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On the bound state spectrum of a many-electron-proton system

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Abstract. An equation is obtained for the pairing amplitude in a many-electron-proton system at finite temperature. It is noted that under certain approximations it can be solved to give temperature-dependent discrete energy spectra.

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We consider the many-electron-proton system subject to the usual electrodynamic interaction

$$-\mathscr{L} = -ie\bar{\psi}_A\gamma_\mu\psi_A A_\mu + ie\bar{\psi}_B\gamma_\mu\psi_B A_\mu, \tag{1}$$

where ψ_A, ψ_B and A_μ are, respectively, the electron, the proton and the photon fields. We work in units with $\hbar = C = 1$ and $e^2/4\pi \equiv \alpha \simeq 1/137$. Assuming homogeneity for the medium, we then consider the two-body Green's function

$$\begin{aligned} K(x_1,x_2;x_3,x_4) &= -\operatorname{Tr}\{\exp[(\Omega+\mu N - H)\beta] \\ &\quad \times T_\tau(\psi_A(x_1)\psi_B(x_2)\bar{\psi}_A(x_3)\bar{\psi}_B(x_4))\} \\ &\equiv -\langle T_\tau(\psi_A(x_1)\psi_B(x_2)\bar{\psi}_A(x_3)\bar{\psi}_B(x_4))\rangle, \end{aligned} \tag{2}$$

where Tr stands for trace and means the sum of the diagonal elements of the operator inside the curly brackets over all possible states of the system at temperature T; T_τ refers to 'time' ordering with respect to the variable τ, beginning with the latest τ on the left, and includes a sign change for odd permutations for fermion operators; H, N and Ω are, respectively, the hamiltonian, the number operator and the thermodynamic potential of the system; μ stands for chemical potential and $\beta = 1/kT$ where k is the Boltzman constant, taken to be unity in the following.

The 'time' dependence of any of the 'Heisenberg' fields in the above is given by

$$\phi(x_i) = \phi(\mathbf{x}_i,\tau_i) = \exp[(H-\mu N)\tau_i]\phi(\mathbf{x}_i)\exp[-(H-\mu N)\tau_i]. \tag{3}$$

The Green's functions F as functions of any of the τ_i have the property

$$F(\tau_i = \beta) = \pm F(\tau_i = 0), \tag{4}$$

and for any $\tau = \tau_i - \tau_j$,

$$F(\tau < 0) = \pm F(\tau + \beta), \tag{5}$$

with the sign $+(-)$ when the τ's are associated with boson (fermion) operators. These functions can be Fourier summed with only even (odd) components for the boson (fermion) case (Abrikosov *et al* 1974; Fetter and Walecka 1971).

Going over to the interaction representation, we have

$$K(x_1, x_2; x_3, x_4) = \mathrm{Tr}\frac{\{\exp[(\mu N - H_0)\beta]\, T_\tau(\psi_A^I(x_1)\psi_B^I(x_2)\bar{\psi}_A^I(x_3)\bar{\psi}_B^I(x_4)S)\}}{\mathrm{Tr}\{\exp[(\mu N - H_0)\beta]S\}}$$

$$\equiv \frac{\langle T_\tau(\psi_A^I(x_1)\psi_B^I(x_2)\bar{\psi}_A^I(x_3)\bar{\psi}_B^I(x_4)S\rangle_0}{\langle S\rangle_0}, \tag{6}$$

where ψ^I are now field operators in the interaction representation and the averaging is done over the states of the system with noninteracting particles. Further,

$$S = S(\beta, 0) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^\beta \cdots \int_0^\beta \mathrm{d}\tau_1 \cdots \mathrm{d}\tau_n\, T_\tau(H_I(\tau_1)H_I(\tau_2)\cdots H_I(\tau_n)), \tag{7}$$

where H_I again refers to the interaction representation. The perturbation series for the Green's function K is thus given by

$$\begin{aligned} K(x_1, x_2; x_3, x_4) = {} & \frac{1}{\langle S\rangle_0} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \int_0^\beta \cdots \int_0^\beta \mathrm{d}\tau_1' \cdots \mathrm{d}\tau_n' \\ & \times \langle T_\tau(\psi_A^I(x_1)\psi_B^I(x_2)\bar{\psi}_A^I(x_3)\bar{\psi}_B^I(x_4) \\ & \times H_I(\tau_1')H_I(\tau_2')\cdots H_I(\tau_n'))\rangle_0. \end{aligned} \tag{8}$$

