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Average Λ-nucleus potentials

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Abstract. A few model Λ-nucleus potentials are proposed which explain the ground state binding energy data of ^{5}He and the p-shell hypernuclei satisfactorily. Potential-II is capable of distinguishing the hypernuclei of the same mass number but of different N and Z values. The dependence of this potential on $(N - Z)$ term indicates that there is a charge symmetry breaking component in ΛN force. Alongwith the earlier density dependent effective Λ-nucleus interaction, these potentials may be used to determine approximately the density distributions of light nuclei. From these potentials an estimate of D_Λ is also made. It is found to be in conformity with the earlier estimates.

Keywords. Hypernuclei; ΛN interaction; (K^-, π^-) reaction; (π^+, K^+) reaction.

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Over the past few decades significant progress has been made in the field of hypernuclei. A vast amount of spectroscopic data on various hypernuclei have been accumulated through (K^-, π^-) reactions (Bruckner *et al* 1975, 1976; Bertini *et al* 1970, 1978; Povh 1980). Recently the single particle energies of Λ particle in various hypernuclei have also been measured in studies of (π^+, K^+) reactions (Chrien 1988). The excitation spectra for the (π^+, K^+) process consists of a series of well defined peaks which are identified with various orbital angular momentum states $s_\Lambda, p_\Lambda, d_\Lambda, f_\Lambda \ldots$ of the Λ hyperon. This has renewed the interest in theorists to further investigate the hypernuclear structure and to learn more about the behaviour of Λ particle inside a hypernucleus. Several authors (Dalitz and Downs 1958; Gal 1975; Bodmar and Murphy 1965, 1966; Bouyssy 1977; Dover 1981) working within different framework of analysis have contributed a lot to our understanding about the ΛN interaction and the structure of the host nucleus.

A hypernucleus is a many-body system and the interaction of a Λ particle with the nucleons inside it is predominantly a two-body interaction. In a hypernucleus, the total Λ-nucleus interaction will, therefore, be a sum over all the pairs of the ΛN interaction. With this total Λ-nucleus interactions, the solution of Schrödinger equation becomes impracticable since the coordinates of Λ particle and those of nucleons are coupled to each other. To get rid of this difficulty, we resort to some approximate methods.

For many-body systems in quantum mechanics one often starts with an average one-body potential V and solve the Schrödinger equation to obtain single particle energies and the wavefunctions. The single particle wavefunctions serve as the basis functions in many-structure calculations. Microscopically, this potential is determined by Hartree–Fock method by averaging the effective two-body interaction. For many nuclear structure calculations, however, the detailed features of this potential may not be so important. In that case we only assume that a one-body potential exists and when we need to use the single particle wavefunctions we often make use of the phenomenological forms for V. These phenomenological average potentials are chosen in such a way so as to reproduce the observed properties of the system. Such model potentials have played a vital role in the studies of hypernuclei and have contributed a great deal of knowledge to our understanding about the hypernuclear structure and the behaviour of Λ particle in a hypernucleus. Since there have been no experiments on Λ-nucleus scattering which could provide us any information about Λ-nucleus potential, the model potentials are determined only from the binding energy data of hypernuclei.

A number of such model Λ-nucleus potentials are available in literature (Dover 1981; Iwao 1980) and the potentials have some shortcomings. Either the choice of the potential is not appropriate or the potential parameters have been obtained in a somewhat arbitrary manner which limit their applicability. Also these potentials are not able to distinguish between the hypernuclei of same A but of different N and Z. This indicates that the overall situation regarding the phenomenological Λ-nucleus potentials is not very satisfactory. Therefore, it seems desirable to reconsider the problem of the determination of the model potentials.

