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Spherical gravitational collapse with photon emission and a generalized Schwarzchild interior solution

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Abstract. The general dynamical equations for perfect fluid filled spheres with an outward flux of photons are derived. The vital role played by the energy density of the free gravitational field in accelerating photon production has been emphasized. It is pointed out that even when the material energy density is finite, the energy density of the free gravitational field can take infinitely large values resulting in vanishing surface area of the star. A generalized Schwarzschild interior solution with conformally flat geometry but with photon emission has been obtained. It is pointed out that the interior conformal coordinate system bears a strong resemblance to the exterior Krushkal coordinates. It is shown that for spherical star the invariant velocity of the fluid particles, falling towards the centre, is proportional to its radius suggesting that the outer envelopes collapse at a faster rate than the core part. It is shown that the interior radiating solution can be matched with generalized Schwarzchild exterior solution.

Keywords. Gravitational collapse; photon emission; energy momentum tensor; trapped surfaces.

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1. Introduction

The qualitative and quantitative nature of spherical gravitational collapse has been discussed in great detail in the literature. It is usual to take the material content of the spheres as consisting of a perfect fluid of proper density ρ and pressure p. The earliest studies on the gravitational collapse of spherical objects with uniform density and vanishing pressure were carried out by Datt (1938) and Oppenheimer and Snyder (1939). They concluded that when all the thermonuclear sources of energy are exhaused a sufficiently heavy star will undergo free collapse resulting in a singularity at the centre. More general situations with uniform density but non-uniform pressure were discussed by Thompson and Whitrow (1967, 1968), Bonnor and Faulkes (1967), Taub (1968), Bondi (1969) and others (For more details, see Kramer *et al* 1980). The non-uniform pressure produces non-vanishing pressure gradients which oppose collapse beyond a certain radius resulting in gravitational bounce and consequent oscillatory motion. The general dynamical equations for spherical gravitational collapse were given by Misner and Sharp (1964).

However, spherical gravitational collapse with escaping photons did not receive the same attention in spite of noteworthy contributions by Michel (1963), Bondi (1964), Misner (1965), Hernandez and Misner (1966) and others. In these studies it was suggested that at some stage of gravitational collapse, the star may eject considerable amount of energy in the form of photons. Michel (1963) argued that an appreciable amount of photon emission will reduce the pressure resulting in the failure of hydrodynamic equilibrium and consequent collapse towards the centre. As the star contracts matter inside gets more and more compressed leading to higher temperatures and even greater rate of neutrino emission. This process will continue till it would not be possible to convert gravitational energy into photons at the rate demanded by the raising temperature after which the star will cool down. Bondi (1964) and Misner (1965) independently developed the formulae for mass loss during the emission of photons. A different formalism more geometrical in nature was given by one of the authors (Krishna Rao 1972) in which the role played by the energy density of the free gravitational field during collapse has been brought to light. On the other hand analytical solutions describing the outward flow of photons were constructed from known static as well as non-static models for qualitative discussion (Vaidya 1951, 1966; Herrera *et al* 1980 and Krori *et al* 1985).

In the present paper we give in § 2, the general dynamical equations for spherical gravitational collapse with photon emission following the procedure of our earlier work (Krishna Rao and Annapurna 1986). A generalized Schwarzchild interior solution with photon emission is given in § 3. The boundary conditions between the interior and exterior radiating solutions are established in § 4 and the concluding remarks given in § 5.

2. The field equations

We assume that the star is filled with a perfect fluid distribution of matter with photon emission along the radial direction and that the photons are neither scattered nor absorbed by the surrounding matter. The energy momentum tensor for such a distribution is given by

$$T_a{}^b = (\rho + p)u_a u^b - p g_a{}^b + N_a{}^b,$$
$$N_a{}^b = q\, k_a k^b, k_a k^a = 0, u_a u^a = 1. \tag{1}$$