Expanding in powers of the coupling constant e, and at the same time making use of the generalized Wick theorem, we can write (8) in the form

$$\begin{aligned} K(x_1, x_2; x_3, x_4) = {} & G^A(x_1, x_3)G^B(x_2, x_4) + e^2 \int \mathrm{d}x_5 \int \mathrm{d}x_6 G^A(x_1, x_5)\gamma_\mu^{[A]} \\ & \times G^B(x_2, x_6)\gamma_\mu^{[B]} D(x_5, x_6) G^A(x_5, x_3) G^B(x_6, x_4) \\ & + \cdots\cdots, \end{aligned} \tag{9}$$

where the integrations over $x_5 = (\mathbf{x}_5, \tau_5)$ and $x_6 = (\mathbf{x}_6, \tau_6)$ mean integrations over all space as usual and over τ_5 and τ_6 between 0 and β. G^A, G^B and D are the free-particle Green's functions or the 'propagators':

$$\begin{aligned} G^A(x, y) &= G^A(x - y) = \langle T_\tau(\psi_A^I(x)\bar{\psi}_A^I(y))\rangle_0, \\ G^B(x, y) &= G^B(x - y) = \langle T_\tau(\psi_B^I(x)\bar{\psi}_B^I(y))\rangle_0, \\ D_{\mu\nu}(x, y) &= D_{\mu\nu}(x - y) = \langle T_\tau(A_\mu^I(x)A_\nu^I(y))\rangle_0. \end{aligned} \tag{10}$$

The electron propagator $G^A(\mathbf{r}, \tau)$ is, for instance, given by

$$G^A(\mathbf{r}, \tau) = \frac{T}{(2\pi)^3} \int \mathrm{d}\mathbf{p} \sum_n \exp(i\mathbf{p}\cdot\mathbf{r} - iw_n\tau) G^A(\mathbf{p}, w_n), \tag{11}$$

$$G^A(\mathbf{p}, w_n) = \frac{1}{i\gamma\cdot\mathbf{p} - \gamma_4(iw_n + \bar{\mu}_A) + m_A}$$
$$= \frac{i\gamma\cdot\mathbf{p} - \gamma_4(iw_n + \bar{\mu}_A) - m_A}{(iw_n + \bar{\mu}_A)^2 - E_p^2} \tag{12}$$

where $\bar{\mu}_A$ is the chemical potential for the electron species in the medium, $E_p = (p^2 + m_A^2)^{1/2}$ and

$$w_n = (2n+1)\pi T; \quad n = 0, \pm 1, \pm 2, \ldots. \tag{13}$$

A similar expression holds for $G^B(\mathbf{r}, \tau)$—we only have to replace m_A and $\bar{\mu}_A$ by m_B and $\bar{\mu}_B$ in the above. The photon propagator is given by

$$D_{\mu\nu}(\mathbf{r}, \tau) = \delta_{\mu\nu}\frac{T}{(2\pi)^3}\int \mathrm{d}\mathbf{p} \sum_n \exp(i\mathbf{p}\cdot\mathbf{r} - iw_n\tau) D(\mathbf{p}, w_n)$$
$$\equiv \delta_{\mu\nu} D(\mathbf{r}, \tau), \tag{11a}$$

$$D(\mathbf{p}, w_n) = \frac{1}{w_n^2 + |\mathbf{p}|^2}, \tag{12a}$$

$$w_n = 2n\pi T; \quad n = 0, \pm 1, \pm 2, \ldots. \tag{13a}$$

Using the 'propagator' approach, eq. (9) can be analysed in terms of Feynman diagrams. There is then a complete parallel between the present case and the one in the normal zero-temperature field theory [Salpeter and Bethe 1951; Gell-Mann and Low 1951]. We then immediately see that the perturbation series can be summed to give an integral equation. Working in the ladder approximation, we get

