Pramana – J. Phys., Vol. 37, No. 5, November 1991, pp. 395–403. © Printed in India.

General relativistic model for energy release of an unstable shell of charged matter in the presence of a dyon core

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MS received 16 August 1990; revised 30 May 1991

Abstract. We construct a formula for the energy released when a spherical shell of charged matter disperses in the presence of a dyon core. Reasons for such a dyon-charged shell configuration on a macroscopic scale stem from the recent speculation concerning the existence of CHAMPS (charged massive particles) along with the possible existence of monopoles produced around the time of nucleosynthesis.

Keywords. Dyon; gamma ray bursts; general relativistic stability.

PACS No. 04·90

1. Introduction

The recent scan of the spectrum of extragalactic radiation (Ressell and Turner 1989) along with the discovery of a new phase of QED (Galdi 1989) wherein new bound state phenomena occur, has prompted the theoretical community to look for alternate mechanisms for the generation of γ ray bursts based on physics outside the usual neutron star scenario. New phases of matter, new gravitational binding mechanisms and extra dimensions to space and time all may figure into the source of the γ ray bursts.

Of course the subject of condensed astrophysical objects in general relativity has been thoroughly studied in the past. The historical problem of the interior Schwarzschild solution (Schwarzchild 1916), charged fluid spheres in general relativity (Bohra and Mehra 1971), (Omote and Sato 1974) and spheres containing degenerate neutrons studied by Cameron (1979) who gave a maximum mass of such objects, represent some of the notable developments in this broad field of interest. Recently charged boson stars discussed by Jetzer (1990a) have been studied with a maximum central density permitted so as to prevent gravitational collapse. Mixed fermion boson stars have been studied by Jetzer (1990b) and non-topological solitons by (Frieman *et al* 1988).

With the recent studies regarding the possibility of charged dark matter (CHAMPS) consisting of massive particles between 20 and 1,000 TeV (Dimopoulus *et al* 1990; De Rujula *et al* 1990) it would be of interest to ask how such matter would behave in the vicinity of a massive central body which carries electromagnetic degrees of freedom such as electric and magnetic charge (Prasad and Sommerfield 1975; Callan 1982). The present theory of inflation discourages production of monopoles in the early universe but there are reasons to believe that monopoles and antimonopoles could be generated around the time of nucleosynthesis due to energetic collisions of massive particles (Hill 1982). If, in fact, such monopoles were produced around the time of nucleosynthesis and an ensemble of them were localized in a small region, they might appear on large scales as a macroscopic accumulation of magnetic charge. Reasoning further, if this magnetic charge accumulation were localized near a concentrated accumulation of electric charged particles in the form of CHAMPS we would have the necessary components of a macroscopic dyon that may influence the surrounding space-time. Through electrostatic repulsion or hydrodynamic motion some of the charged particles might drift into a halo or shell surrounding the dyon to form a dyon-charged shell configuration that may serve as a burst of γ rays upon dispersing. In particular, in what follows, we study the structure of the gravitational field and the electromagnetic field surrounding a macroscopic dyon in the presence of a charged shell. By assuming the normal pressure as zero in the shell we derive a formula for the energy released when such a shell disperses. In the conclusion we comment on how such a dyon with surrounding charged may be detected in the cosmos.

2. Energy released from charged shell in the presence of a dyon core

We begin our analysis by stationing an abelian dyon of macroscopic electric charge e, macroscopic magnetic charge q and mass M_x at $r=0$. For the region $0<r<R_1$ the only field present is the field of the dyon plus the gravitational field generated. For the region $R_2 \geqslant r \geqslant R_1$ we have a charged shell of electric charge density ρ_0; energy density ε_0 and vanishing normal pressure. For the region $r>R_2$ we have a charge-free and matter-free region in the presence of the electromagnetic field and the gravitational field generated by the dyon plus shell.

