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Effect of viscosity of the pressure-transmitting fluid on the metrological characterization of the piston gauge up to 1 GPa

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Abstract. A systematic theoretical investigation has been carried out to study the effect of viscosity of the pressure-transmitting fluids on the measurement of pressure up to 1 GPa using piston gauges. The fluid flow equation is modified to determine the fall rate (v) with pressure (P), taking the pressure-dependent viscosity $\eta(P)$ and clearance between piston and cylinder $[H(P)]$ terms into account. Above 0·4 GPa, the fall rate curve shows the tendency to be pressure independent. The near-constancy of v with P can be avoided with less viscous fluid or by increasing $H(P)$. Finally. the initial clearance obtained from the experimental data of fall rate, shows a weak dependence of pressure, although theoretically it is assumed to be independent of pressure. This weak pressure dependence is attributed to the effect of viscosity of the pressure transmitting fluid.

Keywords. Piston gauge; accurate measurement of pressure; effect of viscosity; pressure transmitting fluid.

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1. Introduction

The accuracy of pressure measurement and its reliability, is very important in a large number of industrial applications, like power generation, oil exploration, aircraft, chemical, synthesis of new materials etc. There are several methods to measure the pressure accurately but the commonly used one in the range up to 1 GPa is the piston gauge. Different types of piston gauges available like the simple piston gauge, the re-entrant piston gauge, the controlled clearance piston gauge etc. and the merits and demerits of these gauges have already been reported in literature (Heydemann and Welch 1975; Sharma and Jain 1986). The mechanism of these piston gauges is essentially balancing the upward force, due to the pressure in the system acting on the area of the piston, with the downward gravitational force of known weights. In these gauges, the determination of the effective area is difficult because of high pressure elastic-distortion effect.

In general, the operation of the piston gauges assumes that the measurement of pressure does not depend on the nature of the pressure-transmitting fluid in the clearance between the piston and cylinder (Heydemann and Welch 1975). However, the extensive studies by Sharma *et al* (1983, 1984) and Newhall *et al* (1979), have shown that the operation of the piston gauge is nearly independent of viscosity over a wide range to some critical value of the order of 60 centipoise (cP) and beyond which the viscosity begins to have a very large influence on the pressure measurement. It is suggested that the dilute lighter liquids like gasoline and the mixture of sebacate with heptane may be used in place of undiluted sebacate as the pressure-transmitting fluid for the controlled clearance piston gauge. As reported by Bandyopadhyay *et al* (1987) and Hilsch and Jager (1989) and also by the general experience that when these lighter pressure-transmitting fluids are used, then a reasonable fall rate for the measurement of pressure is obtained only when the clearance between the piston and cylinder is very small. These small clearances give rise to traces of friction and wear which in the long run may affect the zero pressure effective area of the piston cylinder assembly.

The present studies are to determine theoretically the fall rate of the piston cylinder assembly from the modified fluid flow equation through introducing the pressure-dependent viscosity and the clearance between the piston and cylinder terms. The fall rate obtained with pressure curve, beyond a certain pressure of 0·4 GPa, shows a non-linear behaviour which is consistently observed at all initial clearances studied in the present paper. However, when the viscosity of a mixture of less viscous fluid-like heptane, with normally used oil-like sebacate is considered, this non-linearity of the fall rate with pressure beyond 0·4 GPa disappears. Although it is not possible at this moment to compare the results obtained theoretically and experimentally since the in situ measurement of the initial clearance between the piston and cylinder is difficult, but a close comparison of the theoretical results with the existing experimental data is possible and thus is carried out.

