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Stability of second harmonic minority heating in magnetic mirror systems

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Abstract. A dispersion relation has been derived to study the stability of ion cyclotron (IC) propagation at the second harmonic of the minority component deuterium in a mirror confined plasma that has hydrogen as the majority species. We have worked in the frame of the majority ions; our dispersion relation can thus be used to study IC propagation in a single ion plasma also. Analysis of the dispersion relation yields two modes – one below and the other above the ion gyro-frequency Ω_H of hydrogen. The expression for the growth rate is used to explicitly show that an instability can arise in the plasma when the loss-cone index $j \geqslant 3$; this instability being a result of the coalescing of the two modes supported by the plasma. Unfortunately, the minority component deuterium does not decrease this instability and thus the proposed scheme of IC heating at the second harmonic of the minority component seems unsuited to mirror devices.

Keywords. Ion cyclotron waves; two-ion component plasma; dispersion relation; stability.

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1. Introduction

Ion cyclotron (IC) heating is one of the more promising additional methods for auxilliary heating of fusion plasmas and many successful experiments have been carried out using it (Askinasi *et al* 1989). A number of mirror-confined plasmas have also used IC waves—these include both the fast wave ($\omega > \Omega_i$; ω being the wave angular frequency and Ω_i the ion gyro-frequency) (Ichimaru *et al* 1988) and also the slow wave $\omega < \Omega_i$ (Golovato *et al* 1989). Mirror-confined plasmas are, however, susceptible to micro-instabilities such as the drift cyclotron loss-cone (DCLC) mode (Turner *et al* 1977) and the Alfven ion cyclotron (AIC) mode (Casper and Smith 1982), which unfortunately limit confinement. These instabilities produce fluctuations with frequencies $\omega \approx \Omega_i$ (the ICRF regime). For example, the AIC instability generates IC waves with a frequency slightly less than the minimum ion gyro-frequency in the end cells of a tandem mirror (Smith *et al* 1983).

In addition to being unstable, this frequency regime also has another undesirable feature of turning into alpha particle heating as soon as they are created leading to a large amount of power being absorbed in the outer regions of the plasma. To avoid this drawback second harmonic minority heating of hydrogen or helium has been suggested (Hellsten *et al* 1985). Experimentally, heating at the second harmonic of deuterium had been tried very early (Takahashi *et al* 1977) and more recently at the second harmonic of hydrogen (Hwang *et al* 1983). And interestingly, it has been shown that ICRF heating at the second harmonic of deuterium in a D–T plasma produces an anisotropic tail perpendicular to the magnetic field leading to a significant enhancement of the reaction rate (Krapchev 1985).

This paper examines the stability of second harmonic minority heating in mirror confined plasmas. With this objective, we have derived a dispersion relation for the near perpendicular propagation of IC modes in a plasma containing hydrogen as the majority species and deuterium as the minority constituent. The dispersion relation can be used to study IC propagation in a single-ion plasma also and is valid for any value of the loss-cone index j. The solution of this dispersion relation yields two modes—a low frequency (LF) and a high frequency (HF) mode, below and above the ion gyro-frequency Ω_H of hydrogen. The expression for the growth rate is used to explicitly show that an instability can arise in a single-ion plasma for $j \geqslant 3$. Unfortunately, the addition of the minority component deuterium does not significantly reduce the instability that can exist in a single-ion component plasma and thus other methods have to be sought to stabilize the plasma.

2. The dielectric tensor

We intend mainly to study IC propagation at the second harmonic of the ion gyro-frequency of the minority constituent deuterium in a magnetic mirror. Mirror confined plasmas tend to have a loss-cone velocity distribution except when they are collisional or flow-confined (Golovato *et al* 1989). The loss-cone velocity distribution will thus model all the three constituents of our plasma namely hydrogen (H), deuterium (D) and electron (e).

The elements of the dielectric tensor, for the loss-cone velocity distribution function, have been derived earlier (Chandu Venugopal 1983; hereinafter referred to as I) and thus will not be repeated here.

