be performed by analytical methods. Similar problem for gravitational collapse of rotating bodies to the Kerr's object was carried out by numerical methods (Nakamure *et al* 1980, 1981; Evans 1986, Piran and Stark 1986). The final stage was shown to be either BH or naked singularity. By time inversion, for anticollapse and WHs one can conclude that the dynamics of the WH's matter matched with the Kerr metric can also be strictly investigated.

We shall consider a certain approximation to the complete problem, which allows an analytic solutions and deals with the case of slow rotation neglecting the terms with a^2 (as compared with M^2 and r^2 in geometrized units, $c = G = 1$). Of course, the simplification of the STM structure in such cases is an explicit deficiency of this approach, since the interior horizon ($r = r_-$) and the surface of infinite blue shift ($r = R_-$) are very close at the physical singularity ($r = 0, \theta = \pi/2$) and we can obtain the so-called quasi-Kerr STM (figure 1). For slowly rotating BHs this metric was used by Flores and Lopez (1987).

In order to construct the WH model we must match the interior geometry of expanding and slowly rotating matter with the exterior one. For the latter we take the known Kerr metric in the oblate quasispheroidal Boyer–Lindquist coordinates (Chandrasekhar 1983) and neglect the terms containing the second and higher powers of a:

$$\mathrm{d}s^2 = (1 - 2M/r)\mathrm{d}t^2 - (1 - 2M/r)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2) + (4Ma/r)\sin^2\theta\mathrm{d}t\mathrm{d}\varphi, \tag{1}$$

where M is the mass of the WH. This slowly rotating Kerr metric is in essence the Schwarzschild one with the additional off-diagonal coefficient $g_{t\varphi} = 4Mar^{-1}\sin^2\theta$

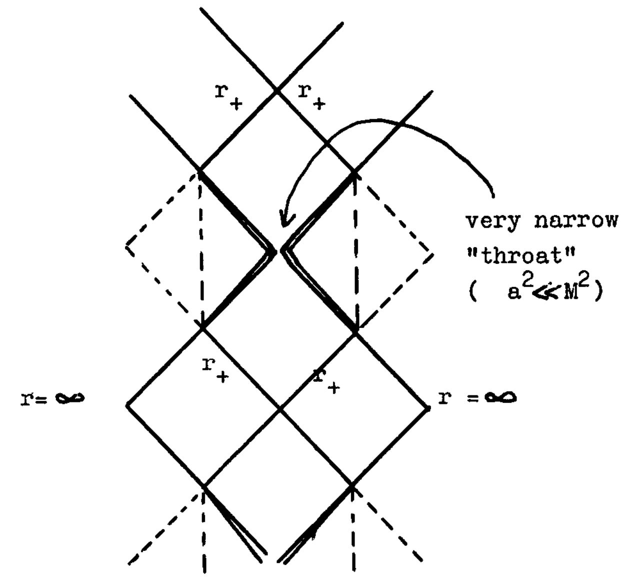

Figure 1. The Penrose diagram for extended STM of slowly rotating Kerr metric. Small difference between r_- and $r = 0$ appears rather essential for causal structure of the space-time.

and has the one horizon $r = 2M$. The off-diagonal terms is singular at $r = 0$, and at the symmetry axis it vanishes. In the extended STM the true singularity will not be purely spacelike (i.e. neither in the usual Kerr case nor in the Schwarzschild one), since when $r \to 0$

$$ds^2 = -(2M/r)dt^2 + (4Ma/r)\sin^2\theta\, dt d\varphi, \tag{2}$$

and $g_{t\varphi}$ contributes significantly. Hence, the structure of STM can be pictured in the form of the Penrose diagram reducing the Kerr diagram when $r = r_- = M - (M^2 - a^2)^{1/2}$ the true singularity occurs, and thus we have the set of submanifolds connected by very narrow throats, since $a \neq 0$ (figure 1).

