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Effects of saturation on optical bistability with coupled surface plasmons

M B PANDE and S DUTTA GUPTA

School of Physics, University of Hyderabad, Hyderabad 500 134, India

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Abstract. We present exact numerical results for a symmetric layered medium (prism/Ag film/nonlinear dielectric/Ag film/prism) where the middle dielectric slab is assumed to have a saturation-type nonlinearity. We show bistable behaviour in the power dependence of the reflectivity of p-polarized light under the condition when coupled surface modes are excited in the structure. Moreover, we study the effect of saturation on the bistable behaviour to show that multivalued character is inhibited by saturation effects. The field distributions corresponding to the minimum reflectivity states of the nonlinear structure are also presented.

Keywords. Saturation; bistability; surface plasmon.

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1. Introduction

Coupled surface plasmons have been the subject of intense theoretical and experimental investigation in the past two decades (Dutta Gupta *et al* 1987; Pande and Dutta Gupta 1990b; Raether 1977). Recently, a new kind of coupling of the surface plasmons in a symmetric layered structure (prism/Ag film/nonlinear dielectric/Ag film/prism) was suggested (Welford and Sambles 1988). The coupled modes of this structure were shown to be characterized by comparable decay rates for both the long range (LR) and short range (SR) excitations. The approximately equal weight of the resonances due to the LR and SR modes was exploited to show optical multistability in a nonlinear structure of the same composition except that the central dielectric was assumed to have the simplest kind of non-linearity, namely, Kerr type non-linearity (Pande and Dutta Gupta 1990a, hereinafter referred to as I). The possibility of such multistable behaviour in a conventional coupling scheme through a metal layer was ruled out. In this paper, we consider a more realistic model of non-linearity incorporating the saturation effects. Our aim is to investigate the following aspects: (a) how the saturation affects the bistable and multistable behaviour, whether it inhibits bistability or not, (b) what are the field distributions corresponding to the minimum reflectivity (which signals the excitation of the surface modes) of the nonlinear structure. These distributions are of special interest and they carry information about the transverse field profiles of the nonlinear coupled surface modes. The organization of the paper is as follows. In § 2 we describe the system and formulate the problem. In the same section we outline the method to obtain the exact numerical results for the reflection coefficient of the nonlinear structure. In § 3 we present the results of our numerical calculations.

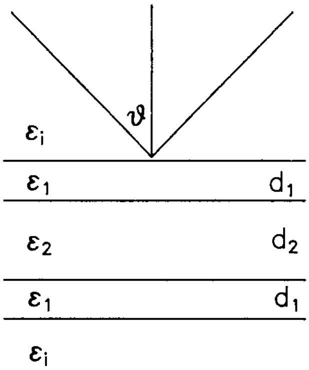

Figure 1. Schematic view of the layered structure.

2. Formulation of the problem

We consider the system (figure 1) consisting of two thin metal films with dielectric constant ε_1 and thickness d_1 separated by a nonlinear dielectric of thickness d_2. The structure is bounded on both sides by a high index prism with dielectric constant ε_i. A p-polarized plane wave of wavelength λ is incident on the structure at an angle θ. Let the non-linearity of the central spacer dielectric layer be given by the field dependent dielectric function (Peschel *et al* 1988)

$$\varepsilon_2 = \varepsilon_{20} + \frac{\alpha|\mathbf{E}|^2}{1 + \dfrac{\alpha}{\varepsilon_{\text{sat}}}|\mathbf{E}|^2} \tag{1}$$

where ε_{20} is the linear part of the dielectric function, α is the non-linearity coefficient, ε_{sat} is the saturation coefficient, and $\mathbf{E}$ is the electric field vector. Note that in the limit of very large $\varepsilon_{\text{sat}} (\varepsilon_{\text{sat}} \to \infty)$ eq. (1) reduces to the standard Kerr non-linearity considered in I. Thus, the results of our earlier calculations are contained in the framework of the present theory. It may also be noted that for practical materials the value of ε_{sat} is typically below 0·1 (Peschel *et al* 1988). Assuming the x-dependence of the field to be $\sim \exp(ik_x x)$ (with $k_x = k_0\sqrt{\varepsilon_i}\sin\theta$, $k_0 = 2\pi/\lambda$) and no variation along z ($\partial/\partial z \simeq 0$), the Maxwell's equations for p-polarized field components in the nonlinear medium can be reduced to the following form