Here ρ and p are the proper density and pressure of the fluid respectively, whereas q denotes the energy density of the photons in the rest frame of the fluid. It was shown by Misner (1965) that $u^a\, T^b_{a;b} = 0$ gives

$$(\rho u^b)_{;b} + p u^b{}_{;b} = -nC = -u^a N_{a;b}^{\ b}, \tag{2}$$

where $C(T, n)$ is the cooling rate per unit amount of matter, T and n being the temperature and baryon number density respectively. Also C is termed as the energy generation rate since the energy which is lost by the fluid appears in $N_a{}^b$ as radiation. Also, the equation of motion takes the form

$$(\rho + p)\dot{u}_a = h_a{}^b p_{,b} - N_{a;b}^{\ b} + n\, C\, u_a, \tag{3}$$

where $\dot{u}_a = u_{a;b} u^b$ and $h_a{}^b = g_b{}^a - u_a u^b$. Further, in view of $k_a N^{ab} = 0, k_{a;b} N^{ab} = 0$, we obtain

$$k_a N^{ab}_{\ ;b} = 0. \tag{4}$$

Writing $\rho = n(1+e)$, where e is the specific internal energy, which does not include the rest mass energy, the first part of (2) takes the form

$$e_{,a}u^a + p(1/n)_{,a}u^a = -C. \tag{5}$$

Similarly the thermodynamic relation $\mathrm{d}e = T\mathrm{d}S - p\,\mathrm{d}V$, where S denotes the entropy and $V(=1/n)$ the specific volume, assumes the form

$$TS_{,a}u^a = -C. \tag{6}$$

We now take the geometry of the star as spherical so that the space-time metric is chosen as

$$\mathrm{d}s^2 = -\exp\lambda\,\mathrm{d}r^2 - R^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2) + \exp\nu\,\mathrm{d}t^2, \tag{7}$$

where λ, R, ν are functions of r and t only. Following the procedure adopted in our earlier work (Krishna Rao and Annapurna 1986) and using the fact in a comoving coordinate system $u^a = (0,0,0,\exp(-\nu/2))$ we give below Einstein's field equations for (1) and (7) as

$$8\pi\,T^1_1 = -8\pi(p+q) = (1/R\exp\lambda)\{(R'^2/R) + R'\nu'\} + (1/R\exp\nu)\{2\ddot{R} + (\dot{R}^2/R) - \dot{R}\dot{\nu}\} + (1/R^2), \tag{8}$$

$$8\pi T^2_2 = 8\pi T^3_3 = -8\pi p = 8\pi\varepsilon - (1/R\exp\lambda)\{2R'' - (R'^2/R) - R'(\lambda' + \nu')\} + (1/R\exp\nu)\{2\ddot{R} - (\dot{R}^2/R) + \dot{R}(\dot{\lambda} - \dot{\nu})\} - (1/R^2), \tag{9}$$

$$8\pi T^4_4 = 8\pi(\rho+q) = -(1/R\exp\lambda)\{2R'' + (R'^2/R) - R'\lambda'\} + (1/R\exp\nu)\{\dot{R}^2/R) + \dot{R}\dot{\lambda}\} + (1/R^2), \tag{10}$$

$$8\pi T^1_4 = 8\pi q\exp\{(\nu-\lambda)/2\} = (1/R\exp\lambda)(2\dot{R}' - R'\dot{\lambda} - \dot{R}\nu'), \tag{11}$$

where

$$8\pi\varepsilon = \exp(-\lambda)\{(R''/R) - (R'/R)^2 - (\nu''/2) - (\nu'^2/4) - (R'\lambda'/2R) + (R'\nu'/2R) + (\lambda'\nu'/4)\} + \exp(-\nu)\{(\ddot{\lambda}/2) + (\dot{\lambda}^2/4) - (\ddot{R}/R) + (\dot{R}/R)^2 - (\dot{R}\dot{\lambda}/2R) + (\dot{R}\dot{\nu}/2R) - (\dot{\lambda}\dot{\nu}/4)\} + (1/R^2), \tag{12}$$

is the eigenvalue of the conformal Weyl tensor (Krishna Rao 1966). When $\varepsilon = 0$ the spherically symmetric space-times can be mapped conformally to flat space-times and all uniform density spheres belong to this class (Krishna Rao 1973). Throughout this paper a prime and overhead dot denote differentiation with respect to r and t respectively.