As the interior metric we use the metric of slowly rotating object collapsing into the Kerr BH. It is explicitly time-dependent in the synchronous coordinates (Flores and Lopez 1987):

$$\begin{aligned} ds^2 = d\tau^2 - (r_0/R_0)^2 dR^2 - R^2(d\Theta^2 + \sin^2\Theta d\Phi^2) \\ + 4Mar_0^{1/2}RR_0^{-2}\sin^2\Theta dR d\Phi, \end{aligned} \tag{3}$$

in which $R = R_0$ determines the surface of anticollapsar and

$$r_0 = r_g^{1/3}[(3/2)(R_0 - \tau)]^{2/3}, \; r_g = 2M. \tag{4}$$

The matching procedure of (1) and (3) is performed as well as in the construction of the canonical WH model (Narlikar *et al* 1974) taking into account the off-diagonal term. In the synchronous coordinates

$$\tau = t \pm \int (2Mr)^{1/2}(r - 2M)^{-1}\, dr \tag{5}$$

we get $g_{r\varphi} = 4Mar^{-1}\sin^2\theta$, and immediate comparison gives at $\Theta = \theta, \Phi = \varphi$:

$$r_0^{1/2} R R_0^{-2} = r^{-1} \tag{6}$$

and at the WH boundary

$$r = R_0 r_0^{-1/2}. \tag{7}$$

Its motion will be constrained by the following equation in dependence on the external coordinates (t, r, θ, φ)

$$(\mathrm{d}r/\mathrm{d}t)_{R=R_0} = F(\tau) \tag{8}$$

in the proper time of the anticollapsar (τ). For this equation let us determine

$$\mathrm{d}r/\mathrm{d}\tau = \tfrac{1}{2} R_0 r_g^{-(1/6)} [3(R_0 - \tau)/2]^{(4/3)} \tag{9}$$

and

$$\mathrm{d}t/\mathrm{d}\tau = (1 - 2M/r)^{-1} = \{1 - r_g^{(2/3)} [3(R_0 - \tau)/2]^{-2/3}\}^{-1} \tag{10}$$

for WH expanding up to infinity (i.e. for the free fall of a collapsing matter reversed in time) from the corresponding geodesics equations (Chandrasekhar 1983).

In the general case (grey hole expanding not up to infinity), one should take into account the values of the total energy (E) and angular momentum of the boundary (L), then

$$\mathrm{d}t/\mathrm{d}\tau = (Er - 2MaL) r^2 (r - 2M), \tag{11}$$

and substitute r by its expression from (4) and (7). Thus from (8)–(10) we obtain

$$(\mathrm{d}r/\mathrm{d}t)_0 = \tfrac{1}{2} R_0 r_g^{-(1/6)} [3(R_0 - \tau)/2]^{-(4/3)} \{1 - r_g^{2/3} [3(R_0 - \tau)/2]^{-2/3}\}. \tag{12}$$

3. Spectral shift of radiation from white hole

In order to calculate the spectral shift we shall use the following familiar expression

$$\frac{\nu}{\nu_0} = \frac{(u_i v^i)_1}{(u_i v^i)_2}, \tag{13}$$

where ν is frequency of reception, ν_0 is the frequency of emission, the subscript 1 denotes emitter, the subscript 2 receiver, v^i and u_i are velocities and directions of emission from WH.

For the remote receiver we assume

$$(v^i)_2 = (1, 0, 0, 0);\ (u_i)_2 = (1, 0, 0, 0), \tag{14}$$

and for the anticollapsar with the metric (3) (Flores and Lopez 1987):

$$(v^i)_1 = (1, 0, 0, (3/2) a (R_0 / R r_0)^2) \tag{15}$$

and components of $(u_i)_1$ can be determined from equations for null geodesics in the

Kerr field with the given approximation

$$(\mathrm{d}r/\mathrm{d}\tau)^2 = E^2 r^2 - (L^2 + Q) + (2M/r)(Q + L^2 - 2aEL), \tag{16}$$

where Q is the fourth Kerr invariant for separation of variables in the corresponding Hamilton–Jacobi equation (Chandrasekhar 1983), and E and L will be now the parameters of emitted photons. Thus,

$$\nu/\nu_0 = [(\mathrm{d}t/\mathrm{d}\tau)_1 - (3/2)ar_1^2 R_0{}^2(\mathrm{d}r/\mathrm{d}\tau)_1]. \tag{17}$$

After some algebra we can derive the expression for spectral shift in the dependence on the location of emission point, r_1, WH surface, R_0, and characteristics of an emitted photons: E, L, Q, which can be connected with the frequency and the 'reversed' impact parameters:

$$\nu/\nu_0 = (r_1 E - 2aML)(r_1 - 2M)^{-1} r_1^{-2} - (3/2)ar_1^2 R_0^{-2} \times [E^2 r_1^2 - (L^2 + Q) + (2M/r_1)(L^2 + Q - 2aEL)]^{1/2}. \tag{18}$$

We show the cumbersome dependence on various parameters according to this expression in the plots (figures 2–4), which indicate that the spectral shift can be very large for specific values of a, L, E, M, etc. It increases rapidly when emission occurs from deep points of the interior and appears greater for bigger WH when $R_0 \sim M$. Parameter of rotation gives the essential contribution in the spectral shift and makes it less. The effect of energetics of emitted photons is seen only for small shifts when $\nu/\nu_0 < 1$ (figure 2).