2. Theoretical discussion

Let us consider a piston of known diameter ($2r_p$) fitted into a matching cylinder of known diameter ($2r_c$) filled with fluid as shown schematically in figure 1. The piston and the cylinder are exactly of revolving form. The piston is loaded with known weights and rotated to relieve friction and to assure concentricity. Considering the flow of the fluid between the piston and cylinder in the laminar flow approximation, the volumetric flow can be given by (Zhokhovski 1959)

$$Q(P) = -[(\pi r_p H^3(P,x)(\mathrm{d}P/\mathrm{d}x)]/6\eta(P) \tag{1}$$

where $H(P,x)$ is the clearance between piston and cylinder at a pressure P at a point x (shown in figure 1), $(\mathrm{d}P/\mathrm{d}x)$ is the gradient of pressure across the vertical engagement length (l) as shown in figure 1 and $\eta(P)$ is the coefficient of the viscosity of the pressure transmitting fluid. Further, the effective area is defined as (Zhokhovski 1959),

$$A_{\mathrm{eff}}(P) = [A_p(P) + A_c(P)]/2 = A_0(1 + \lambda P) \tag{2}$$

$$A_0 = \pi r_p^2[1 + H(0)/r_p] \tag{3}$$

where A_0 is the effective area at normal atmospheric pressure, $H(0)$ is the initial clearance and λ is known as the pressure distortion coefficient of the combined piston cylinder assembly. In this investigation, the piston and cylinder are made of tungsten carbide and stainless steel. The average diameter of the piston and the ratio of outer to inner diameter of the cylinder, are 2·522 mm and 10·691 mm, respectively. Inserting the elastic constants of these materials, taken from the literature, one obtains,

$$A_p(P) = A_p(0)(1 - 0{\cdot}055 \times 10^{-5}P) \tag{4}$$

$$A_c(P) = A_c(0)(1 + 0{\cdot}633 \times 10^{-5}P) \tag{5}$$

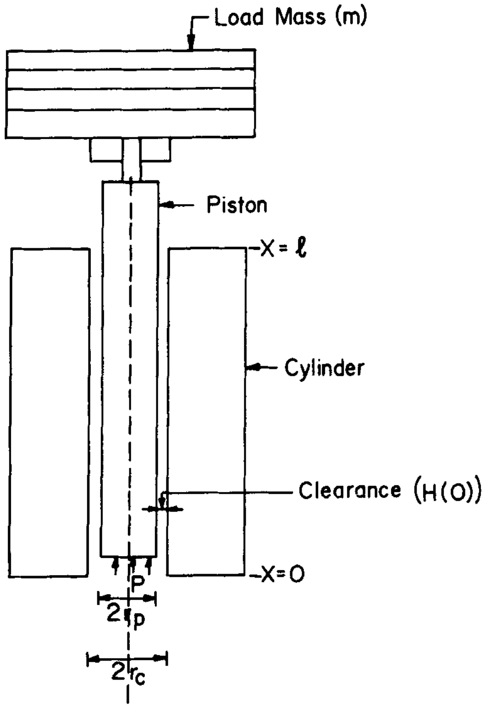

Figure 1. Schematic diagram of a simple piston cylinder assembly.

where P is expressed in MPa. This indicates that λ_{th} for the present piston gauge is $0{\cdot}289 \times 10^{-5}\,\mathrm{MPa}^{-1}$. It may be mentioned here that experimental value (λ_{ex}) obtained is always more than that of the theoretical value, λ_{th}. Table 1 shows the values of these parameters λ_{ex}, λ_{th} and also $\Delta\lambda = \lambda_{ex} - \lambda_{th}$ reported by various internationally recognized metrological laboratories. It is clear from this table that a definite value of $\Delta\lambda$ is always found irrespective of the materials of the piston and cylinder. This difference $\Delta\lambda$ has been a subject of great interest and discussion.

In order to explain this difference ($\Delta\lambda$), Bass (1978) has proposed a model where the pressure distortion coefficient is obtained by considering a dynamic variation of clearance [$H(x)$] with pressure $P(x)$. Normally, it is assumed that $P_s/P = 0{\cdot}5$ where P_s is the pressure in the annulus between the piston and cylinder and P is the pressure below the piston. Welch and Bean (1984) have shown, by measuring the pressure profile in the clearance, that $P_s/P = 0{\cdot}842$ which determines pressure distortion coefficient $\lambda_{th} = 0{\cdot}434 \times 10^{-5}\,\mathrm{MPa}^{-1}$ and is in good agreement with the experimental value. However, in the Bass model (1978), the viscosity of the pressure-transmitting fluid is considered as constant, and eventually, Welch and Bean (1984) have studied up to 210 bar where the contribution of viscosity of pressure-transmitting fluid is negligible, therefore, the application of the Bass model is quite appropriate in this range. In the high pressure region where the viscosity contribution is quite appreciable in the normally used pressure-transmitting fluids, Bass model has limited applicability for estimation.