As in our previous work, it is easy to compute the expression for $\exp(\lambda)$ by taking the combination $\{(10)+(8)-(9)\}$. Thus

$$\exp\lambda = R'^2/\Gamma^2 = R'^2 \Big/ \left[1 + U^2 - \frac{8\pi}{3}(\rho+\varepsilon)R^2\right], \tag{13}$$

where, by definition, we write

$$\Gamma = \exp(-\lambda/2)R' = D_r R = \left[1 + U^2 - \frac{8\pi}{3}(\rho+\varepsilon)R^2\right]^{\frac{1}{2}}, \tag{14}$$

$$U = u^a(\partial R/\partial x^a) = \exp(-\nu/2)\dot{R} = D_t R. \tag{15}$$

The operators D_r and D_t used above are unit derivatives in orthogonal space-like and time-like directions respectively. Thus, the operator

$$D_k = D_r + D_t, \tag{16}$$

is the derivative along a radial light ray and

$$D_k R = \Gamma + U, \tag{17}$$

plays an important role in the discussion of trapped null surfaces.

By a suitable choice of the comoving coordinate r the baryon conservation law takes the form

$$\exp(-\lambda/2) = 4\pi n R^2. \tag{18}$$

Thus, combining (14) and (18), we get

$$\Gamma = \exp(-\lambda/2)R' = 4\pi n R^2 R'. \tag{19}$$

Also (5) and (6) respectively take the form

$$D_t e = -p D_t(1/n) - C, \tag{20}$$

$$D_t S = -C/T. \tag{21}$$

It is easy to obtain the various equations which gövern the evolution of the system and we list them below:

$$D_t(\lambda/2) = (\partial U/\partial R) - (L/R), \tag{22}$$

$$D_t \Gamma = U D_r(\nu/2) + (L/R), \tag{23}$$

$$D_t\{4\pi(\rho+\varepsilon)R^3/3\} = -4\pi R^2 p U - L(U+\Gamma), \tag{24}$$

$$D_r\{4\pi(\rho+\varepsilon)R^3/3\} = 4\pi R^2 \rho \Gamma + L(U+\Gamma), \tag{25}$$

$$(\rho+p)D_r(\nu/2) = -D_r p - nC, \tag{26}$$

$$D_t(nR^2) = -nR^2(\partial U/\partial R) + (nRL/\Gamma), \tag{27}$$

$$D_t U = -\Gamma^2(\rho+p)^{-1}(\partial p/\partial R) - (4\pi/3)(\rho+\varepsilon+3p)R - nC\Gamma(\rho+p)^{-1} - (L/R), \tag{28}$$

$$D_k L = 4\pi R^2 nC - (2L/R)\{D_r U + \Gamma D_r(\nu/2) - (L/R)\}, \tag{29}$$

where $L = 4\pi R^2 q$. The five equations—(15), (20), (27), (28) and (29) – are called dynamical equations (Hernandez and Misner 1966).

We may mention here that the present formalism is more suitable to distinguish truly relativistic situations in which the energy density of the free gravitational field plays a crucial role during collapse. In this connection we note that the contribution from the field energy in relation to the material energy can be raised substantially by choosing an appropriate coupling constant (instead of unity which we have chosen here for simplicity). Such a field energy when converted into thermal energy ($\varepsilon \propto T^4$) results in higher temperatures in the star's core and thereby accelerating the photon production. So, the star can shed considerable amount of its mass in the form of outgoing photons. The above arguments can be substantiated from (24) which is nothing but the energy conservation equation. For a collapsing fluid $U < 0$, and we note from the second term on the rhs of (24) that outside the Schwarzschild-like surface $D_k R = \Gamma + U > 0$, so that the photon flux travels along the radial null direction with the fundamental velocity. The first term on the rhs of (24), that is, $-4\pi R^2 pU$, gives the work done by the pressure forces on the collapsing boundary surface of the star.

The role played by the energy density of the free gravitational field in highly relativistic situations can further be understood by considering the phenomenon of formation of trapped surfaces (Penrose 1965). For the spherically symmetric case the condition for the formation of trapped surfaces is given by $D_k R < 0$ (Misner 1967). For a collapsing fluid, with $U \leqslant 0$, after making use of (14) and (15) in (17) we get

$$D_k R = \left\{ 1 - \frac{8\pi}{3}(\rho + \varepsilon)R^2 \right\} \Big/ (\Gamma - U) < 0.$$

Since the denominator, of the above equation is positive, we write the above condition in terms of the surface area $4\pi R^2$ of the spherical object as $(4\pi R^2)^{-1} < 2(\rho + \varepsilon)/3$.