In the present work we attempt to find some phenomenological Λ-nucleus potentials which may prove to be useful in hypernuclear studies. The choice of these potentials is made on the ground that the shape of the overall potential for Λ depends on the shape of the nuclear density and that of the effective ΛN potential and will roughly be proportional to the shape of density distribution of the core nucleus. The strong density dependent effects may be simulated by the potential parameters. We, therefore, try a potential of the form of density distribution. Firstly we consider a Woods-Saxon shape for the average potential

$$V(r) = \frac{-V_0}{1+\exp[(r-R)/a]} \tag{1}$$

with the following form of R, $R = (r_{0_1} + r_{0_2}A^{1/3})A^{1/3}$, where r_{0_1} and r_{0_2} are constants. With the similar form for the half density radius in a two-parameter Fermi charge distribution, Negele (1970) was able to explain the proton charge density distribution throughout the periodic table. This form of R, therefore, seems to be very appropriate for our purpose. To determine the values of the parameters we fit the ground state binding energy data of p-shell hypernuclei treating V_0, r_{0_1}, r_{0_2} and a as parameters. In fitting the energies we consider only ${}^{5}_{\Lambda}$He, ${}^{8}_{\Lambda}$Li, ${}^{10}_{\Lambda}$Be, ${}^{11}_{\Lambda}$B, ${}^{12}_{\Lambda}$B, ${}^{13}_{\Lambda}$C, ${}^{14}_{\Lambda}$C and ${}^{15}_{\Lambda}$N hypernuclei. We will, however, predict the B_Λs of the remaining hypernuclei from the best fit parameters. The results of the fit are given in table 1. The parameter values corresponding to the best fit are $V_0 = 31{\cdot}24$ MeV, $r_{0_1} = 0{\cdot}4384$ fm, $r_{0_2} = 0{\cdot}29$ fm and $a = 0{\cdot}666$ fm.

The binding energies are explained quite satisfactorily. The values of the potential parameters, especially, V_0 and a are very reasonable. With this potential the B_Λ's of the remaining hypernuclei are calculated. The results are given in table 2. Except for ${}^{9}_{\Lambda}$Be the binding energies for rest of the hypernuclei are reproduced correctly. The large difference, in ${}^{9}_{\Lambda}$Be is due to its unstable ^{8}Be core and should be treated in the alpha cluster model.

This potential, despite being successful in explaining the B_Λ data, is not able to distinguish between the hypernuclei of the same A but different N and Z. It gives the

Table 1. Fitted Λ binding energies.

Hypernucleus	Experimental B_Λ (MeV)	Theoretical B_Λ (MeV) with potential I	II
${}^{5}_{\Lambda}He$	$3{\cdot}12 \pm 0{\cdot}02$	3·14	3·00
${}^{8}_{\Lambda}Li$	$6{\cdot}80 \pm 0{\cdot}30$	6·94	7·28
${}^{10}_{\Lambda}Be$	$9{\cdot}11 \pm 0{\cdot}22$	9·21	9·43
${}^{11}_{\Lambda}B$	$10{\cdot}24 \pm 0{\cdot}05$	10·25	10·24
${}^{12}_{\Lambda}B$	$11{\cdot}37 \pm 0{\cdot}06$	11·22	11·25
${}^{13}_{\Lambda}C$	$11{\cdot}69 \pm 0{\cdot}12$	12·13	11·93
${}^{14}_{\Lambda}C$	$12{\cdot}17 \pm 0{\cdot}33$	12·97	12·78
${}^{15}_{\Lambda}N$	$13{\cdot}59 \pm 0{\cdot}15$	12·77	13·37

Table 2. Calculated B_Λ values of *p*-shell hypernuclei.

Hypernucleus	Experimental B_Λ (MeV)	Calculated B_Λ with potential I	II
${}^{6}_{\Lambda}He$	$4{\cdot}18 \pm 0{\cdot}10$	4·43	4·72
${}^{7}_{\Lambda}Li$	$5{\cdot}58 \pm 0{\cdot}03$	5·71	5·81
${}^{7}_{\Lambda}Be$	$5{\cdot}16 \pm 0{\cdot}08$	5·71	5·33
${}^{8}_{\Lambda}He$	$7{\cdot}16 \pm 0{\cdot}70$	6·94	7·70
${}^{8}_{\Lambda}Be$	$6{\cdot}84 \pm 0{\cdot}05$	6·94	6·86
${}^{9}_{\Lambda}Li$	$8{\cdot}53 \pm 0{\cdot}15$	8·11	8·59
${}^{9}_{\Lambda}Be$	$6{\cdot}71 \pm 0{\cdot}04$	8·11	8·22
${}^{9}_{\Lambda}B$	$7{\cdot}88 \pm 0{\cdot}15$	8·11	7·85
${}^{10}_{\Lambda}B$	$8{\cdot}89 \pm 0{\cdot}12$	9·21	9·11
${}^{12}_{\Lambda}C$	$10{\cdot}76 \pm 0{\cdot}19$	11·22	11·00
${}^{16}_{\Lambda}O$	$13{\cdot}00 \pm 2{\cdot}00$	14·51	13·93

same B_Λ for hypernuclei having the same A but different N and Z values (isobaric nuclei). To remove this discrepancy we use the following form for R in eq. (1)

$$R = C_1 + C_2 A^{1/3} - C_3[(N-Z)/A],$$

where C_1, C_2 and C_3 are constants.