In an earlier paper, Sharma *et al* (1988b) have shown that the radial clearance term $H(P, x)$ at pressure P can be expressed in terms of the elastic constants in the case

Table 1. A comparison of λ_{ex} and λ_{th} of the piston gauges for the different laboratories.

Laboratory	Materials of piston and cylinder	λ_{ex} (10^5) (MPa^{-1})	λ_{th} (10^5) (MPa^{-1})	$\Delta\lambda = \lambda_{ex} - \lambda_{th}$ (10^5) (MPa^{-1})	Ref.
PTB (Germany)	WC and steel	0.34	0·29	0·05	Jager (1985)
NPL (England)	Steels	0·43	0·32	0·11	Dadson *et al* (1965)
NIST (USA)	Steels	0·42	0·32	0·10	Welch and Bean (1984)

of controlled clearance piston gauge as

$$H(P,x) = H_0 + KP - r_p dP_j \tag{6}$$

where H_0 is the initial clearance at zero measured pressure between the piston and cylinder and $K/r_p = 0{\cdot}344 \times 10^{-5}\,\mathrm{MPa}^{-1}$ and d is the pressure distortion coefficient of the jacket cylinder. Thus defining $H(0) = H_0 - r_p dP_j$ and modifying for a simple piston cylinder, i.e., $P_j = 0$ and introducing the compression of the piston due to pressure, (6) can be written as

$$H(P,x) = H(0) + KP - K'P \tag{7}$$

where K' is the compression of the piston due to the pressure $K'/r_p = 0{\cdot}035 \times 10^{-5}\,\mathrm{MPa}^{-1}$ and in both the cases r_p is expressed in mm. In the second term of (7), P represents the pressure distribution in the annulus $[P(x)]$, while in the third term P represents pressure below the pistons $(P = P(0))$. Introducing these terms in (1) and assuming that pressure-dependent viscosity of the used liquid follows an exponential-type of variation $\eta(P) = \eta(0)\exp(\mathrm{CP})$ and $Q = \pi r_p^2 v$, where v, is the average fall rate of the piston, (1) can be rewritten as,

$$v\int_0^l \mathrm{d}x = -(1/(6\eta(0)r_p))\int_{\mathrm{P}}^{0}(H(0) + KP - K'P)^3\exp(-\mathrm{CP})\mathrm{d}P. \tag{8}$$

Equation (8) can be solved by different methods including the finite elemental analysis, however the numerical method of integration has been followed, the integrated equation as obtained can be written as,

$$\begin{aligned} v = \frac{1}{6\eta(0)lCr_p}\Bigg[& H(0) - K'P)^3 + 3\frac{K}{C}(H(0) - K'P)^2 + 6\left(\frac{K}{C}\right)^2(H(0) - K'P) \\ & + 6\left(\frac{K}{C}\right)^3 - \exp(-\mathrm{CP})\Bigg\{(H(0) + (K - K')P)^3 \\ & + 3\frac{K}{C}(H(0) + (K - K')P)^2 + 6\left(\frac{K}{C}\right)^2(H(0) \\ & + (K - K')P) + 6\left(\frac{K}{C}\right)^3\Bigg\}\Bigg]. \end{aligned} \tag{9}$$