$$K(x_1, x_2; x_3, x_4) = G^A(x_1, x_3) G^B(x_2, x_4) + e^2 \int \mathrm{d}x_5 \int \mathrm{d}x_6$$
$$\times G^A(x_1, x_5)\gamma_\mu^{[A]} G^B(x_2, x_6)\gamma_\mu^{[B]} D(x_5, x_6)$$
$$\times K(x_5, x_6; x_3, x_4). \tag{14}$$

The ladder approximation can easily be dispensed with to get the exact integral equation, but the latter is not tractable and will therefore not be discussed here.

Equation (3) and the homogeneity of the medium yield the result that $K(x_1, x_2; x_3, x_4)$ depends on x_i only through $(X - X')$, x and x' where

$$X = \mu_A x_1 + \mu_B x_2, \quad X' = \mu_A x_3 + \mu_B x_4, \quad x = x_1 - x_2, \quad x' = x_3 - x_4 \tag{15}$$

and

$$\mu_A = \frac{m_A}{m_A + m_B}, \quad \mu_B = \frac{m_B}{m_A + m_B}. \tag{15a}$$

We may therefore write

$$K(x_1, x_2; x_3, x_4) = \frac{T^3}{(2\pi)^6}\iiint \mathrm{d}\mathbf{p}_1 \mathrm{d}\mathbf{p}_2 \mathrm{d}\mathbf{p}_3 \sum_{n_1}\sum_{n_2}\sum_{n_3} \exp[i\mathbf{p}_1(\mathbf{X} - \mathbf{X}')$$
$$- iw_{n_1}(\tau_x - \tau_{x'})] \exp[i\mathbf{p}_2\cdot\mathbf{x} - iw_{n_2}\tau]$$
$$\times \exp[-i\mathbf{p}_3\cdot\mathbf{x}' + iw_{n_3}\tau'] K(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; w_{n_1}, w_{n_2}, w_{n_3}), \tag{16}$$

where we have put

$$\tau_x = \mu_A \tau_1 + \mu_B \tau_2, \quad \tau_{x'} = \mu_A \tau_3 + \mu_B \tau_4, \quad \tau = \tau_1 - \tau_2, \quad \tau' = \tau_3 - \tau_4. \tag{17}$$

We can Fourier transform (14) to get

$$\begin{aligned} K(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; w_{n_1}, w_{n_2}, w_{n_3}) = & \frac{1}{T}\delta^3(\mathbf{p}_2 - \mathbf{p}_3)\delta w_{n_2}, w_{n_3} G^A(\mu_A \mathbf{p}_1 + \mathbf{p}_2, \mu_A w_{n_1} + w_{n_2}) \\ & \times G^B(\mu_B \mathbf{p}_1 - \mathbf{p}_2, \mu_B w_{n_1} - w_{n_2}) \\ & + \frac{T}{(2\pi)^3} e^2 \int \mathrm{d}\mathbf{p} \sum_n G^A(\mu_A \mathbf{p}_1 + \mathbf{p}_2, \mu_A w_{n_1} + w_{n_2}) \\ & \times \gamma_\mu^{[A]} G^B(\mu_B \mathbf{p}_1 - \mathbf{p}_2, \mu_B w_{n_1} - w_{n_2}) \gamma_\mu^{[B]} \\ & \times D(\mathbf{p}_2 - \mathbf{p}, w_{n_2} - w_n) K(\mathbf{p}_1, \mathbf{p}, \mathbf{p}_3; w_{n_1}, w_n, w_{n_3}). \end{aligned} \tag{18}$$

Note that n_1 here takes even values and all other n's take odd values.