$$\mathrm{d}\tilde{E}'_x/\mathrm{d}\xi = -D\tilde{E}''_y \tag{2}$$

$$\mathrm{d}\tilde{E}''_x/\mathrm{d}\xi = D\tilde{E}'_y \tag{3}$$

$$\mathrm{d}\tilde{E}'_y/\mathrm{d}\xi = (A_1A_2 - B^2)^{-1}[A_2(C\tilde{E}''_x + F\tilde{E}'_y) - B(-C\tilde{E}'_x + F\tilde{E}''_y)] \tag{4}$$

$$\mathrm{d}\tilde{E}''_y/\mathrm{d}\xi = (A_1A_2 - B^2)^{-1}[-B(C\tilde{E}''_x + F\tilde{E}'_y) + A_1(-C\tilde{E}'_x + F\tilde{E}''_y)] \tag{5}$$

where the dimensionless variable ξ is defined as $\xi = k_0 y$. The single (double) primes in (2)–(5) denote the real (imaginary) part of the dimensionless field components (for example, $\tilde{E}'_x = \sqrt{\alpha}\,\mathrm{Re}(E_x)$). The quantities A_1, A_2, B, C, D and F in (2)–(5) are defined as follows:

$$A_1 = (\varepsilon_2 M + 2\tilde{E}'^2_y), A_2 = (\varepsilon_2 M + 2\tilde{E}''^2_y), B = 2\tilde{E}'_y\tilde{E}''_y,$$

$$C = \varepsilon_2\eta M, D = (\eta - \varepsilon_2/\eta), F = 2D(\tilde{E}'_x\tilde{E}''_y - \tilde{E}''_x\tilde{E}'_y), \eta = k_x/k_0 \tag{6}$$

where M is given by the expression

$$M=\left[1+\frac{\tilde{E}_x'^2+\tilde{E}_x''^2+\tilde{E}_y'^2+\tilde{E}_y''^2}{\varepsilon_{sat}}\right]^2. \tag{7}$$

Note that in the limit $\varepsilon_{sat}\to\infty$, $M=1$ and the set of equations (2)–(5) reduce to the set investigated in I. It is clear from (2)–(5), that the form of non-linearity given by (1) makes it practically impossible to reach any analytical solution for the field profiles. Thus we are forced to integrate (2)–(5) numerically. The numerical integration of the above set requires the initial data at $y=0$, which is obtained from the boundary condition (continuity of the tangential field components $\tilde{E}_x$ and $\tilde{H}_z$). The boundary conditions at $y=0$ can be reduced to the following equations.

$$\tilde{E}_x^{0-}=\tilde{E}_x^{0+} \tag{8}$$

$$\tilde{E}_y^{0-}=\eta\tilde{H}_z^{0+}\left[\varepsilon_{20}+\frac{|\tilde{E}|^2}{1+\dfrac{1}{\varepsilon_{sat}}|\tilde{E}|^2}\right]^{-1}. \tag{9}$$

The method of calculating the reflection coefficient for the nonlinear structure is analogous to that of I and we do not repeat the arguments here. In the next section we present the results pertaining to the saturation effects and field profiles.

3. Numerical results and discussions

We define the dimensionless intensities $U_i=|\tilde{H}_i|^2$, $U_r=|\tilde{H}_r|^2$ and $U_f=|\tilde{H}_f|^2$, where the subscripts i, r and f refer to the incident, reflected and transmitted waves respectively. We present the results for the reflection coefficient $R(=U_r/U_i)$ for the system parameters given by $\lambda=1{\cdot}06\,\mu$m, $\varepsilon_i=6{\cdot}145$, $\varepsilon_1=-67{\cdot}03+2{\cdot}44i$, $d_1=0{\cdot}045\,\mu$m and $\varepsilon_{20}=2{\cdot}54$. We have chosen the value of the nonlinear spacer layer thickness d_2 such that the coupled mode splitting is prominent. Thus, for $d_2=1\,\mu$m, the coupled modes occur at $\theta_{SRSP}=39{\cdot}85°$ (SR mode) and $\theta_{LRSP}=41{\cdot}45°$ (LR mode). We have calculated the reflection coefficient for the system when the angle of incidence is chosen close to the SR resonance, namely, $\theta=40{\cdot}5°$, for various values of the parameter ε_{sat} ranging from 10^5 to 0·06. The results are shown in figure 2 where we have plotted the reflection coefficient R as a function of the incident power. It is clear from figure 2 that decrease in the parameter ε_{sat} (i.e. increase in the saturation effects) leads to a shrinkage of the bistability loop and the behaviour finally reduces to a monostable one. Note also that the curve corresponding to the fictitiously large value of $\varepsilon_{sat}\sim10^5$ coincides with the one for a Kerr nonlinear system (I). It is also clear that a decrease in ε_{sat} leads to a shift of the minima to larger input intensities. This implies that the nonlinear surface modes are excited for larger input intensities with increasing saturation effects. The limiting case $\varepsilon_{sat}\to0$ leads to a linear behaviour ($\varepsilon_2=\varepsilon_{20}$ as $\varepsilon_{sat}\sim0$) depicted by a horizontal line at $R=0{\cdot}934$ implying the insensitivity to the incident intensity.

