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Regge relation and geometrization of fundamental constant

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Abstract. We show that the Regge relation between angular momentum and mass is due to curved space time description of basic interactions. We try to understand the geometrization of h and e in the light of the relation.

Keywords. Regge relation; Planck constant; geometrization.

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1. Introduction

Recently there has been a lot of discussion on the Regge-like relation between the angular momentum J and the mass M of a wide range of celestial bodies (Sabbata and Sivaram 1988; Wesson 1979, 1981; Brosche 1980). It is observed that a wide variety of celestial objects satisfy the relation

$$J = pM^2 \tag{1}$$

where $p = 3{\cdot}10^{-16}\,\mathrm{g}^{-1}\,\mathrm{cm}^2\,\mathrm{s}^{-1}$. In the hadronic regime, one also encounters a relation like (1). It is found that all hadrons lie on a trajectory known as Regge trajectory given by the relation

$$J_h = \alpha_0 + \alpha' M^2 \tag{2}$$

where α_0 and α' are the intercept and the slope of the trajectory respectively. In the strong gravity formulation of strong interaction (Salam and Strathdee 1977; Salam 1977; Biswas *et al* 1983, 1989), the relation (2) is formulated as

$$J_h \sim (G_f/\hbar c) M^2 \tag{3}$$

at sufficiently high energy with $P_h \simeq (G_f/\hbar c) \simeq 1(\mathrm{GeV/c^2})^{-2}$. Now it is argued, seeing the resemblance of (1) and (3), that the strong gravity serves as connecting link for the universal relation between moments of celestial bodies and spin of elementary particles. This aspect has been elucidated by Sabbata and Sivaram from various footings (Sabbata and Sivaram 1988). In this paper we show that the tensor dominance (spin 2^+) in the interaction lies in the origin of the relations (1) and (3) which provide a way of the geometrization of $\hbar$ and e built up in the geometric structure of space time. Recently Ross (1989) has argued that one can build Planck's constant into the structure of space time on the assumption that the torsional defect that intrinsic spin produces in the geometry is a multiple of Planck length. Our attempt is based on the relations (1) and (3). The empirical observations like the relations (1) and (3) are well tested (Biswas *et al* 1983, 1989; Sivaram and Sinha 1977, 1979). The origin may be understood as follows.

2. Motion of spinning top

Let us discuss the motion of a spinning top in a given Riemannian background (Hojman and Hojman 1976). The relativistic spherical top in Minkowski background was discussed in length by Hanson and Regge (1974) and spinning tops in external fields by Hojman and Regge (1976). The Lagrangian is constructed from the invariants

$$a_1 = g_{\mu\nu} u^\mu u^\nu = u^2, \tag{4a}$$

$$a_2 = g_{\mu\nu} g_{\alpha\beta} \sigma^{\alpha\mu} \sigma^{\nu\beta} = \operatorname{tr}\sigma^2, \tag{4b}$$

$$a_3 = g_{\mu\nu} g_{\alpha\beta} g_{\gamma\delta} u^\mu \sigma^{\nu\alpha} \sigma^{\beta\gamma} u^\delta = u\sigma\sigma u, \tag{4c}$$

$$a_4 = g_{\mu\nu} g_{\rho\tau} g_{\alpha\beta} g_{\gamma\delta} \sigma^{\delta\mu} \sigma^{\nu\rho} \sigma^{\tau\alpha} \sigma^{\beta\gamma}, \tag{4d}$$

such that

$$L_0 = (a_1)^{\frac{1}{2}} \left(\frac{a_2}{a_1}, \frac{a_3}{a_1^2}, \frac{a_4}{a_1^2} \right). \tag{5}$$