This condition is sufficient, but not necessary, in order that outgoing light rays be trapped. We note that large positive values of ε leads to trapped surfaces with smaller R. We further note that even when the material energy density ρ is finite, infinite space-time curvatures ($\varepsilon \to \infty$) lead to vanishing surface area of the star.

From the general system of equations given above one can obtain simple but meaningful solutions by appropriate choice of the physical as well as geometric variables. For example one can consider the case $p = 0$ but $\varepsilon \neq 0$. Eventhough the physical problem has been simplified by taking the material content of the sphere as a pressureless dust which is inhomogeneous the geometry still poses insurmountable difficulties to obtain an analytical solution in the closed form. However, the case $\varepsilon = 0$, $p \neq 0$ is more easy to handle (Krishna Rao 1969, 1972). In the next section we present one such solution which is a generalization of the well known Schwarzschild interior solution.

3. Generalized Schwarzschild interior solution

The curvature coordinate system in which the Schwarzschild interior solution is expressed presents problems if we consider its generalization with photon emission. However, in the conformally flat coordinate system it offers a simple generalization. Also, in this new coordinate system light geometry is that of a flat Minkowski continuum so that the photons emitted from the core of the star travel outwards without deflection in spite of the fact that the spacetime is inhomogeneous due to non-uniform pressure. Mathematically, the conformally flat form offers the advantage that we shall be dealing with a single function $\psi(\eta, \tau)$.

It is known that the conformally flat form of the Schwarzschild interior solution is given by (Krishna Rao and Patel 1972)

$$ds^2 = \psi^2(\eta, \tau)(-d\eta^2 - \eta^2 d\theta^2 - \eta^2 \sin^2\theta d\phi^2 + d\tau^2), \tag{30}$$

where

$$\psi(\eta,\tau) = (2\pi\rho_0/3)^{\frac{1}{2}}\eta(P + P^{-1}), \tag{31}$$

$$P = (\eta)^{-1}\{(1+\eta^2)^{\frac{1}{2}} - 1\}\exp\{(1+\eta^2)^{\frac{1}{2}} + \tau\} \tag{32}$$

ρ_0 being the constant proper density of the model. The pressure p_0 is given by

$$8\pi p_0 = 4\psi\{2\pi\rho_0/3(1+\eta^2)\}^{\frac{1}{2}}\cosh\{(1+\eta^2)^{\frac{1}{2}} + \tau\} - 8\pi\rho_0. \tag{33}$$

It is interesting to note that the velocity $w(=\mathrm{d}\eta/\mathrm{d}\tau)$ of the material particles in the conformally flat coordinate system is given by

$$w = -\eta/(1+\eta^2)^{\frac{1}{2}}, \tag{34}$$

showing that the particles rush towards the centre ($\eta = 0$) with almost the velocity of light ($C = 1$). Their velocities diminish till they are instantaneously at rest with respect to the origin and then gain as they move away from the origin asymptotically tending to the velocity of light. In the (η,τ) – plane, $P =$ constant are rectangular hyperbolas and thus the conformal coordinate system bears a strong resemblance to the exterior Krushkal coordinate system.

We now obtain the generalization of the Schwarzschild interior solution by using an earlier result (Krishna Rao 1969, 1972). Thus, writing in place of ψ a new function μ such that

$$\mu(\eta,\tau) = \psi(\eta,\tau) + F^*(\eta,u), \tag{35}$$

where $F^*(\eta,u) = F(u) + \eta\bar{F}(u)$, $u = \tau - \eta$ and an overhead bar for F denotes a differentiation with respect to u, the resulting space-time satisfies equations (1). We note that F^* is a retarded function and the term $\eta\bar{F}(u)$ gives the 'correction' for retardation. The expressions for the fluid density ρ, pressure p and neutrino flux density q are given by