3. The approximation scheme

We are interested in the near perpendicular ($k_\parallel \ll k_\perp$, k being the wave vector) propagation of IC modes with a frequency around the second harmonic of the ion gyro-frequency of deuterium (that is, $\omega \sim 2\Omega_D$). We shall work in the frame of the majority hydrogen ions; to these ions this would be a wave with a frequency around its gyro-frequency Ω_H (that is, $\omega \sim \Omega_H$). In present ICRF heating scenarios $L_\perp$ (which is defined below) $\ll 1$ (Fuchs and Bers 1989) while the ion plasma frequency ω_{pi} is generally very much greater than the ion gyro-frequency (i.e. $\omega_{pi}^2 \gg \Omega_i^2$). To these we add the simplifying assumption that the plasma is approximately temperature isotropic not only among the different constituents but also with respect to the temperature parallel ($T_\parallel$) and perpendicular ($T_\perp$) to the magnetic field. These conditions lend themselves to the following ordering scheme (in terms of a small parameter ε):

$$\gamma_H = 1 - Z_H^2 \sim \varepsilon, \theta, L_\perp, L_\parallel \text{ and } (1/\overline{\omega_{pH}^2}) \sim \varepsilon$$

$$T_\perp/T_\parallel \sim 1 \text{ and } m_e/m_H \sim \varepsilon^2 \tag{1}$$

where

$$Z_H = (\omega/\Omega_H), \theta = (k_\parallel/k_\perp), L_{\perp(\parallel)} = (k_\perp^2 T_{\perp(\parallel)}/\Omega^2 m) \text{ and } \overline{\omega_{pH}^2} = (\omega_{pH}^2/\Omega_H^2). \quad (2)$$

Here m denotes the mass and T the temperature of the constituents.

4. Dispersion relations

4.1 *Propagation at the second harmonic of deuterium in a hydrogen-deuterium plasma*

We derive in this section a dispersion relation using the ordering scheme of (1) for the near perpendicular propagation of IC waves at the second harmonic of the minority component deuterium ion's gyro-frequency in a hydrogen-deuterium plasma.

The expressions for the various tensor elements using this ordering scheme and containing terms of order ε^{-1}, ε^0 and ε are lengthy even for an anisotropic Maxwellian plasma (Landau and Cuperman 1971; hereinafter referred to as II). They have also been derived earlier for a single ion (hydrogen) loss-cone plasma considering only the real part of the plasma dispersion function which occurs in the tensor elements (I).

From the ordering scheme (1) we can show that the asymptotic expansion of the plasma dispersion function is needed (I, II). The new expressions are even lengthier since we are dealing with a two-ion plasma and considering both the real and imaginary parts of the plasma dispersion function for all the constituents. We shall thus not give them here. On substituting these expressions into the formula for the dispersion relation (II) and carrying out the lengthy but straightforward algebraic simplification, we arrive at the dispersion relation which can be written as

$$\text{Re}\, D(\omega, k_\parallel, k_\perp) + i\, \text{Im}\, D(\omega, k_\parallel, k_\perp) = 0. \quad (3)$$

The expression for Re $D(\omega, k_\parallel, k_\perp)$ is

$$A\gamma_H^2 - B\gamma_H + C - D = 0 \quad (4)$$

where

$$A = 1 + \frac{20}{9}N_{DH} + \frac{4}{3}N_{DH}^2 - \frac{2}{3}N_{DH}\frac{L_{\perp H}}{\beta_\perp}$$

$$B = 1 + \frac{4}{3}N_{DH} - \frac{L_{\perp H}}{\beta_\perp} - \delta$$

with

$$\delta = [(8/3)N_{eH} + 2N_{eH}T_{\perp eH} - (L_{\perp H}/\beta_\perp)] * L_{\perp H} - (2/\overline{\omega_{pH}^2}) + (1/6)N_{DH}L_{\perp D}$$

$$C = \frac{L_{\perp H}^2}{4}\frac{(j-2)}{(j+1)}$$

and

$$D = \frac{4\theta^2 L_{\parallel H}}{\gamma_H}[1 + (4/3)N_{DH} - (L_{\perp H}/\beta_\perp)].$$

In the above

$$\beta_\perp = \frac{4\pi N_H T_{\perp H}}{B_0^2} \text{ with } \frac{L_{\perp H}}{\beta_\perp} \sim 1 + N_{DH}.$$

Also

$$N_{eH} = (N_e/N_H),\ N_{DH} = (N_D/N_H) \text{ and } T_{\perp,eH} = (T_{\perp e}/T_{\perp H})$$

N denotes the number densities and B_0 the ambient magnetic field.