We also studied the longitudinal and transverse field profiles $|\tilde{E}_x|$ and $|\tilde{E}_y|$ in the nonlinear medium corresponding to the minimum reflectivity states (see figure 2) of the structure. For more clarity we normalized these field distributions with respect

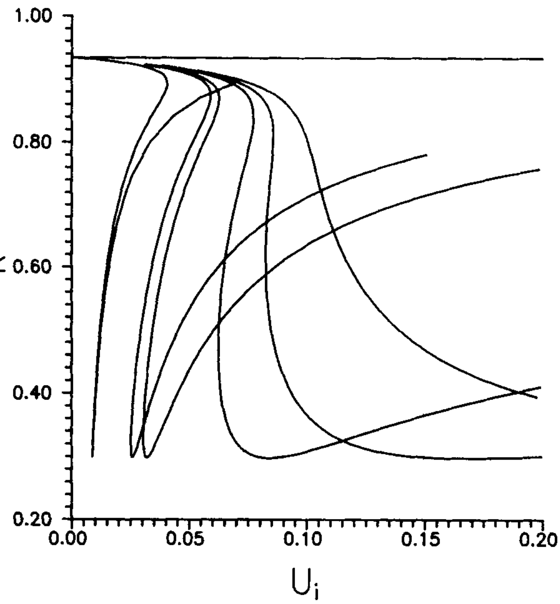

Figure 2. Reflection coefficient R as a function of input intensity U_i for $d_2 = 1{\cdot}0\,\mu m$ and $\theta = 40{\cdot}5°$. The curves from left to right are for values of $\varepsilon_{sat} = 10^5$, 0·1, 0·09, 0·07, 0·065 and 0·06. The other parameters are $\lambda = 1{\cdot}06\,\mu m$, $\varepsilon_i = 6{\cdot}145$, $\varepsilon_1 = -67{\cdot}03 + 2{\cdot}44i$, $d_1 = 0{\cdot}045\,\mu m$ and $\varepsilon_{20} = 2{\cdot}54$.

to the magnitude of the incident amplitude (i.e. $\sqrt{U_i}$). The results are shown in figures (3a) and (3b) where we have plotted $|\tilde{E}_x|/\sqrt{U_i}(|\tilde{E}_y|/\sqrt{U_i})$ as a function of the normalized length y/d_2. It is clear from figure 3a that the double hump structure for the field distribution for $|\tilde{E}_x|$, characteristic of the Kerr nonlinear case (see the curve for largest ε_{sat}) still persists when finite saturation effects are taken into account. But it gets more and more flat at the centre of the slab for increasing saturation. In the limit $\varepsilon_{sat} \to 0$ we recover the distribution for the linear system (Welford and Sambles 1988) characterized by a single maximum at the centre of the slab (not shown). On the other hand, the transverse field profile $|\tilde{E}_y|$ does not get affected much when saturation comes into play (see figure 3b). Note the proximity of the curves in figure 3b for all values of ε_{sat}. This may be explained taking into account the dispersive character of the non-linearity. Field induced dispersion is likely to affect the longitudinal component rather than the transverse component (this is also apparent from (2)–(5)).

In what follows, we investigate the effect of saturation on multistability which was reported for a Kerr nonlinear structure (I). We choose $d_2 = 1{\cdot}8\,\mu m$ and $\theta = 41{\cdot}3°$. We show the dependence of R on the incident intensity in figure 4 for various values of ε_{sat}, namely, $\varepsilon_{sat} = 10^5$, 0·08, 0·06 and 0·03. Again we recover the results for the Kerr nonlinear case for large ε_{sat}. A decrease in ε_{sat} leads to a shift of the curves to higher intensities (see figure 4). There is an overall broadening of the resonances as a result of which the finer details corresponding to the Kerr nonlinear case (large ε_{sat}) are lost. The dominant role is played by the non-linearity induced mode even for strong saturation whereas the coupled mode resonances are lost. We also studied the field distributions for the longitudinal and transverse components corresponding to the non-linearity induced modes (left most minima of the curves in figure 4). They show