The momentum p^μ and spin $S^{\mu\nu}$ are given by (Hojman and Hojman 1976)

$$P^\mu = -\frac{\partial L_0}{\partial u_\mu} = -2u^\mu L_1 - 2\sigma^{\mu\nu}\sigma_{\nu\lambda} u^\lambda L_3, \tag{6a}$$

$$S^{\mu\nu} = -\frac{\partial L_0}{\partial \sigma_{\mu\nu}} = -4\sigma^{\nu\mu} L_2 - 2(u^\mu \sigma^{\nu\lambda} u_\lambda - u^\nu \sigma^{\mu\lambda} u_\lambda) L_3 - 8\sigma^{\nu\lambda}\sigma_{\lambda\tau}\sigma^{\tau\mu} L_4, \tag{6b}$$

where

$$L_i = \partial L_0 / \partial a_i. \tag{6c}$$

The top is described by its position $x^\mu(\tau)$ and $e^\mu_{(\alpha)}(\tau)$ to specify its orientation. The following are the useful relations.

$$\text{Velocity} = u^\mu = \mathrm{d}x^\mu/\mathrm{d}\tau, \tag{7}$$

$$\text{angular velocity} = \sigma^{\mu\nu} = \eta^{(\alpha\beta)} e^\mu_{(\alpha)} \dot{e}^\nu_{(\beta)} = -\sigma^{\nu\mu}, \tag{8}$$

$$(\text{Tetrad}),\ \tau = \dot{e}^\mu_{(\alpha)} = \frac{De^\mu_{(\alpha)}}{D\tau} \equiv \frac{de^\mu_{(\alpha)}}{d\tau} + \Gamma^\mu_{\lambda\rho}(x) e^\lambda_{(\alpha)} u^\rho. \tag{9}$$

The equations of motion that follow from (4) to (9) are

$$\frac{DP^\mu}{D\tau} = 1/2 R^\mu_{\nu\alpha\beta} u^\nu S^{\alpha\beta}, \tag{10}$$

$$\begin{aligned} \frac{DS^{\mu\nu}}{D\tau} &= S^{\mu\lambda}\sigma^\nu_\lambda - \sigma^{\mu\lambda} S^\nu_\lambda, \\ &= P^\mu u^\nu - P^\nu u^\mu. \end{aligned} \tag{11}$$

For details the reader is referred to Hanson and Regge (1974) and Hojman (1975). The Lagrangian L_0 is so chosen that it is a homogeneous function of degree one in velocities and is invariant under the change of time variable $\tau' = \tau'(\tau)$, usually called reparametrization invariant. From the equations of motion (10) and (11), it can be shown that $M^2 = P_\mu P^\mu$ and $J^2 = 1/2 S^{\mu\nu} S_{\mu\nu}$ are constants and using the constraint

$$S^{\mu\nu} P_\nu = 0 \tag{12}$$

we find

$$P_\mu P^\mu - f(\tfrac{1}{2} S^{\mu\nu} S_{\mu\nu}) = 0. \tag{13}$$

If the geometry described by $g_{\mu\nu}$ possesses some symmetry, one can show that

$$C_\xi = P^\mu \xi_\mu - \tfrac{1}{2} S^{\mu\nu} \xi_{\mu\cdot\nu}, \tag{14}$$

is a constant where ξ_μ is a Killing vector associated with the symmetry, i.e.,

$$\xi_{\mu;\nu} + \xi_{\nu;\mu} = 0. \tag{15}$$

Associated with these constraints one can choose

$$x^0 - \tau = 0, \tag{16}$$

$$e^\mu_{(0)} - P^\mu / M = 0, \tag{17}$$

as gauge.

If we consider the motion of the top in e.m. background described by

$$L = L_0 - e A^\mu u_\mu, \tag{18}$$

we again get the relation (13) provided

$$P^\mu \to \pi^\mu, \tag{19}$$

where

$$\pi^\mu = P^\mu - e A^\mu.$$

It is worthwhile to point out that (12), (13), (16) and (17) are the constraints. The above discussion concludes that if a particle motion (having spin J) is described in a spin 2 background, the emergence of Regge-like behaviour between spin and mass seems to be a general consequence. Thus the origin of the relations like (1) and (3) is quite general and is an indication of a connecting link between microphysics and macrophysics. It may be mentioned that the existence of Regge trajectory for free relativistic top with $g_{\mu\nu} = \eta_{\mu\nu}$ was also shown by Hanson and Regge (1974). This is quite natural because of tensor character of (4), (10) and (11).