3. Experimental arrangement

The primary standard in our laboratory is a controlled clearance piston gauge. It has the provision of going up to 1·4 GPa with different piston cylinder assemblies. The details of the experimental method for the metrological characterization of the gauge namely the determination of the cube root of fall rate as a function of jacket pressure have been carried out as usual (Sharma *et al* 1984). In brief, the fall rate of the piston at a given pressure is measured using a linear voltage displacement transducer (LVDT) with digital read out and having an usual resolution of 0·01 mm and a sensitivity better than 1 mV/V. The analog output of LVDT is directly recorded on the X, Y-t chart recorder. The temperature of the gauge is measured by a platinum resistance thermometer placed near the bottom of the cylinder. The piston is rotated at an arbitrarily chosen prefixed uniform speed which is driven by a synchronous motor and the rotational frequency is measured with an infrared detector. A calibrated bourdon tube gauge was used to measure the jacket pressure. The effective area and the pressure distortion coefficient of the secondary standards are obtained through a zig zag exercise of crossfloating the secondary standard against the primary standard.

4. Discussion

The fall rate of the piston as a function of pressure can be obtained from (9) provided the initial clearance $H(0)$ is known. However, there is no direct experimental method available to determine the value of the clearance between the piston and cylinder better than a few microns. The diameter of the piston can be measured with three dimensional coordinator with high accuracy but the inner diameter of the cylinders which are used in this high pressure region is of the order of 2–2·5 mm and cannot be measured as accurately (0·1 μm) as the outer diameter of the piston. Therefore, $H(0) = r_c - r_p$ which is of the order of a few micrometers or even less cannot be measured with that precision.

The theoretical fall rates from (9) are obtained for a simple piston cylinder, the radius of the piston (r_p) is 1·261 mm, the engagement length is taken to be 10 mm and the $H(0)$ is varied from 1·5 μm to 2·1 μm. The values of the viscosity of the pressure-transmitting fluid for the liquid diethylhexyl sebacate are taken, after applying the temperature correction, from pressure viscosity report of ASME (1952). The value of η_0 thus obtained is 17 cP and $C = 9{\cdot}03 \times 10^{-5}\,\mathrm{MPa}^{-1}$, K and K' are substituted as discussed. Figure 2 shows the computed fall rate of the piston as a function of pressure with different values of $H(0)$. It can be seen from the figure that the fall rate gradually increases with the increase of pressure up to 0·4 GPa, beyond that there is no appreciable change in the fall rate with pressure and this nonlinear behaviour of the curve has been observed at all initial clearances ranging from 1·5 to 2·1 μm. Interestingly, this near-constancy of fall rate above 0·4 GPa using same pressure transmitting fluid, has already been reported by Sharma *et al* (1983, 1984) in the controlled clearance piston gauge and it is essentially shown there that stall jacket pressure (Pz) with measured pressure (P) curve follows a non-linear behaviour above approximately the same pressure of 0·4 GPa. Later, Bandyopadhyay *et al* (1987) and Hilsch and Jager (1989) have confirmed the observation of Sharma *et al* (1983) and Yamamoto *et al* (1985) have, in fact, tried to correlate critical jacket pressures

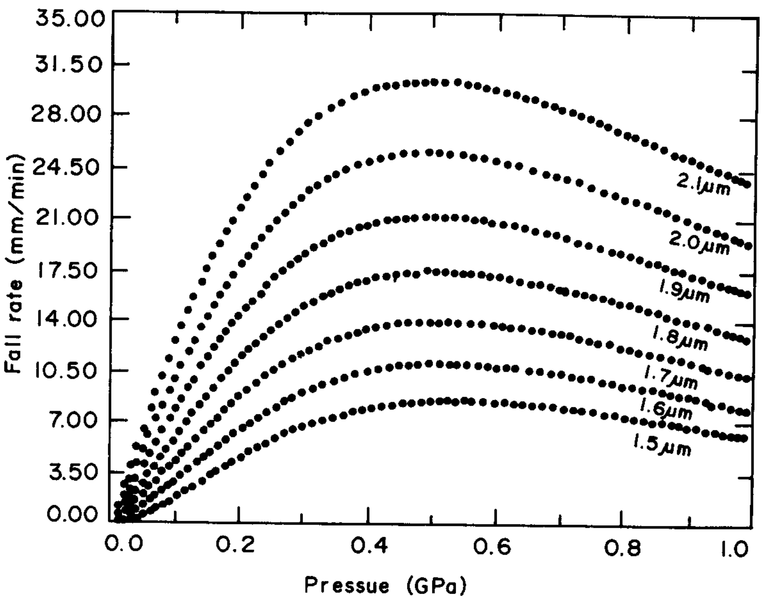

Figure 2. Computed fall rate versus pressure curve at different initial clearances.