We now observe that for any given system the average of an operator taken over the grand canonical ensemble at temperature T can be replaced by its average over the stationary state with energy E equal to the average energy of the system at that temperature [Gorkov 1958]. So we consider

$$K(x_1, x_2; x_3, x_4) = -\langle n | T_\tau(\psi_A(x_1)\psi_B(x_2)\bar{\psi}_A(x_3)\bar{\psi}_B(x_4)) | n \rangle. \tag{19}$$

For $\tau_1, \tau_2 > \tau_3, \tau_4$, we get

$$K(x_1, x_2; x_3, x_4) = -\sum_m \langle n | T_\tau(\psi_A(x_1)\psi_B(x_2)) | m \rangle \langle m | T_\tau(\bar{\psi}_A(x_3)\bar{\psi}_B(x_4)) | n \rangle, \tag{20}$$

where the state $|m\rangle$ has one more A and one more B particle than the state $|n\rangle$. If the energies of these states are E_m and E_n, respectively, we can write

$$E_m = E_n + \bar{\mu}_A + \bar{\mu}_B + \varepsilon \tag{21}$$

where $\varepsilon > (m_A + m_B)$, except if we have a bound state situation.

The dependence on x_i's can easily be factored in (20) to give

$$\langle n | T_\tau(\psi_A(x_1)\psi_B(x_2)) | m \rangle = \frac{1}{\sqrt{v}} \exp[i\mathbf{p}_1 \cdot \mathbf{X} - \varepsilon \tau_x] F_{p_1, \varepsilon}(\mathbf{x}, \tau),$$

$$\langle m | T_\tau(\bar{\psi}_A(x_3)\bar{\psi}_B(x_4)) | n \rangle = \frac{1}{\sqrt{v}} \exp[-i\mathbf{p}_1 \cdot \mathbf{X}' + \varepsilon \tau_{x'}] \bar{F}_{p_1, \varepsilon}(\mathbf{x}', \tau'). \tag{22}$$

We now extract out the contribution to (20) corresponding to the bound state situation. We get

$$\begin{aligned} K(x_1, x_2; x_3, x_4) &= \tilde{K}(x_1, x_2; x_3, x_4) + \cdots\cdots, \\ \tilde{K}(x_1, x_2; x_3, x_4) &= \frac{-1}{(2\pi)^3} \int \frac{\mathrm{d}\mathbf{p}_1}{2\varepsilon_{p_1}} \exp[i\mathbf{p}_1 \cdot (\mathbf{X} - \mathbf{X}') - \varepsilon_{p_1} \tilde{\tau}] \\ & \quad \times F_{p_1, \varepsilon_{p_1}}(\mathbf{x}, \tau) \cdot \bar{F}_{p_1, \varepsilon_{p_1}}(\mathbf{x}', \tau'); \quad \tilde{\tau} = \tau_x - \tau_{x'}. \end{aligned} \tag{23}$$

Keeping in mind the periodicity in the various τ-variables [see (5)], we can now Fourier expand $\tilde{K}$. We get

$$\tilde{K}(x_1, x_2; x_3, x_4) = \frac{-T}{(2\pi)^3} \int \frac{d\mathbf{p}_1}{2\varepsilon_{p_1}} \sum_{n_1} \exp[i\mathbf{p}_1 \cdot (\mathbf{X} - \mathbf{X}') - iw_{n_1}\tilde{\tau}] \times G(w_{n_1}) F_{p_1, \varepsilon_{p_1}}(x, \tau) \bar{F}_{p_1, \varepsilon_{p_1}}(\mathbf{x}', \tau'), \tag{24}$$

where

$$G(w_{n_1}) = \int_0^\beta \exp[(iw_{n_1} - \varepsilon_{p_1})\tilde{\tau}] d\tilde{\tau} = \frac{\exp[-\varepsilon_{p_1}\beta] - 1}{iw_{n_1} - \varepsilon_{p_1}}. \tag{25}$$