The expression for Im $D(\omega, k_\parallel, k_\perp)$ is

$$\begin{aligned}\operatorname{Im} D(\omega, k_\parallel, k_\perp) = -\gamma_H^2 \Bigg\{ & \left[B - \frac{\gamma_H}{2}\left(\frac{1}{2} + \frac{L_{\perp H}}{\beta_\perp} + \frac{4}{9} N_{DH}\right) - \frac{2C}{\gamma_H} \right] e_1 \\ & + \left[N_{DH}\left(1 + \frac{4}{3} N_{DH} - \frac{L_{\perp H}}{\beta_\perp}\right) L_{\perp D} \right] e_2 \\ & + \frac{1}{\gamma_H}\left[2 N_{eH} \frac{T_{\perp e}}{T_{\parallel e} * (j+1)} L_{\perp e} \right] \tilde{e}_0 \Bigg\} \end{aligned} \tag{5}$$

where, in general,

$$e_n = (\pi/8)^{1/2} \frac{1}{(\theta^2 L_\parallel)^{1/2}} \exp\left[-\frac{(Z-n)^2}{2\theta^2 L_\parallel} \right]$$

and $\tilde{e}_0 = -e_0$. This re-definition is necessary to make $\tilde{e}_0$ positive as it is intrinsically negative due to the factor $(L_{\parallel,e})^{1/2}$ in the above expression (II).

As a check on our dispersion relation we note that for $N_{DH} = 0$, (4) is identical to that for a single ion plasma (I); in addition it reduces to the dispersion relation for an anisotropic Maxwellian plasma for $j = 0$ (II). Similarly Im $D(\omega, k_\parallel, k_\perp)$ reduces to the corresponding expression in II for $N_{DH} = 0$ and $j = 0$. However inspecting (4) we find the first three terms to be of $\sim \varepsilon^2$, we need to set

$$(1 + (4/3)N_{DH} - (L_{\perp H}/\beta_\perp)) \sim \varepsilon.$$

Unfortunately the D term is of order ε^3 and thus does not contribute to (4). Thus our dispersion relation for the near perpendicular propagation of IC modes has the final form

$$A\gamma_H^2 - B\gamma_H + C = 0. \tag{6}$$

Similarly inspecting (5) we find the deuterium contribution to be an order of magnitude less than the other two terms (from hydrogen and electrons) and thus has to be dropped from it though it has the very attractive feature of contributing to damping for $Z_H < 1{\cdot}0$. The expression for the growth rate Z_{iH} can be calculated from the well known formula and is given by

$$Z_{iH} = \frac{\begin{aligned}-\gamma_H^2\{[(A\gamma_H^2 - C) - (\gamma_H^2/2)(1/2 + (L_{\perp H}/\beta_\perp) + (4/9)N_{DH})]e_1 \\ + [2N_{eH}(T_{\perp e}/T_{\parallel e}*(j+1))L_{\perp e}]\tilde{e}_0\}\end{aligned}}{2Z_H(A\gamma_H^2 - C)} \tag{7}$$

4.2 *Propagation at the second harmonic in a deuterium plasma*

For a comparative study we now consider the propagation of IC waves at $2\Omega_D$ in a plasma containing deuterium alone. We use the same ordering as (1) except that now

$$4 - Z_D^2 \sim \varepsilon \text{ and } m_e/m_D \sim \varepsilon^2 \text{ where } Z_D = \omega/\Omega_D.$$

Proceeding as outlined in §4·1 we arrive, after a lengthy simplification, the dispersion relation for these IC waves which can also be written in the form of (3). However, the relevant expressions now are

$$\mathrm{Re}\, \mathrm{D}(\omega, k_{\parallel}, k_{\perp}) = \frac{5}{3} \frac{L_{\perp \mathrm{D}}}{(4 - Z_{\mathrm{D}}^2)} - 1 = 0 \tag{8}$$

and

$$\mathrm{Im}\, \mathrm{D}(\omega, k_{\parallel}, k_{\perp}) = 5/6\, L_{\perp \mathrm{D}} e_2. \tag{9}$$

As a check on (8), we point out its similarity, to another dispersion relation derived earlier for the anisotropic Maxwellian distribution (Cuperman and Metzler 1971).