$$\begin{aligned} 8\pi\rho = 8\pi\rho_0 &+ (8\pi\rho_0/3)^{\frac{1}{2}}[6F^*\sinh\{(1+\eta^2)^{\frac{1}{2}} + \tau\} \\ &+ \eta(P - P^{-1})\{\bar{F} + [(1+\eta^2)^{\frac{1}{2}} + \tau]\bar{\bar{F}}\}] + (\bar{F}^2 + 2F\bar{\bar{F}}), \end{aligned} \tag{36}$$

$$8\pi p = 4\mu\{2\pi\rho_0/3(1+\eta^2)^{\frac{1}{2}}\cosh\{(1+\eta^2)^{\frac{1}{2}} + \tau\} - 8\pi\rho, \tag{37}$$

$$4\pi q = \eta\mu\bar{\bar{\bar{F}}}, \tag{38}$$

where $-k_1 = k_4$, $k_2 = k_3 = 0$. It may be noted that the neutrino emission has no effect on the fluid velocity w given by (34). Also, it is easy to see that $(\rho_0 + p_0)/\psi = (\rho + p)/\mu$.

In (28), term $(-L/R)$ is termed as the gravitational induction field (Lindquist *et al* 1965) and it is directed towards the centre of force. In the present case the contribution of the induction field is given by $-\mathrm{d}^2(\eta\bar{F})/\mathrm{d}u^2$. Also, the induction field contributes for the increase in the energy of the fluid particles of the medium. We express the invariant velocity of the fluid particles as

$$U = u^a\{\partial(\eta/\mu)/\partial x^a\} = (\psi/\mu)u^a\{\partial(\eta/\psi)/\partial x^a\} - (\eta/\mu)^2 u^a\{\partial(F^*/\eta)/\partial x^a\}, \tag{39}$$

where $u^1 = -\mu\eta$ and $u^4 = \mu(1+\eta^2)^{1/2}$. In (39), the first term in the last expression represents the contribution from the original non-radiating solution and in the case of the static Schwarzschild interior solution it vanishes identically, so that in the present case

$$U = -(\eta/\mu)[F^* + \{\eta + (1+\eta^2)^{1/2}\}\bar{F}^*]. \tag{40}$$

Noting that (η/μ) is the invariant radius of the sphere, it is significant that U is proportional to this quantity suggesting that the outer envelopes of the star fall towards the centre much faster than the core part.

4. Boundary conditions

It is well known that the exterior geometry of spherically symmetric radiating star is given by Vaidya's (1953) metric,

$$ds'^2 = \{1 - 2M(u')/r\}\,du'^2 + 2du'\,dr - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{41}$$

where M is an arbitrary non-increasing function of the retarded time u'. However, it will be convenient, for our present purpose, to write the exterior radiating solution in the form given by Synge (1957). Thus, using a double null coordinate system, we write the exterior Schwarzschild radiating metric as

$$ds'^2 = 2f\,du'\,dv' - r^2(d\theta^2 + \sin^2\theta\, d\phi^2), \tag{42}$$

where

$$f = 2\alpha(v')(\partial r/\partial u'), \tag{43}$$

$$(\partial r/\partial v') = \{-\alpha(v') + \beta(v')/r\}, \tag{44}$$

α and β being arbitrary functions.

Similarly, we express our interior radiating solution given in § 3 as

$$ds^2 = \mu^{-2}[2du\,dv - \{(v-u)^2/2\}(d\theta^2 + \sin^2\theta\, d\phi^2)], \tag{45}$$

where

$$\mu = [(\pi\rho_0/3)^{1/2}(v-u)(P+P^{-1}) + F(u) + 2^{-1/2}(v-u)\bar{F}(u)],$$
$$P = (v-u)^{-1}[\{2+(v-u)^2\}^{1/2} - 2^{1/2}]\exp[2+(v-u)^2+(v+u)]/2^{1/2}. \tag{46}$$

We shall now show that the exterior and interior radiating solutions given by (42) and (45) can be matched on the boundary

$$f(y^i) = (y^1 - y^4) - 2^{1/2}\mu a(y^4) = 0. \tag{47}$$

Since we are using admissible coordinates on both sides of the boundary we have to prove only that the metric tensor and its first derivatives are continuous across the boundary given by (47). We identify the points on both sides of the boundary as

$$v' = A(v), \quad u' = B(u) \tag{48}$$

there being no change in the angular coordinates θ and ϕ.