We will employ the spherically symmetric metric

$$(\mathrm{d}s)^2 = e^{\nu}(\mathrm{d}x^4)^2 - e^{\lambda}(\mathrm{d}r)^2 - r^2(\mathrm{d}\theta)^2 - r^2\sin^2\theta(\mathrm{d}\phi)^2 \tag{1}$$

along with the energy momentum tensor of matter for $R_1 \leqslant r \leqslant R_2$.

$$T_4^4 = \varepsilon,\ T_1^1 = 0 \text{ (vanishing normal pressure)}$$
$$T_2^2 = T_3^3 = -P. \tag{2}$$

We also have the proper electric charge density ($\rho_0 =$ constant) for $R_2 \geqslant r \geqslant R_1$ in the interior of the charged shell. The electromagnetic field in all regions will be described by the lagrangian

$$\mathscr{L} = -\frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\sqrt{-g} - J^{\mu}A_{\mu}\sqrt{-g} \tag{3}$$

and field equation

$$\frac{\partial}{\partial x^{\nu}}(\sqrt{-g}\,F^{\mu\nu}) = 4\pi\sqrt{-g}\,J^{\mu} \tag{4}$$

($J^{\mu} = 4$ current), and the condition,

$$\frac{\partial}{\partial x^{\nu}}(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) = 0 \tag{5}$$

which is a statement of the existence of the potential for all $r \neq 0$. The energy momentum tensor of the electromagnetic field is

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial \mathscr{L}}{\partial g^{\mu\nu}} = \frac{1}{16\pi} g_{\mu\nu}(F_{\alpha\beta}F^{\alpha\beta}) - \frac{1}{4\pi}F_{\mu\alpha}F_{\nu}{}^{\alpha}. \tag{6}$$

For the region $R_1 \leqslant r \leqslant R_2$ we will use the approximation $e^{\lambda} \simeq e^{\nu} \simeq 1$ in (4) to simplify the solution for the electric field. For the spherically symmetric Einstein equations we have in all regions

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 T_4^4$$

$$\frac{1}{r^2}(e^{-\lambda} - 1) + \frac{1}{r}e^{-\lambda}\nu' = -\frac{8\pi G}{c^4}(T_1^1)$$

$$e^{-\lambda}\left(\frac{\nu''}{2} - \frac{\lambda'\nu'}{4} + \frac{(\nu')^2}{4} + \frac{(\nu' - \lambda')}{2r}\right) = -\frac{8\pi G}{c^4}T_2^2 = -\frac{8\pi G}{c^4}T_3^3. \tag{7}$$

In (7) for the region $0 < r < R_1$ we have $T^{\mu\nu}$ due to the electromagnetic field of the dyon at $r = 0$, for $R_2 \geqslant r \geqslant R_1$ we have the energy momentum tensor due to the matter as specified by (2) and the electromagnetic field of dyon and charged shell, and for $R_2 < r$ we have the energy momentum tensor of electromagnetic field of dyon plus shell.

For the magnetic field of the dyon we have from (5) for $R_1 > r > 0$ ($F_{23} = r^2 \sin\theta B_r$)

$$\frac{\partial}{\partial r}(\varepsilon^{4123} r^2 \sin\theta B_r) = 0$$

$$B_r = \frac{q}{r^2} \text{ for } 0 < r < R_1 \tag{8}$$

(q = magnetic charge of macroscopic dyon.)

For the electrical field for $0 < r < R_1$ we have

$$\frac{\partial}{\partial r}\left(\frac{1}{4\pi}F^{41}\sqrt{-g}\right) = 0 \tag{9}$$

using $F_{14} = E(r)$, $\lambda + \nu = 0$ for $0 < r < R_1$ which follows from the $\begin{pmatrix}1\\1\end{pmatrix}$ and $\begin{pmatrix}4\\4\end{pmatrix}$ component of the Einstein equations, (9) gives

$$\frac{\partial}{\partial r}(r^2 E_r) = 0,\ E_r = \frac{e}{r^2} \tag{10}$$

(e = electric charge of macroscopic dyon).

For the energy momentum tensor for $0 < r < R_1$ we have

$$T_1^1 = T_4^4 = \frac{E_r^2 + B_r^2}{8\pi}.$$

Using the $\binom{4}{4}$ Einstein equation from (7) gives for $0<r<R_1$

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 \left(\frac{e^2}{8\pi r^4} + \frac{q^2}{8\pi r^4}\right)$$

or

$$e^{-\lambda} = 1 - \frac{2GM_x}{rc^2} + \frac{Gq^2}{r^2c^4} + \frac{Ge^2}{r^2c^4} = e^{\nu}. \tag{12}$$

Here M_x = mass of macroscopic dyon at $r=0$ Eqs (8), (10) and (12) specify the total electric and magnetic field configuration for $0<r<R_1$.