Further, we have

$$F_{p_1, \varepsilon_{p_1}}(\mathbf{x}, \tau) = \frac{T}{(2\pi)^3} \int d\mathbf{p}_2 \sum_{n_2} \exp[i\mathbf{p}_2 \cdot \mathbf{x} - iw_{n_2}\tau] \cdot F_{p_1, \varepsilon_{p_1}}(\mathbf{p}_2, w_{n_2}),$$

$$\bar{F}_{p_1, \varepsilon_{p_1}}(\mathbf{x}', \tau') = \frac{T}{(2\pi)^3} \int d\mathbf{p}_3 \sum_{n_3} \exp[-i\mathbf{p}_3 \cdot \mathbf{x}' + iw_{n_3}\tau'] \cdot \bar{F}_{p_1, \varepsilon_{p_1}}(\mathbf{p}_3, w_{n_3}). \tag{26}$$

We thus get

$$K(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3; w_{n_1}, w_{n_2}, w_{n_3}) = \frac{-1}{(2\pi)^3} \frac{1}{2\varepsilon_{p_1}} \frac{(\exp[-\varepsilon_{p_1}\beta] - 1)}{(iw_{n_1} - \varepsilon_{p_1})} F_{p_1, \varepsilon_{p_1}}(\mathbf{p}_2, w_{n_2}) \times \bar{F}_{p_1, \varepsilon_{p_1}}(\mathbf{p}_3, w_{n_3}) + \cdots\cdots. \tag{27}$$

where the terms not explicitly written on the right hand side are all regular at $iw_{n_1} = \varepsilon_{p_1}$, the analytic continuation from discrete iw_{n_1} to continuous w being implied.

We now multiply (18) by $(iw_{n_1} - \varepsilon_{p_1})$ and take the limit $iw_{n_1} \to \varepsilon_{p_1}$. The inhomogeneous term drops out and we are left with

$$F_{p_1, \varepsilon_{p_1}}(\mathbf{p}_2, w_{n_2}) = \frac{T}{(2\pi)^3} e^2 \int d\mathbf{p} \sum_n G^A\left(\mu_A \mathbf{p}_1 + \mathbf{p}_2, \frac{\mu_A \varepsilon_{p_1}}{i} + w_{n_2}\right) \times \gamma_\mu^{[A]} G^B\left(\mu_B \mathbf{p}_1 - \mathbf{p}_2, \frac{\mu_B \varepsilon_{p_1}}{i} - w_{n_2}\right) \gamma_\mu^{[B]} \times D(\mathbf{p}_2 - \mathbf{p}, w_{n_2} - w_n) F_{p_1, \varepsilon_{p_1}}(\mathbf{p}, w_n). \tag{28}$$

This equation for the pairing amplitude $F_{p_1, \varepsilon_{p_1}}(\mathbf{p}_2, w_{n_2})$ is an eigenvalue equation in ε_{p_1} and the eigenvalues of ε_{p_1} determine the bound state spectrum of the system.

On substituting for G^A etc., we can write (28) more explicitly in the form

$$F_{p_1, \varepsilon_{p_1}}(\mathbf{p}_2, w_{n_2}) = \frac{T}{(2\pi)^3} e^2 \int d\mathbf{p} \sum_n \times \frac{1}{[i\gamma \cdot (\mathbf{p}_2 + \mu_A \mathbf{p}_1) - \gamma_4(\mu_A \varepsilon_{p_1} + iw_{n_2} + \bar{\mu}_A) + m_A]^{[A]}} \gamma_\mu^{[A]} \times \frac{1}{[-i\gamma \cdot (\mathbf{p}_2 - \mu_B \mathbf{p}_1) - \gamma_4(\mu_B \varepsilon_{p_1} - iw_{n_2} + \bar{\mu}_B) + m_B]^{[B]}} \gamma_\mu^{[B]} \times \frac{1}{|\mathbf{p}_2 - \mathbf{p}|^2 + (w_{n_2} - w_n)^2} F_{p_1, \varepsilon_{p_1}}(\mathbf{p}, w_n). \tag{29}$$