One of particular cases $L = 1$ and $Q = 0$ is shown in figure 6 and its expression

$$\nu/\nu_0 = (r_1 E - 2aM)/r_1^2(r_1 - 2M) - (3/2)ar_1^2 R_0^{-2} \times [E^2 r_1^2 - 1 + (2M/r_1)(1 - 2aE)]^{1/2} \tag{19}$$

indicates the contribution of small rotation in the second additive; the first additive includes, mainly, the gravitational contribution which diverges at $r_1 \to 0$ and $r_1 \to 2M$.

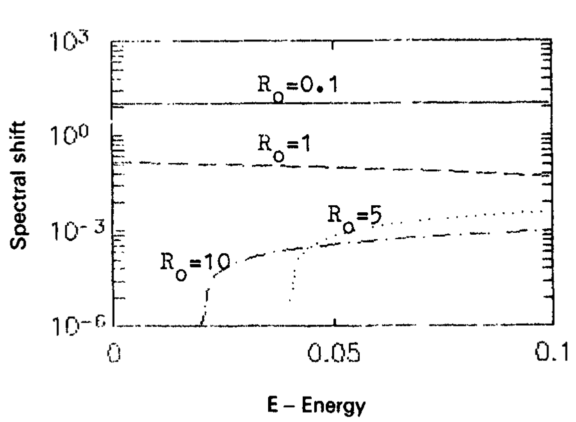

Figure 2. Spectral shift vs energy of photons (E) for WHs of different sizes (R_0). Parameters in the geometrized units: $Q = 1$; $L = 1$; $M = 1$; $a = 0{\cdot}1$.

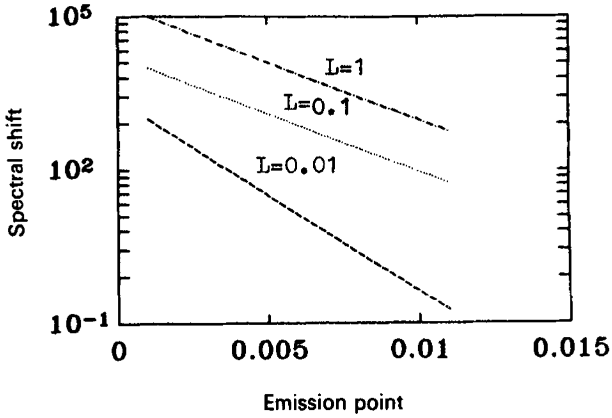

Figure 3. Spectral shift vs location of emission point (r_1) for some values of orbital momentum of emitted photons (L); $Q = 0{\cdot}01$; $M = 1$; $E = 0{\cdot}001$; $R_0 = 0{\cdot}001$.