The expression for the growth/damping rate $Z_{i\mathrm{D}}$ is now given by

$$Z_{i\mathrm{D}} = -\frac{(4 - Z_{\mathrm{D}}^2)^2}{4Z_{\mathrm{D}}} e_2. \tag{10}$$

5. Discussion

We shall now consider our dispersion relations. To simplify our discussion we consider a single-ion plasma; our conclusions are, however, valid for the two-ion plasma also. In a single-ion plasma $A = 1$ and the discriminant $B^2 - 4AC$ can become negative, leading to a pair of complex conjugate roots (and hence an instability) if and only if $j \geqslant 3$. Thus a single-ion plasma can become unstable if the loss-cone index j exceeds the critical value of 3. As regards (7) let $j = 0$ also. The term $(-C)$ is now positive definite. Since A is now equal to 1, the ion term within the curly brackets can become negative only if $L_{\perp \mathrm{H}}/\beta_{\perp} > 1{\cdot}5$; a situation we shall avoid. Thus the IC modes are damped for $j = 0$ in agreement with the earlier conclusion for an anisotropic Maxwellian plasma (II). Again inspecting (7) we find that both hydrogen and electrons contribute to the instability with increasing j: the factor $(j-2)/(j+1)$ in the ion term C increases from 0·25 for $j = 3$ to 1 for $j = \infty$, while the factor $(j+1)$ in the denominator of the electron term increases correspondingly from 0·25 to ∞. Numerical computation is, however, needed to find out the exact region of instability.

As regards the dispersion relation in a pure deuterium plasma we find, from (8), that the propagation characteristics of the modes are independent of the loss-cone index j for waves at the second harmonic of the deuterium ion gyro-frequency. Also the dispersion relation is linear and does not exhibit the mode conversion characteristic of (6). And from (10) it is obvious that the modes around $2\Omega_{\mathrm{D}}$ will always be damped. These conclusions also hold for second harmonic propagation in a plasma containing hydrogen alone.

6. Results

We use the parameters of typical mirror experiments namely $n_{\mathrm{H}} = 4 \times 10^{12}\,\mathrm{cm}^{-3}$, $B_0 = 10\,\mathrm{kG}$ and $\beta_{\perp} = 0{\cdot}035$ (Smith *et al* 1983 and Smith *et al* 1984). r_L denotes the ion Larmour-radius while $T_{\perp e\mathrm{H}}$ is set equal to 1 for simplicity in all our computations.

Figure 1 is a plot of the dispersion relation (6) for N_{D}, the deuterium density equal to 0. It depicts the variation Z_{H} versus $k_{\perp} r_{\mathrm{LD}}$ for two values of j namely 0 (indicated

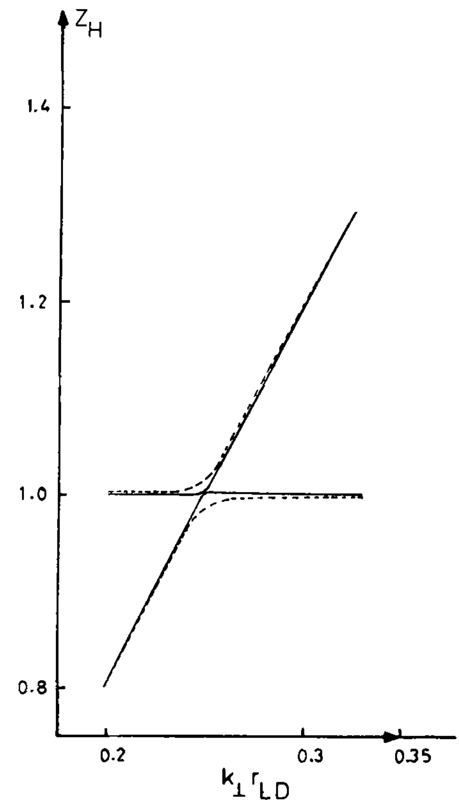

Figure 1. Plot of Z_H versus $k_\perp r_{LD}$ in a single ion hydrogen plasma for $j = 0$ (indicated by dotted lines and representing a Maxwellian plasma) and $j = 5$ with $T_{\perp eH} = 1{\cdot}0$.

by dotted lines) and 5 in a single ion (hydrogen) plasma. For $j = 0$ (Maxwellian plasma), the two modes are well separated indicating that the plasma is stable. However, for $j \geqslant 3$ (6) has complex roots in the coalescing region and the plasma can become unstable.

The addition of the minority component does not introduce any substantial change in this characteristic of the dispersion relation. However, there is a shift of the coalescing region towards a higher $k_\perp r_{LD}$ as the deuterium density is increased for a given j. The dispersion relation was also studied as a function of the temperature ratio between the two-ion species namely, $T_{\perp H}/T_{\perp D}$. While the general features of the dispersion diagram are still the same, the coalescing region is shifted towards a lower $k_\perp r_{LD}$ as this temperature ratio is increased from 0·5 to 2·5.