or simply as

$$[i\gamma\cdot(\mu_A \mathbf{p}_1 + \mathbf{p}_2) - \gamma_4(\mu_A \varepsilon_{p_1} + iw_{n_2} + \bar{\mu}_A) + m_A]^{[A]}$$
$$\times [i\gamma\cdot(\mu_B \mathbf{p}_1 - \mathbf{p}_2) - \gamma_4(\mu_B \varepsilon_{p_1} - iw_{n_2} + \bar{\mu}_B) + m_B]^{[B]} \cdot F_{p_1, \varepsilon_{p_1}}(\mathbf{p}_2, w_{n_2})$$
$$= \frac{T}{(2\pi)^3} e^2 \int d\mathbf{p} \sum_n \frac{\gamma_\mu^{[A]} \gamma_\mu^{[B]}}{(w_{n_2} - w_n)^2 + |\mathbf{p}_2 - \mathbf{p}|^2} F_{p_1, \varepsilon_{p_1}}(\mathbf{p}, w_n). \tag{30}$$

Let us now go over to a simplified situation in which $\mathbf{P}_1 = 0$. This corresponds to the particles A and B moving in the medium with equal and opposite three-momenta. Let the pair-energy in that case be denoted by M. Equation (30) then takes the form

$$[i\gamma\cdot\mathbf{p}_2 - \gamma_4(\mu_A M + iw_{n_2} + \bar{\mu}_A) + m_A]^{[A]}$$
$$\times [-i\gamma\cdot\mathbf{p}_2 - \gamma_4(\mu_B M - iw_{n_2} + \bar{\mu}_B) + m_B]^{[B]} \cdot F_M(\mathbf{p}_2, w_{n_2})$$
$$= \frac{T}{(2\pi)^3} e^2 \int d\mathbf{p} \sum_n \frac{\gamma_\mu^{[A]} \gamma_\mu^{[B]}}{(w_{n_2} - w_n)^2 + |\mathbf{p}_2 - \mathbf{p}|^2} \cdot F_M(\mathbf{p}, w_n). \tag{30a}$$

In the limit of instantaneous Coulomb interaction, the denominator on the right hand side reduces to $|\mathbf{p}_2 - \mathbf{p}|^2$ and $\gamma_\mu^{[A]}\gamma_\mu^{[B]}$ is replaced by $\gamma_4^{[A]}\gamma_4^{[B]}$. The expression on the left hand side can then be written as a function of $\mathbf{p}_2$ alone. Going over to nonrelativistic kinematics, we can then reduce (30a) to the form

$$\phi_M(\mathbf{p}_2) = \frac{T}{(2\pi)^3} e^2 \int \frac{d\mathbf{p}}{|\mathbf{p}_2 - \mathbf{p}|^2} \sum_n \frac{1}{(iw_n - \varepsilon_p^A + \mu_A w + \bar{\mu}_A)}$$
$$\times \frac{1}{(-iw_n - \varepsilon_p^B + \mu_B w + \bar{\mu}_B)} \phi_M(\mathbf{p}), \tag{31}$$

where we have put

$$M = m_A + m_B + w = m_A + m_B - |w|,$$
$$\varepsilon_p^A = p^2/2m_A, \quad \varepsilon_p^B = p^2/2m_B. \tag{32}$$

Performing the sum over n in (31), we obtain

$$\varphi_M(\mathbf{p}_2) = \frac{\alpha}{4\pi^2} \int \frac{d\mathbf{p}}{|\mathbf{p}_2 - \mathbf{p}|^2} \frac{Q_B(w, p^2)}{[w + \bar{\mu}_A + \bar{\mu}_B - p^2/2\mu]} \phi_M(\mathbf{p}), \tag{33}$$

where

$$Q_B(w, p^2) = \tanh\left(\frac{\mu_A w - \varepsilon_p^A + \bar{\mu}_A}{2T}\right) + \tanh\left(\frac{\mu_B w - \varepsilon_p^B + \bar{\mu}_B}{2T}\right). \tag{34}$$

Equations (31) and (33) have been proposed earlier [Malik *et al* 1989]. The derivation given here provides the correct many-body interpretation of these equations.

Equation (33) can be solved nonperturbatively in the high temperature regime to give temperature-dependent discrete energy spectra and has application to the soft X-ray emission data from the Sun and other hot bodies [Malik *et al* 1989].

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