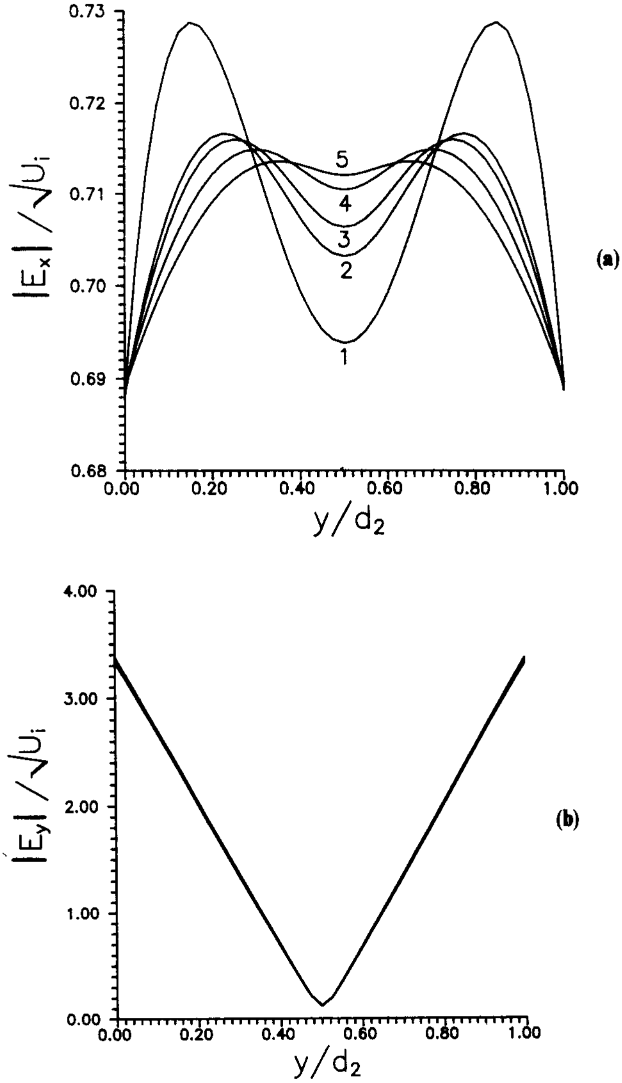

Figure 3. Normalized field distributions (a) $|\tilde{E}_x|\sqrt{U_i}$ and (b) $|\tilde{E}_y|/\sqrt{U_i}$ in the central dielectric layer corresponding to the minima of reflectivity in figure 2. Curves from 1 to 5 in figure 3a are for $\varepsilon_{sat} = 10^5$, 0·1, 0·09, 0·07 and 0·065 respectively. Other parameters are as in figure 2.

a dip close to the centre of the nonlinear slab. Moreover, there is an asymmetry with respect to the centre of the nonlinear slab, implying that the modes reside close to one of the metal/nonlinear dielectric interfaces.

4. Conclusions

In conclusion, we have investigated the effects of saturation on the bistability and multistability with coupled surface plasmons in a symmetric layered medium. We have demonstrated that bistable/multistable behaviour is inhibited by saturation. We

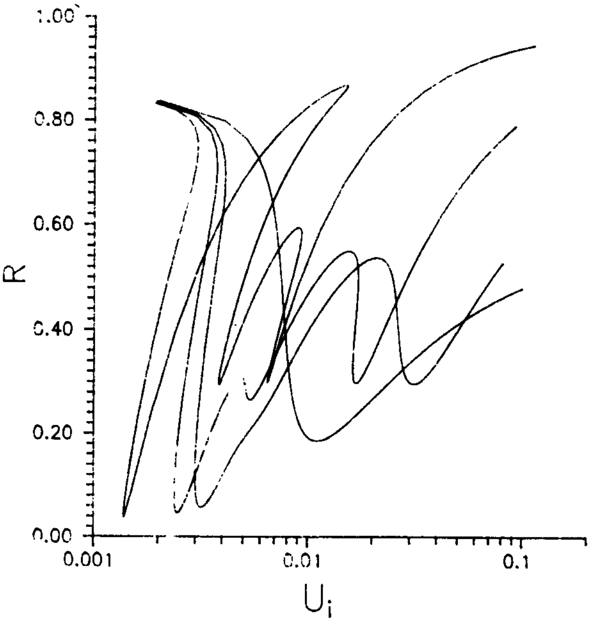

Figure 4. Reflection coefficient R as a function of the input intensity U_i for $d_2 = 1{\cdot}8\,\mu$m and $\theta = 41{\cdot}3°$. Curves from left to right are for values of $\varepsilon_{sat} = 10^5$, 0·08, 0·06 and 0·03. The other parameters are as in figure 2.

also studied the saturation induced changes in the field profiles in the nonlinear medium.

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