Figure 2 compares the growth rates in two types of plasmas for $j = 5$, $T_{\perp H}/T_{\perp D} = 2{\cdot}0$ and $\theta = 0{\cdot}025$, this value of θ being chosen so as to be consistent with our ordering scheme. A comparison of plot (a) (for single ion plasma) with plots (b) and (c) (which are for a plasma containing 4% and 8% of deuterium respectively as a minority species and in which a wave with $\omega \approx 2\Omega_D$ propagates) shows that the growth rate is larger for 4% deuterium and slightly less for 8% of deuterium. For other values of the minority ion concentration between one and ten per cent, the growth rate is slightly less but no marked reduction was noticed. Also as can be seen from the figure the instability shifts to a higher $k_\perp r_{LD}$ with the addition of deuterium.

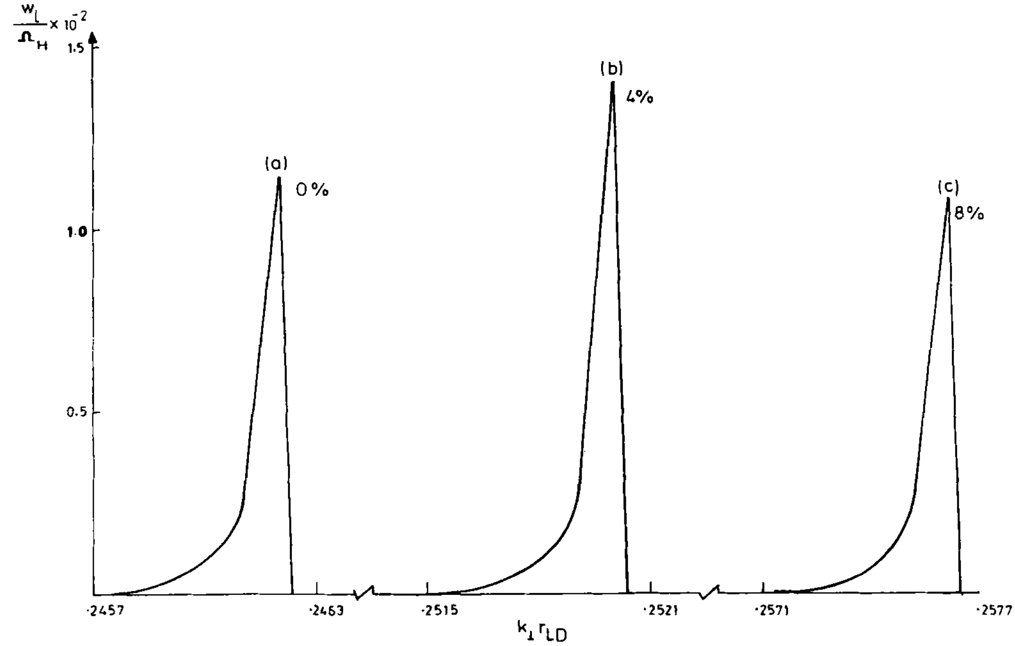

Figure 2. Plot of Z_{iH} versus $k_\perp r_{LD}$ for $j=5$, $\theta = 0{\cdot}025$ $T_{\perp D}/T_{\perp H} = 2{\cdot}0$ and $T_{\perp eH} = 1{\cdot}0$. Plot (**a**) is for a single ion plasma of hydrogen while plots (**b**) and (**c**) are for a plasma containing 4% and 8% of deuterium respectively.

The growth rate was also studied by varying $T_{\perp H}/T_{\perp D}$ from 0·5 to 2·5. While the general characteristics were the same as in figure 2, there was a shift in the region of wave growth towards lower values of $k_\perp r_{LD}$ as $T_{\perp H}/T_{\perp D}$ was increased from 0·5 to 2·5. This shift in the region of wave growth is consistent with the results of the real part of the dispersion relation.

7. Conclusions

We have in this paper derived a dispersion relation to study the stability of the second harmonic minority heating scheme in a mirror confined plasma. The dispersion relation derived can also be used to study IC propagation in a single-ion loss-cone plasma. It has been shown, both analytically and numerically, that a loss-cone plasma is unstable to IC modes when the loss-cone index exceeds the critical value of 3. Experimentally, it is well-known that IC instabilities produce waves with $\omega \approx \Omega_i$; our computed results also show the instabilities to be confined to a narrow region around Ω_i. Unfortunately, the proposed scheme of heating at the second harmonic of the minority component deuterium also has the undesirable feature of being unstable in the ICRF regime though it may have other desirable features.

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