A similar form for the half density radius in a two-parameter Fermi charged distribution has been found to explain the charge densities throughout the periodic table. Our Woods-Saxon potential with this form for R will, hereafter be referred to as potential II. To determine the parameters of this potential we fit the B_Λ data of the same hypernuclei treating V_0, C_1, C_2, C_3 and a as parameters. The fitted energies are given in table 1. The parameter values corresponding to the best fit are $V_0 = 32{\cdot}93$ MeV, $C_1 = -0{\cdot}6691$ fm, $C_2 = 1{\cdot}3007$ fm, $C_3 = -0{\cdot}145$ fm and $a = 0{\cdot}5517$ fm. The energies are explained satisfactorily as with potential-I and the values of the potential parameters V_0 and a are also very reasonable.

With this potential we calculate the binding energies of the remaining hypernuclei.

The results are given in table 2. The calculated energies are in good agreement with the observed values. The calculated energies of isobaric hypernuclei are also in good agreement with the observed ones.

The potential-II has one important feature that is its dependence on $(N-Z)$ term. This term is directly related to small charge symmetry breaking effects. For this potential the fit is made for nuclei of different A but excluding the isobaric nuclei. The predicted energies of the isobaric nuclei ($A=7,8,9$) with the same $(N-Z)$ dependence are then the same as the observed energies (table 2). This agreement is really significant and clearly indicates that there are some charge symmetry breaking effects in the interaction.

One may, however, question the validity of this result since it does not involve the spin-spin term which may be of the same order of magnitude as the charge symmetry breaking effects. A detailed and precise study of spin-spin term in ΛN force was carried out by Millener *et al* (1985) and has been found to be very small. The inclusion of this term in the present analysis may only slightly change the values of the potential parameters. This will, however, not change our conclusion.

In our earlier analysis (Ahmad *et al* 1985) it was remarked that B_Λ analysis of a hypernucleus can be used to determine the density distribution of its core nucleus. In a later investigation (Mian 1987) the size parameter of the shell model density of some p-shell nuclei was also determined. However, this method is applicable only when the density distribution of the nucleus contains only one parameter. For multiparameter densities such as the two or three parameter Fermi distribution or the sum of gaussians, it becomes very difficult to determine these density parameters since we have with us only one datum point i.e. B_Λ which has to be fitted to determine all the parameters. However, with the determination of this potentials one can also obtain the multiparameter densities of nuclei. The procedure is as follows: the radial part of the Schrödinger equation for Λ particle moving in the Woods-Saxon potential (eq. 1) is

$$\frac{-\hbar^2}{2m}\frac{d^2u(r)}{dr^2}+V(r)u(r)=Eu(r), \tag{2}$$

where E is the binding energy of Λ particle. The expectation value of E i.e., $\langle E\rangle$ is given as

$$\langle E\rangle=-\frac{\hbar^2}{2m}\int_0^\infty \frac{u}{r}\frac{d^2}{dr^2}\left(\frac{u}{r}\right)r^2 dr+\int_0^\infty Vu^2 dr. \tag{3a}$$

When the Λ-nucleus potential is density-dependent such as the one given by (6) or those given in Mian (1987), Ahmad *et al* (1985), Millener *et al* (1988), the expectation value of E then becomes

$$\langle E_d\rangle=-\frac{\hbar^2}{2m}\int_0^\infty \frac{v}{r}\frac{d^2}{dr^2}\left(\frac{v}{r}\right)r^2 dr+\int_0^\infty V_d v^2 dr \tag{3b}$$

where V_d is a density-dependent effective Λ-nucleus potential and $v(r)$ is the radial wavefunction of Λ, obtained by solving the Schrödinger equation

$$-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}v(r)+V_d(r)v(r)=E_d v(r). \tag{4}$$