3. $\hbar$-geometrization

In view of the recent interest to find a connection between micro and macrophysics, it is suggested that the large scale structure of the universe is determined by microphysics through the primordial density fluctuation of the early universe (see Aurilia *et al* 1987 and references therein). While most of the authors seem busy with the scenario of cosmic inflation, there are a few (Sabbata and Sivaram 1988; Biswas *et al* 1989; Biswas

1990) who take hadronic microcosmos as a laboratory to understand the large scale structure of the universe. The elucidation of the relations (1) and (3) with respect to (i) Blackett relation connecting the magnetic moment H and J of celestial objects, i.e.,

$$R = (G^{1/2}/c)J \tag{20}$$

(ii) the angular momentum of white dwarfs and neutron stars, (iii) gluon bremsstrahlung mechanism, (iv) the theoretical explanation of (20) through strong gravity ($G_f \sim 10^{38} G_N$) and torsion suggests that the dimensionless fine structure constant α appears not only typical for particle physics but also for gravitational physics. All of them provide support to the possibility of unifying gravitation with other fundamental interactions.

The macrophysics is described by Newton's constant G_N and the gravitation field $g_{\mu\nu}$ (spin 2^+). The microphysics deals with the elementary particles and their interaction. Theories related to microphysics may be the GUT$_s$, QCD or even the string theories. Our suggestion is that if we consider hadrons interacting via strong gravity (i.e. by exchange of massive spin 2 tensor mesons) with $G_f \sim 10^{38} G_N$, it is possible to treat hadrons just like celestial objects via relation (1) with $G_N \to G_f$. Thus the tensor dominance in strong interaction provide the connecting link between microphysics and macrophysics, the origin of which may be traced back to the existence of the relation like (13). The coupling constant α_{gut} of GUT or α_g of QCD are not suitable for describing the empirical relations (1) and (3). These aspects are also discussed by Sabbata and Sivaram (1988). There are so many citations for the relations (1) and (3) in Sabbata and Sivaram (1988). We mention one of them to elucidate our stand. For a typical main sequence of stars i.e., sun, white dwarfs. neutrons stars, the angular momentum J is given by

$$J \simeq (10^{76} - 10^{78})\hbar. \tag{21}$$

To find the explicit dependence of J on G_N we note that the mass and radius for a typical main sequence of star are given by (Sabbata and Sivaram 1988)

$$M_S \simeq (\hbar c/G_N m_p^2)^{3/2} m_p, \tag{22}$$

$$R_S \simeq (\hbar^2/m_e e^2)(\hbar c/G_N m_p^2)^{\frac{1}{2}}. \tag{23}$$

For a typical rotational velocity

$$V_S \simeq (G_N M_S/R_S)^{\frac{1}{2}}, \tag{24}$$

the angular momentum ($J = M_S V_S R_S$), using (22)–(24), is given by

$$J_S \simeq \left(\frac{\hbar c}{G_N m_p^2}\right)^{5/2} (m_p^3/m_e)^{\frac{1}{2}} \left(\frac{G_N}{\hbar c}\frac{\hbar c}{e^2}\right)^{1/2} \hbar,$$

$$\sim G_N{}^{-2}\hbar \simeq 10^{78}\hbar. \tag{25}$$

In the above equations, $m_p =$ mass of proton and $m_e =$ mass of electron. For white dwarfs and neutron stars (Sivaram 1984), the angular momentum is in the range as in (21).