We now study the region $R_1 \leqslant r \leqslant R_2$, for the electromagnetic equation we have from (4) with $J^4 = \rho_0(\mathrm{d}x^4/\mathrm{d}S) = \rho_0 e^{-\nu/2}$, ($\rho_0$ = constant charge density)

$$\frac{\partial}{\partial r}(r^2 E e^{-(\lambda+\nu)/2}) = 4\pi\rho_0 r^2 e^{\lambda/2}. \tag{13}$$

Using the approximation in (13) outlined earlier for $R_2 \geqslant r \geqslant R_1 (e^{\lambda} \simeq e^{\nu} \simeq 1)$ (13) gives

$$r^2 E = \frac{4\pi\rho_0 r^3}{3} + c, \tag{14}$$

upon matching (14) to (10) at $r = R_1$ gives

$$E = \frac{4\pi\rho_0}{3}\left(\frac{r^3 - R_1^3}{r^2}\right) + \frac{e}{r^2} \text{ for } R_1 \leqslant r \leqslant R_2. \tag{15}$$

The magnetic field for $R_1 \leqslant r \leqslant R_2$ is still

$$B_r = \frac{q}{r^2} \tag{16}$$

from (5), here the condition

$$\frac{\partial}{\partial x^{\nu}}(\varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}) = 0$$

holds for $R_1 \leqslant r \leqslant R_2$($q$ = macroscopic magnetic charge of central dyon).

From the form of the energy momentum tensor for the electromagnetic field in (6) and the field as described by (15) and (16) for $R_1 \leqslant r \leqslant R_2$ along with the energy momentum tensor of matter for $R_1 \leqslant r \leqslant R_2$ from (2) we have for the first of (7) for $R_1 \leqslant r \leqslant R_2$

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G r^2}{c^4}\left[\varepsilon_0 + \frac{q^2}{8\pi r^4} + \frac{1}{8\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3)\right)^2\right] \tag{17}$$

here we have used the approximation $e^{\lambda} \simeq e^{\nu} \simeq 1$ in rhs of (17) for T_4^4. Integrating (17) from R_1 to r gives

$$re^{-\lambda} - (re^{-\lambda})R_1 = r - R_1 - \frac{8\pi G}{c^4}\int_{R_1}^{r} r^2\left[\varepsilon_0 + \frac{q^2}{8\pi r^4} + \frac{1}{8\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3)\right)^2\right]\mathrm{d}r. \tag{18}$$

In (18) we substitute (12) at $r = R_1$. Again using (2) with vanishing normal pressure and the second of the Einstein equations in (7) with $T_1^1 = T_4^4$ for the electromagnetic component of the energy momentum tensor we have

$$\nu' = \frac{1}{r}(e^\lambda - 1) - \frac{8\pi G}{c^4} r e^\lambda \left[\frac{q^2}{8\pi r^4} + \frac{1}{8\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3)\right)^2 \right] \tag{19}$$

or

$$\nu = \int^r \frac{1}{r}(e^\lambda - 1)\mathrm{d}r - \frac{8\pi G}{c^4}\int^r r e^\lambda \left[\frac{q^2}{8\pi r^4} + \frac{1}{8\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3)\right)^2 \right] \mathrm{d}r + c. \tag{20}$$

The constant in (20) is found by matching (20) to (12) at $r = R_1$.

Our next task is to calculate the transverse pressure from the third of the equations in (7). From the electromagnetic energy momentum tensor for $R_1 \leqslant r \leqslant R_2$ in (6) along with (15) and (16) for E, B we have in combination with (2) for $R_1 \leqslant r \leqslant R_1$

$$T_2^2 = -P - \frac{E^2}{8\pi} - \frac{B^2}{8\pi}. \tag{21}$$

(where we again use $e^\lambda \simeq e^\nu \simeq 1$ to calculate the electromagnetic component of T_2^2). Substituting in the third part of (7) gives for the transverse pressure

$$P = \frac{c^4}{8\pi G}\left[-\frac{1}{2r}e^{-\lambda}\lambda' + \frac{1}{2r}\nu' e^{-\lambda} + \frac{1}{4}(\nu')^2 e^{-\lambda} + \frac{\nu''}{2}e^{-\lambda} - \frac{1}{4}\nu'\lambda' e^{-\lambda} \right]$$
$$- \frac{q^2}{8\pi r^4} - \frac{1}{8\pi}\left[\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3) \right]^2. \tag{22}$$

Thus from (18), (20) and (22) we have the solution for the metric and the transverse pressure in the region $R_1 \leqslant r \leqslant R_2$.