with the viscosity of the pressure-transmitting liquid. Thus the present theoretically observed near-constancy of the fall rate with pressure is in accordance with the experimental observations.

Figure 3 shows the variation of the fall rate as a function of $H(0)$ for two different pressures using the same pressure-transmitting media. It may not be out of place to mention that the larger fall rate may cause less stability in equilibrium pressure, while the smaller rate results in mechanical friction at the matching surface of the piston cylinder assembly. Therefore, in designing the matching piston cylinder assembly, the lapping of the surfaces has to be perfected in such a way that the fall rate is optimum. From our experience, we find that a reasonably good performance can be obtained if the optimum fall rate is 1–2 mm/min and the corresponding initial clearance is 1–2 μm.

The viscosity of the sebacate oil increases rapidly with the increase of pressure (ASME report 1952) and on dilution with lower viscous fluids, the change of viscosity with pressure is restricted to a reasonably low value and in that case the viscosity may be treated as constant. Assuming constant value of viscosity with pressure, (8) can be simplified as,

$$v = [1/(24\eta(0)Klr_p)][(H(0) + P(K - K'))^4 - (H(0) - K'P)^4]. \tag{10}$$

Determining the viscosity of the mixture of 65% heptane and 35% sebacate (Sharma *et al* 1984), the fall rates have been obtained following (10). Figure 4 shows the computed fall rate as a function of pressure for a given $H(0)$ with pure fluid, and for lower value of $H(0)$ with the mixture. It is clear from the figure that even at the lower initial clearance, the fall rate curve shows a linear behaviour only in the mixture, while for pure fluids, as already discussed, a distinct non-linearity has been observed. This establishes that in the mixture where the viscosity does not vary with pressure, the pressure-transmitting fluid has no effect on the measurement of pressure and is

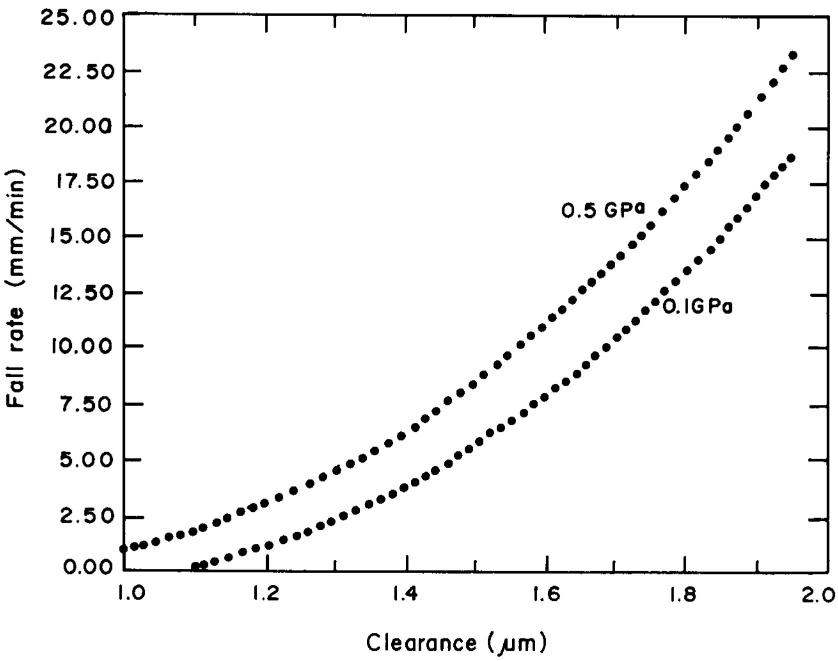

Figure 3. Computed fall rate versus initial clearance curve at two different pressures.