If our guess (i.e., the strong gravity provides the connecting link) is correct, we should get the angular momentum of hadron from (21) and (25) when we set

$G_f \sim 10^{38} G_N$. From (21) and (25) we get (all the other factors remaining same)

$$J_{\text{hadron}} \sim G_f^{-2} \hbar \sim \frac{1}{10^{76}} G_N^{-2} \hbar \sim \hbar, \tag{26}$$

as G_f for strong gravity is 10^{38} times larger than G_N. There are various other considerations e.g., Jean mass for galaxies, radius of star and Hubble radius etc. which give the corresponding quantities for hadrons when $G_N \to G_f$ (Sabbata and Sivaram 1988). Henceforth we take $G_N = G$.

As suggested by Wesson, while in microphysics i.e., particle physics, three constants (e, h, c) are involved from which one can form a dimensionless number α, but in gravitation we have only two constants (G, c) which are not sufficient to form a set in the sense that we cannot form dimensionless numbers. But if we consider the relation (1), we can form a dimensionless number β (a new universal constant) such that

$$\beta = G/cp. \tag{27}$$

The observed value of G, c and p fixes β, equal to the fine structure constant (Sabbata and Sivaram 1988) i.e.,

$$\beta \simeq \alpha. \tag{28}$$

So the relation

$$\alpha = (e^2/4\pi\hbar c) \sim (G/pc), \tag{29}$$

suggests that we expect a relation

$$e^2 = f(L_P)G, \tag{30}$$

$$\hbar = f(L_P)(p/4\pi), \tag{31}$$

in order to suit the empirical observation (29). Here $f(L_P)$ is a function of Planck length L_P. It is suggested that any attempt at unification of gravity with quantum physics inevitably leads to the Planck length

$$L_P = (\hbar G/c^3)^{1/2}, \tag{32}$$

usually interpreted as defining the scale at which quantum corrections to general relativity are expected to become important (Sivaram 1986). We suggest that the relations (30) and (31) are the basis of geometrization of h and e. The correspondence is

$$(e^2, h) \to (G, p). \tag{33}$$

The geometric structure that builds the charge from G will also be responsible to build h out of p. The hints of unification of all interactions lie in (G, c, p) or in α, or in α_U, the fine structure constant of unified theory of four interactions. The simulation of charge from extra dimension has been shown by Chyba (1985) and Weinberg (1983a, b). The Kaluza–Klein description provides a natural explanation for the quantization of charge and is related to the quantization of extra dimension.

To see the connection between e and G, we consider a charged complex field with action

$$S_\phi = \int d^5x(-g^5)^{1/2}(\partial_A\phi)(\partial_B\phi^+)g^{AB}, \tag{34}$$

in the Kaluza–Klein description. Here $A, B = 0, 1, 2, 3, 5$. The Fourier component of ϕ with nontrival dependence $\phi^{(n)}\exp(inx^5R_5)$ will behave as a particle of charge

$$q = (16\pi G)^{1/2}\frac{n}{R_5}\cdot\frac{\hbar}{c}. \tag{35}$$

Introducing $\hbar, c$ we write (35) as (with $n = 1$)

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{4\hbar G}{c^3R_5^2}, \tag{36}$$

$$R_5 = \frac{2}{\alpha^{1/2}}\left(\frac{\hbar G}{c^3}\right)^{1/2} = \frac{2}{\alpha^{1/2}}L_P \sim 3{\cdot}7\,10^{-32}\,\text{cm}. \tag{37}$$

In the above description, the Kaluza–Klein vacuum is taken as $M^4 \times S^1$ and R_5 is the radius of the extradimensional S^1 circle.