We now consider the region $R_2 < r$ (outside the shell), from (5) and (9) we find

$$E = e_1/r^2 \tag{23}$$

(e_1 = total electric charge of dyon plus shell)

$$B = q/r^2. \tag{24}$$

We have used the condition $\lambda + \nu = 0$ for $r > R_2$ which follows from the Einstein equations and the electromagnetic energy momentum tensor. We now match (23) to (15) at $r = R_2$ to find

$$e_1 = e + \frac{4\pi\rho_0}{3}(R_2^3 - R_1^3). \tag{25}$$

Equation (25) relates the charge observed for $r > R_2$ to the charge for the central dyon plus the charge of the shell.

For the metric for $r > R_2$ we insert (23) and (24) into (6) and use the first of the Einstein equations in (7) to give

$$\frac{d}{dr}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} r^2 \left(\frac{q^2}{8\pi r^4} + \frac{e_1^2}{8\pi r^4} \right), \tag{26}$$

integrating (26) we have for $r > R_2$

$$e^{-\lambda} = 1 - \frac{2G}{rc^2}(M_x + M_s) + \frac{Gq^2}{r^2 c^4} + \frac{Ge_1^2}{r^2 c^4} = e^{\nu}. \tag{27}$$

Here the constant of integration is

$$\frac{2G(M_x + M_s)}{c^2}$$

where M_s is the mass of the shell.

To find an expression for M_s we use (18) with upper limits at $r = R_2$, for $(e^{-\lambda})_{R_1}$ we substitute (12), and for $(e^{-\lambda})_{R_2}$ we substitute (27) at $r = R_2$, the right hand side of (18) entails known quantities, when these substitutions are made in (18) we obtain

$$\begin{aligned} M_s = \frac{4\pi}{c^2} \bigg[& \frac{\varepsilon_0}{3}(R_2^3 - R_1^3) - \frac{e^2}{8\pi R_2} + \frac{e\rho_0}{6}(R_2^2 - R_1^2) + \frac{e\rho_0 R_1^3}{3}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) \\ & + \frac{2\pi\rho_0}{45}(R_2^5 - R_1^5) - \frac{2\pi\rho_0^2 R_1^6}{9}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) - \frac{2\pi\rho_0^2 R_1^3}{9}(R_2^2 - R_1^2) \bigg] \\ & + \frac{e_1^2}{2R_2 c^2}. \end{aligned} \tag{28}$$

As mentioned, in calculating (28) we have approximated $e^{\lambda} \simeq e^{\nu} \simeq 1$ in the T_4^4 component of the energy momentum tensor for $R_1 < r < R_2$. This approximation is good as long as the shell is far enough removed from the dyon core and the shell itself does not generate a huge gravitational field.

We now calculate the energy released when the charged shell disperses. Actually, the electrostatic energy generated in the shell is positive but the gravitational energy may be of opposite sign so as to compete with the electrostatic term. For a sphere of non-vanishing normal pressure, Wright (1964) has calculated the gravitational binding energy and demonstrated that greater than a certain radius the sphere gives up energy upon formation and the sphere is stable. In a separate note using the (Florides 1974) solution for a sphere of vanishing normal pressure we have shown that the sphere with vanishing normal pressure has a negative binding energy and thus energy would be released upon its dispersing (Wolf 1990).

The combination of the electrostatic repulsion and the condition of vanishing normal pressure will both generate a destabilizing effect on the charged shell and generate a negative binding energy or said differently, the shell will have a greater energy over and above that of the proper energy before formation and the shell will release energy upon dispersing.

The energy density of the shell is related to the proper mass density and the pressure by $\varepsilon_0 = \bar{\rho}_0 c^2 + P$($\bar{\rho}_0$ = proper mass density), here we have assumed two non-relativistic degrees of freedom to the kinetic energy which is a result of vanishing normal pressure. Thus

$$\bar{\rho}_0 c^2 = \varepsilon_0 - P. \tag{29}$$

The total energy of the shell prior to formation is

$$E_{\text{before}} = \int_{R_1}^{R_2} (\varepsilon_0 - P) 4\pi r^2 e^{\lambda/2} \mathrm{d}r. \tag{30}$$

Here $4\pi r^2 e^{\lambda/2}\, \mathrm{d}r$ is the element of proper volume, and $\varepsilon_0 - P$ the proper rest energy density that does not change upon formation of the shell.