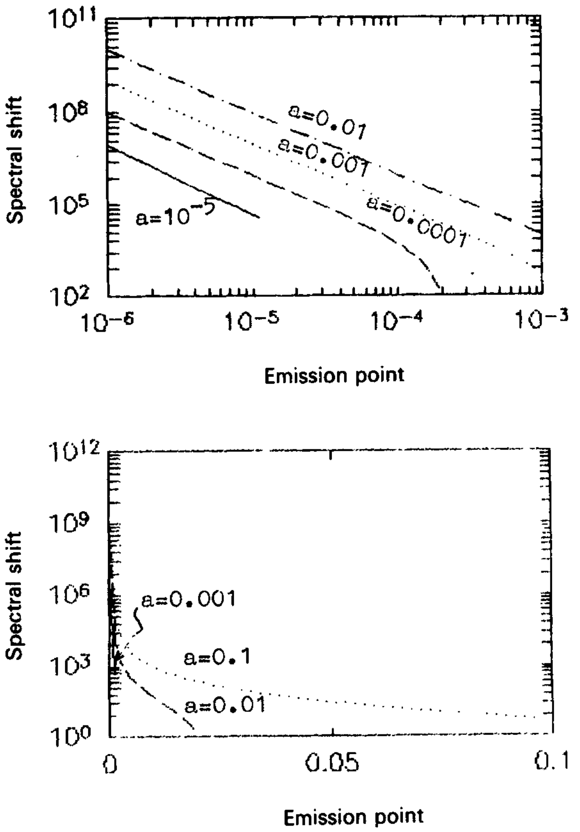

Figure 4. (a, b) Spectral shift vs location of emission point (r_1) for different values of the WH's rotating momentum (a); $Q = 1$; $L = 1$; $M = 1$; $R_0 = 0{\cdot}1$.

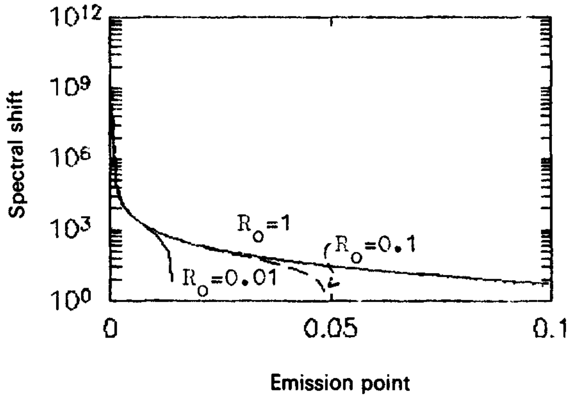

Figure 5. Spectral shift vs location of emission point (r_1) for different size WHs (R_0); $Q = 1$; $L = 1$; $M = 1$; $a = 0{\cdot}1$.

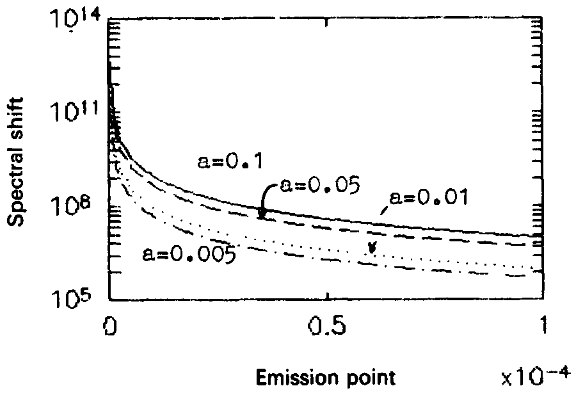

Figure 6. Particular case of the spectral shift dependence ($Q = 0$) on location of emission point for different values of the WH's rotating momentum. $L = 1$; $M = 1$; $R_0 = 10$.

One can conclude that the small rotation can essentially influence upon the value of spectral shift and give differences from the Schwarzschild case.

It is of interest also to consider the case when $L = 2aE$ and $Q = 0$ and gravitational and rotational contributions are maximal:

$$\nu/\nu_0 = E[(r_1 - 4a^2M)/r_1^2(r_1 - 2M) - (3/2)ar_1^2R_0^{-2}(r_1^2 - 4a^2)^{1/2}], \tag{20}$$

The rotation parameter can be $a = r_1/2$, and the second term then will vanish completely while the other value of a when $r_1 = 4a^2M$ gives a negative result. Evidently, this means the absence of radiation for these parameters which could reach a remote observer. Let us notice that the singularity at the event horizon when the emission point coincides with r_g occurs in all these formulas that pick out WHs from other expanding bodies.

If we omit parameters of emitted photons corresponding to rotating field, i.e. L and Q in (18), we can emit up to infinity only for $r_1 > 2M$. Hence, photons without rotating (orbital) momentum can be radiated only above the horizon, when a WH, in fact, transforms into a trivial expanding body, and such photons do not reflect features of STM.

Thus, small rotation makes it possible to build a WH model in an analytic form including different peculiarities of radiation in a rotating gravitational field, and the spectral shift can be considered as the significant characteristic of such rotation available for the observational justification in astrophysics as powerful energy extractions localized in space and transient. Grey and white holes in models with extended STM can be enlisted in explaining a variety of phenomena taking into account features of complete manifolds with cosmological horizon (Geyer 1980; Lake and Weiss 1977; Gurin 1986). For example, they ought to be one more physical realisation of the ideas on superdense D-bodies of Ambartsumian (1986).

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