Thus,

$$\alpha = \frac{4\hbar G}{c^3R_5^2} = \frac{G}{(c^2R_5^2/4\hbar)\cdot c}. \tag{38}$$

Hence if our geometrization is correct we should get

$$p = \frac{c^2R_5^2}{4\hbar} \sim 0{\cdot}5 \times 10^{-16}\,\text{g}^{-1}\,\text{cm}^2\,\text{s}^{-1}, \tag{39}$$

with the substitution $R_5 = 3{\cdot}7 \times 10^{-32}$ cm. This order of estimate justifies our approach. In view of Weinberg's derivation of charges for extra dimension (Weinberg 1983), it is suggested that the geometry that builds e^2 from G is also responsible for building h from p, a constant of macrophysics. From (36) we get

$$e^2 = \frac{16\pi\hbar^2G}{c^2R_5^2}, \tag{40}$$

and hence from (28)

$$f(L_P) = \alpha\frac{4\pi\hbar^2}{c^2}\frac{1}{L_P^2}. \tag{41}$$

If the Kaluza–Klein description has anything to say about geometrization as in (30) and (31), we should be able to generate the Regge relation (1) or (3) from this description. This is indeed the case. We have shown elsewhere that (Biswas *et al* 1989) if we assume that the higher dimensional space time has a Robertson–Walker type metric

$$\begin{aligned} ds^2 &= dt^2 - R_5^2(t)d\varphi^2, \\ &= c(\eta)(d\eta^2 - d\varphi^2), \end{aligned} \tag{42}$$

and take $c(\eta) = A + B \tanh \rho\eta$, it is possible to generate the Regge spectrum

$$w^2 = J^2/R_5^2. \tag{43}$$

With $J^2 = n^2 + \text{constant } (A \pm B)$. In (42) to (43), η is the conformal time given by

$$t = \int^t R_5(\eta')\mathrm{d}\eta'. \tag{44}$$

The discussions leading to (38) as well as (43), relate to the behaviour of space time at length of the order of $R_5 \sim$ Planck length. As suggested by Misner *et al* (1973) the space time is likely a topological foam on this distance scale. In the $\hbar$ geometrization of Ross (1989) a quantized defect in space time is measured by a quantity $\mathscr{L}^a$ called closer failure. In a space time, the torsion has an intrinsic geometric meaning. Suppose in a space time with the connection $\Gamma^\alpha_{\beta\gamma}$, we traverse a closed circuit using parallel displacement of vectors. For two vectors U and V, in a closed loop

$$U^a\nabla_a - V^a\nabla_a U^c - [U, V]^c = 0, \text{ (no torsion)}$$

$$\neq 0, \text{ (torsion present)}. \tag{45}$$

The r.h.s. of (45) is represented as $S^c_{ab}U^a V^b$ and is not equal to zero when the torsion is present. $\mathscr{L}^\alpha$ is related to the torsion

$$S^\alpha_{\beta\gamma} = \Gamma^\alpha_{[\beta\gamma]}, \tag{46}$$

by

$$\mathscr{L}^\alpha = \int S^\alpha_{\beta\gamma}\mathrm{d}A^{\beta\gamma}. \tag{47}$$

The geometrization of $\hbar$ is built up by the assumption

$$\mathscr{L}^0 = nL_p. \tag{48}$$

The relation (48) with

$$\int \mathscr{L}^0 \mathrm{d}x^3 = (nL_p)(L_p), \tag{49}$$

gives

$$S^3 \sim n\hbar. \tag{50}$$

The intrinsic spin S^α is defined by Weinberg's relation

$$S^\alpha = \tfrac{1}{2}\varepsilon^{\alpha\beta\gamma\delta}J_{\beta\gamma}u_\delta. \tag{51}$$

The above discussion indicates that $\hbar$ enters into the geometry through Planck length. In our approach we have geometrization through (39) where $R_5(\sim$ Planck length) and p serve as geometrical entities to build h. In this sense our geometrization is interesting because it takes care of empirical observation (1) and (3) and simulates the charge from the same geometric structure. What we find is that the quantization of the extra compact space dimension explain the numerical value of charge quantum

$$e = \left(\frac{16\pi G\hbar^2}{c^2R_5^2}\right)^{1/2} = \left(\frac{16\pi\hbar G}{4p}\right)^{1/2}, \tag{52}$$

or both e and h from the geometric constants G, p, c. This is what we call the geometrization of h and e.

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