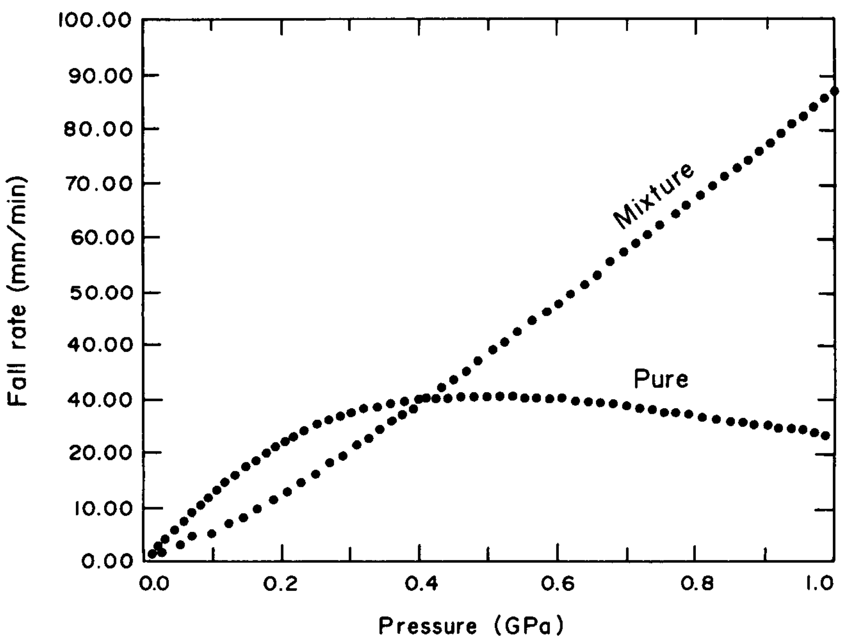

Figure 4. Computed fall rate versus pressure curve for both the pure fluid and the mixture.

in accordance with the experimental results by Sharma *et al* (1984) in the controlled clearance piston gauge.

From the above discussion, it is clear that in the absence of the reliable data of $H(0)$ because of the lack of precise and in situ experimental determination, the quantitative estimation of the effect of viscosity of the pressure-transmitting fluid on

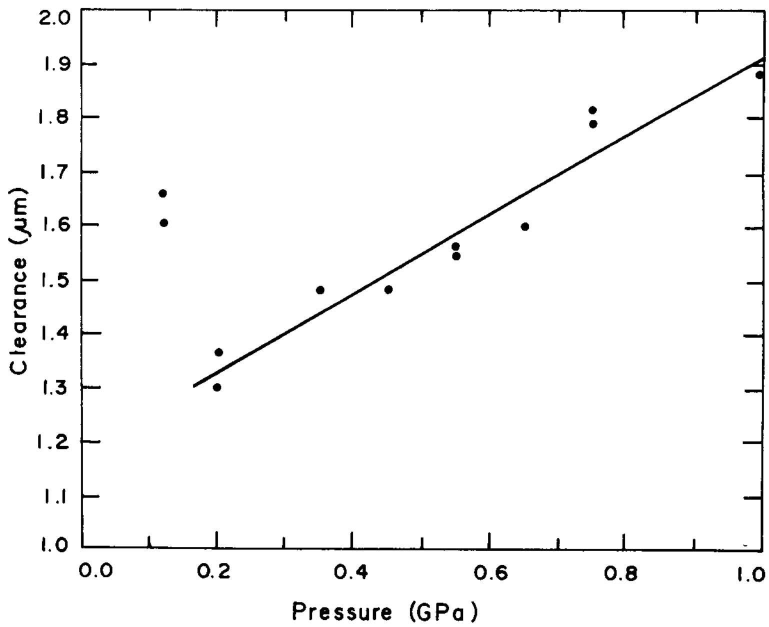

Figure 5. Initial clearance computed from the experimentally obtained fall rate data, is shown as a function of pressure.

the effective area and thereby the measured pressure, is difficult. However, the experimentally obtained fall rate with pressure could be used to compute the value of $H(0)$ as a function of pressure using (9).