Thus, the boundary has representation in terms of the interior and exterior coordinates as below:

Int: $$f(y^i) = (y^1 - y^4) - 2^{1/2}\mu a(y^4) = 0; \quad y^1 = v, y^2 = \theta, y^3 = \phi, y^4 = u, \tag{49}$$

Ext: $$f(y'^i) = (y'^1 - y'^4) - 2^{1/2}\mu a(y'^4) = 0; \quad y'^1 = v', y'^2 = \theta, y'^3 = \phi, y'^4 = u'. \tag{50}$$

Now, the continuity of $g_{22}(= g_{33})$ and $g_{14}(= g_{41})$ give the following two conditions respectively:

$$(v - u)\mu^{-1} = 2^{1/2} r, \tag{51}$$

$$\mu^{-2} = 2\alpha(A(v))\tilde{A}\bar{B}^2(\partial r/\partial B), \tag{52}$$

where an overhead tilde denotes differentiation with respect to v. We can eliminate r from (51) and (52) giving a relation between μ, α, A and B.

The continuity of the first derivatives of the metric tensor can be proved with the help of the second fundamental form. We can calculate the vector normal to the surface given by (47) and then the coefficients of the second fundamental form. Thus, after some long and tedious calculations, we get in terms of the interior metric

$$\Omega_{11} = (a\tilde{\tilde{\mu}}/2\mu X^{1/2}) + (\widetilde{\overline{a\mu}}/\mu X^{3/2})(2^{-1/2} - a\tilde{\mu})^2 - (\tilde{\mu}/\mu^2 X^{1/2})(2^{-1/2} - a\tilde{\mu}) \tag{53}$$

$$\Omega_{14} = (\widetilde{\overline{a\mu}}/2\mu X^{1/2}) + (\bar{\mu}/\mu^2 X^{1/2})(2^{-1/2} - a\tilde{\mu}) + (\overline{\overline{a\mu}}/\mu X^{3/2})(2^{-1/2} - a\tilde{\mu})^2 \tag{54}$$

$$\Omega_{41} = (\widetilde{\overline{a\mu}}/2\mu X^{1/2}) - (\tilde{\mu}/\mu^2 X^{1/2})(2^{-1/2} + \overline{a\mu}) + (a\tilde{\tilde{\mu}}/\mu X^{3/2})(2^{-1/2} + \overline{a\mu})^2 \tag{55}$$

$$\Omega_{44} = (\bar{\mu}/\mu^2 X^{1/2})(2^{-1/2} + \overline{a\mu}) + (\overline{\overline{a\mu}}/2\mu X^{1/2}) - (\widetilde{\overline{a\mu}}/\mu X^{3/2})(2^{-1/2} + \overline{a\mu})^2 \tag{56}$$

where

$$X = 2(2^{-1/2} - a\tilde{\mu})(2^{-1/2} + \overline{a\mu}).$$

Similarly, we calculate Ω'_{11}, Ω'_{14}, Ω'_{41} and Ω'_{44} in terms of the exterior metric and after making use of (51) and (52), find that

$$\Omega_{11} = \Omega'_{11}, \quad \Omega_{44} = \Omega'_{44}, \quad \Omega_{14} = \Omega'_{14}, \quad \Omega_{41} = \Omega'_{41}$$

showing that (42) and (45) can be joined on a common boundary given by (47).

5. Conclusion

From the general dynamical equations governing the collapse of spherically symmetric bodies with photon emission, we note that the energy density of the free gravitational field, (which is always coupled to the material energy density), helps to raise substantially the inner temperature of the star thereby accelerating photon production.

Also, even when the material energy density is finite, the energy density of the free gravitational can take infinitely large values leading to the formation of trapped surfaces. The formalism developed here helps to distinguish the mathematically simpler cases where the interior geometry of the star is conformally flat. We have shown that in such cases a direct generalization of the Schwarzschild interior solution with photon emission can be obtained. Further, the strong resemblance between the conformal coordinates used for the description of the interior metric and the exterior Krushkal coordinates suggest that the generalization presented here is the most natural one. Also, our result that, in the absence of the free gravitational field, the outer envelopes of the star collapse towards the centre faster than the core is physically plausible.

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