Since the two expectation values should be equal or almost equal

$$\langle E \rangle - \langle E_d \rangle = 0$$

or

$$-\frac{\hbar^2}{2m}\left(ru\frac{d^2}{dr^2}\left(\frac{u}{r}\right) - rv\frac{d^2}{dr^2}\left(\frac{v}{r}\right)\right) + (u^2 V - v^2 V_d) = 0. \qquad (5)$$

Equations (2), (4), (6) and (5) can now be used to determine density distributions of p-shell nuclei. The experimental B_Λ is substituted for E_d in (4) and $v(r)$ is found for the assumed values of density parameters occurring in potential V_d. This $v(r)$ is then substituted in (5). The density parameters are then varied and the best values of these parameters for which $v(r)$ satisfies conditions 5, then gives the correct density of the nucleus. This method is an approximate way of determining the density since there is no apparent relationship of Woods–Saxon potential with the density of the core nucleus. It can still find some application at least in those cases where the densities of nuclei cannot be determined by any other known method. For an exact determination of densities one should know the relation of density with the effective single particle potential. Determination of such relationship requires the knowledge of basic interactions and use of an appropriate many-body theory and will not be attempted here.

For theoretical estimate of D_Λ we use the following expressions which is due to Dabrowski and Kohler (1964) $D_\Lambda = -(V + V_R)$, where V is the single particle model potential for Λ and V_R is the rearrangement potential. In phenomenological analysis, the rearrangement term V_R is neglected and the depth of the single particle model potential is identified with D_Λ. This gives $D_\Lambda = 31{\cdot}244$ MeV and 32·93 MeV, for potentials I and II, respectively. These estimates are slightly higher than the earlier D_Λ estimates and this small discrepancy is understandable. The depth of the single particle potential can be identified with D_Λ only when the potential were obtained by fitting the B_Λ data of heavier hypernuclei whose cores form the nuclear matter. In the present analysis, however, the potentials have been obtained by fitting the B_Λ data of only light nuclei. These potentials do not even reproduce the binding energy of heavy hypernuclei. Therefore, the identification of their depth with D_Λ is bound to give some error though very small. This error can further be minimized if we make some allowance for V_R. A good approximation for V_R in the present case is to assume $V_R \simeq -xV$, where $x = 0{\cdot}15$ is the ratio of correlation volume per nucleon (Rozynek and Dabrowski 1979). This gives $D_\Lambda = 26{\cdot}56$ MeV and 28·0 MeV, respectively, for the two potential which are quite reasonable.

Finally, we try the following for the average Λ-nucleus potential

$$V(r) = \frac{C_0\rho(r)}{1 + \alpha\rho(r)}[1 + C_1\rho^{2/3}(r) + C_2\rho(r)]. \qquad (6)$$

This form was used by Dover and Giai (1972) for the central part of N-nucleus scattering potential. With the parameters C_2 and α equal to zero this form coincides with our earlier folded potential. We are trying out it here just to see what values these additional parameters would have in hypernuclei. We fit the ground state B_Λ data of $A \geqslant 5$ hypernuclei. The data are fitted well. The best fit parameters are $C_0 = 36{\cdot}2\ \mathrm{MeV\,fm^3}$, $C_1 = -1{\cdot}88\ \mathrm{fm^2}$, $C_2 = -0{\cdot}0793\ \mathrm{fm^3}$, $\alpha = -0{\cdot}732\ \mathrm{fm^3}$.

Contrary to the nuclear case the values of the parameters C_2 and α are quite small.

The values of C_0 and C_1 are almost the same as in our earlier analysis. The parameter is related with the effective mass of Λ particle and its small value in the present analysis indicates that the effective mass of Λ is almost equal to its free mass.

We have determined the potentials which are able to explain the ground state B_Λ data of all p-shell hypernuclei satisfactorily. Potential-II is capable of even distinguishing the hypernuclei of the same A but of different N, Z. This characteristic had not been possessed by any of the earlier potentials. The dependence of this potential on $(N - Z)$ term indicates the presence of a charge symmetry breaking component in ΛN force. Alongwith our density dependent potential, these potentials can be used to determine the densities of light nuclei. Besides explaining the B_Λ data, these potentials may also serve as a guideline for determining the average single particle potentials for other hyperons such as Σ, $\Xi{\cdot}\Lambda_c$ etc.

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