The measured fall rates as a function of pressure are obtained from a series of repetitive observations. These data are evaluated with the assumption that the elastic constants assumed to be independent of pressure. Figure 5 shows the value of $H(0)$ as a function of pressure obtained from (9). In deriving the fall rate equation, it is assumed that the initial clearance is independent of pressure. However, figure 5 indicates that $H(0)$ has a pressure dependence apart from the lowest pressure points which are due to the fact at low pressure region the uncertainty in the measurement of pressure increases. Leaving these two points and fitting them in the linear form,

$$H(0) = H(0) + r_p \Delta\lambda P \tag{11}$$

gives $H(0) = 1{\cdot}186 \times 10^{-3}$mm and $r_p\Delta\lambda = 0{\cdot}07291 \times 10^{-5}\,\text{MPa}^{-1}$mm and this gives rise to $\Delta\lambda = 0{\cdot}0578 \times 10^{-5}\,\text{MPa}^{-1}$. This $\Delta\lambda$ when added to λ_{th} gives λ_{total} as $0{\cdot}347 \times 10^{-5}\,\text{MPa}^{-1}$ and this is close to the $(0{\cdot}35 \times 10^{-5}\,\text{MPa}^{-1})$. Therefore this extra dependence of pressure when the viscosity of the pressure-transmitting fluid has been taken into account, may be attributed to hidden contribution which could not be obtained from the simple elastic theory. It is interesting to note that the $A_0 = \pi r_p^2 (1 + H(0)/r_p) = 5{\cdot}00021\,\text{mm}^2$ is in close agreement with the A_0 obtained when it is processed through another experimental method, namely the method of cross floating of the two gauges. Incidentally, these values have been used for the calibrations of our piston gauges (NPL-500 and NPL-280). These were used in the fourth phase of the international comparison in the pressure range 20–100 MPa (200 bar to 1 kbar) organized by CCM under BIPM, Paris. The agreement of the deviations of the observed values of Ap from the appropriate least squares best fit straight line is within + 5 ppm in the whole pressure region and the zero pressure effective area (A_0) with respect to the reference value is $-51{\cdot}5$ ppm and the deviation of the pressure distortion coefficient from the reference value is $-0{\cdot}47 \times 10^{-6}\,\mathrm{MPa}^{-1}$ (Legras *et al* 1991). The estimated uncertainty of our measurement in A_0 is ± 74 ppm and therefore this agreement is well within our estimated uncertainty statement. The recent bilateral intercomparison with PTB(FRG) in the pneumatic pressure range 0·4 to 4·0 MPa may also be discussed where the effective area of the NPL-4 standard from the main standard agrees within + 1 ppm with the value derived from the calibration using the PTB-4 standard (Sharma *et al* 1988a, b). Similarly, the recent intercomparison with NIST(USA) in the pressure range 28–280 MPa shows that the agreement of effective area of the respective national standard is within + 3 ppm (Sharma *et al* 1991).

3. Conclusions

The present work shows the effect of viscosity of the pressure-transmitting fluid on the fall rate of the piston as a function of pressure. Theoretically obtained non-linearity in the fall rate with pressure beyond 0·4 GPa at all initial clearances studied in the present work, seems to agree well with the experimental observations of Sharma *et al* (1983, 1984) in the controlled clearance piston gauge. It is shown that the non-linearity in fall rate disappears when a mixture of less viscous fluid is used. Further, the experimentally obtained fall rates with pressure have been used to determine the initial clearance from the theoretical equation and it turns out that the initial clearance shows a pressure dependence, although it is assumed to be independent of pressure. This dependence may be due to the effect of viscosity. It is also shown that the total distortion coefficient (λ_{total}) taking this additional term into account, is close to the experimentally obtained pressure distortion coefficient.

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