The energy of the shell over and above the initial proper rest energy is

$$E = M_s c^2 - \int_{R_1}^{R_2} (\varepsilon_0 - P) 4\pi r^2 e^{\lambda/2} \mathrm{d}r. \tag{31}$$

To evaluate the second term in (31) we insert P given by (22) with $e^{-\lambda}$ given by (18) and v' given by (19), to obtain v'' we differentiate (19) for $R_1 < r < R_2$ to obtain

$$\begin{aligned} v'' = &-\frac{1}{r^2}(e^{\lambda} - 1) + \frac{1}{r} e^{\lambda}\lambda' - \frac{8\pi G}{c^4} e^{\lambda}\lambda' \left[\frac{q^2}{8\pi r^3} + \frac{r}{8\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3) \right)^2 \right] \\ &- \frac{8\pi G}{c^4} e^{\lambda} \left[-\frac{3q^2}{8\pi r^4} + \frac{1}{8\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3) \right)^2 \right. \\ &\left. + \frac{r}{4\pi}\left(\frac{e}{r^2} + \frac{4\pi\rho_0}{3r^2}(r^3 - R_1^3) \right)\left(-\frac{2e}{r^3} + \frac{4\pi\rho_0}{3} + \frac{8\pi R_1^3 \rho_0}{3r^3} \right) \right]. \end{aligned} \tag{32}$$

When P, $e^{\lambda/2}$ and M_s from (28) are substituted into (31) we obtain an expression for the energy released upon dispersing of the charged shell in terms of the energy density of the shell ε_0, the charge density of the shell ρ_0, the dyon parameters e, q, M_x and the inner and outer radius.

3. Conclusion

The above calculation represents a simplified model of a charged shell surrounding an abelian dyon. Even though the constituent dyons of the macroscopic dyon are non-abelian at small length scales, at large distances the macroscopic dyon would appear as abelian since the colour and heavy GUT gauge fields would die off for large r and leave the residual effective abelian electric and magnetic field (Prasad and, Sommerfield 1975). Also the approximation of $e^{\lambda} \simeq e^{\nu} \simeq 1$ in the interior of the shell made in order to calculate the energy momentum tensor would only be valid far from the dyon core and in the presence of a shell that does not generate a huge gravitational field.

To look for such macroscopic dyons surrounded by charged matter (CHAMPS) we would have two provisional ways to identify them. Firstly the metric exterior to the shell depends on M_x (mass of dyon) and the mass of the shell (M_s) as well as the total electric charge and magnetic charge through (27). If bursts of radiation from the dispersing of such a shell were analyzed the red shift of a photon before the burst would depend on M_x, M_s, q, e_1, through (27), while the red shift at the end of the burst would depend on M_x, q, e through (12) since the contents of the shell would be dispersed and not contribute to the metric at the end of the burst. Thus a change in the red shift would signal a decrease in the number of parameters describing the configuration. Secondly if a series of these shells were formed around macroscopic dyons, and a series of energy burst were observed the energy output of each would depend on the central dyon parameters through (31) provided ρ_0, ε_0, R_1, R_2 are roughly the same in the series, thus if we could analyze the energy output versus the three unknowns e, q, M_x we might find a correlation with the parametric dependence of (31) on e, q and M_x. It should also be mentioned as was done by (Frampton *et al* 1989) that the characteristic spectrum of a charged fermion in the field of a microscopic dyon differs from that of the hydrogen spectrum in a specific way. Such dyon-fermion characteristic spectrum might reveal the presence of a microscopic dyon in an astrophysical setting and the collective effects of an ensemble of these dyons could further be searched for. In summary the red shift of light coming from a "dyon-charged shell" prior to and after dispersing would serve as probe to identify parameters in (27) and (12) (M_s = mass of shell, q = central macroscopic dyon magnetic charge, e_1 = electric charge of dyon plus shell, M_x = mass of central macroscopic dyon, e = electric charge of central dyon). Also the possible identification of a series of these configurations might be made by finding a correlation of the total energy released when the dyon charged shell disperses with the parameters M_x, q, e in (31). Lastly as noted in the introduction the existence of charged massive particles (CHAMPS) predicted by particle theory may provide the necessary ingredients for forming such dyon-charged shell configurations (Dimopoulous *et al* 1990; DeRujula *et al* 1990). The only experimental signature for such "CHAMPS" would be super-heavy isotopes that result when CHAMPS replace ordinary protons and neutrons in nuclei.

Acknowledgements

The author thanks the physics departments at Williams College and Harvard University for the use of their facilities.

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