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Some new radiating Kerr-Newman solutions

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Abstract. Three exact non-static solutions of Einstein-Maxwell equations corresponding to a field of flowing null radiation plus an electromagnetic field are presented. These solutions are non-static generalizations of the well known Kerr-Newman solution. The current vector is null in all the three solutions. These solutions are the electromagnetic generalizations of the three generalized radiating Kerr solutions discussed by Vaidya and Patel. The solutions discussed by us describe the exterior gravitational fields of rotating radiating charged bodies. Many known solutions are derived as particular cases.

Keywords. Radiating Kerr-Newman metric; Einstein-Maxwell equations; pure radiation field.

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1. Introduction

Many investigators have shown interest in obtaining generalizations of different static or stationary metrics to non-static situations, especially, in relation to certain astrophysical problems, namely when the energy density of the radiation emitted by the source is not negligible (i.e. supernovae, quasistellar radio source etc.).

The non-static generalization of the well known Schwarzschild exterior solution and the Reisser-Nordstrom solution are given by Vaidya (1951) and Bonner and Vaidya (1970) respectively. Kerr (1963) derived a stationary generalization of Schwarzschild exterior solution which describes the exterior gravitational field of a rotating body. When the rotation parameter vanishes Kerr solution reduces to the Schwarzschild solution. The electromagnetic generalization of the Kerr solution is obtained by Newman *et al* (1965). This solution is known as the Kerr-Newman solution in the literature. The Kerr-Newman metric has proven to be of great interest in the gravitational theory and its applications to astrophysics. The gravitational field surrounding a charged rotating star cannot be described by the Kerr-Newman metric except in the approximation in which one neglects the energy density of the emitted radiation. Therefore it would be interesting to derive metrics which describe the gravitational field surrounding radiating charged bodies.

Many non-stationary generalizations of the Kerr and the Kerr-Newman solutions are available in the literature. Some non-static generalizations of Kerr metric have been discussed by Vaidya and Patel (1973), Vaidya (1974) and Herlt (1980). A radiating Kerr-Newman solution is discussed by Patel and Koppar (1986). The space-times of all these solutions are of Kerr-Schild (1965) type. Vaidya *et al* (1976) have discussed some radiating Kerr solutions which are not of Kerr-Schild type.

Vaidya and Patel (1990) have presented three generalized radiating Kerr solutions described by the metrics which are not of the Kerr-Schild type.

In the present paper, we give the electromagnetic generalizations of the three solutions discussed by Vaidya and Patel (1990) in the form of exact solutions of Einstein-Maxwell equations

$$R_{ik} = -8\pi[E_{ik} + \sigma\omega_i\omega_k], \quad \omega_i\omega^i = 0 \tag{1}$$

$$F^{ik}_{;k} = 4\pi J^i \tag{2}$$

where semicolon indicates covariant differentiation. Here ω_i is null vector, E_{ik} is the electromagnetic energy tensor and $\sigma\omega_i\omega_k$ is the tensor arising out of flowing null radiation. F_{ik} and J^i denote respectively the electromagnetic field tensor and the current four vector.

2. The metric and the electromagnetic field

We take the line element in the form (Vaidya and Patel 1989)

$$\mathrm{d}s^2 = 2(\mathrm{d}u + g\sin\alpha\,\mathrm{d}\beta)(\mathrm{d}t + H\sin\alpha\,\mathrm{d}\beta) - 2L(\mathrm{d}u + g\sin\alpha\,\mathrm{d}\beta)^2 - M^2(\mathrm{d}\alpha^2 + \sin^2\alpha\,\mathrm{d}\beta^2) \tag{3}$$

with

$$g = g(\alpha), \quad H = H(\alpha), \quad L = L(u, \alpha, t), \quad M = M(u, \alpha, t). \tag{4}$$

Here it should be noted that Vaidya and Patel (1989) have discussed a metric which is slightly general than metric (3). In their metric $g = g(\alpha, u)$ and $H = H(u, \alpha, t)$ with $\partial g/\partial u = -\partial H/\partial t$. Their solution describes the field of a radiating Kerr particle in the Einstein universe. When $H = 0$, metric (3) reduces to the metric discussed by Vaidya *et al* (1976) in connection with radiating Kerr solutions.

We now introduce the following tetrad in the space-time described by metric (3):

$$\theta^1 = \mathrm{d}u + g\sin\alpha\,\mathrm{d}\beta, \quad \theta^2 = M\,\mathrm{d}\alpha, \quad \theta^3 = M\sin\alpha\,\mathrm{d}\beta$$
$$\theta^4 = \mathrm{d}t + H\sin\alpha\,\mathrm{d}\beta - L\theta^1. \tag{5}$$

In terms of the above tetrad, metric (3) takes the simple form

$$\mathrm{d}s^2 = 2\theta^1\theta^4 - (\theta^2)^2 - (\theta^3)^2 = g_{(ab)}\theta^a\theta^b \tag{6}$$

where $g_{(ab)}$ are tetrad components of the metric tensor g_{ik}. Here and in what follows the bracketed indices indicate tetrad components with respect to tetrad (5).

With the aid of Cartan's equations of structure one can find the tetrad components R^a_{bcd} of the curvature tensor for metric (3) and tetrad (5). From R^a_{bcd} one can find the tetrad components $R_{(ab)} = R^c_{abc}$ of the Ricci tensor for metric (3). All these computational details are given by the Vaidya and Patel (1989). Therefore we shall not go into these details here.

It is well known that the Kerr-Newman metric is of Kerr-Schild form. For the Kerr-

Schild type space-times $g_{ik} = \eta_{ik} + H\xi_i\xi_k$ and $R_{ik}\xi^i\xi^k = 0$ where ξ_i is a geodetic null shear-free congruence, H is a function of co-ordinates and $\eta_{ik} = \text{diag}\{1, -1, -1, -1\}$ (see Vaidya 1974). For electrovac Kerr-Schild space-times, we must have $E_{ik}\xi^i\xi^k = 0$. If we take the electromagnetic 4-potential A_i as a scalar multiple of the null radiation flow vector ξ_i, then the above requirement is fulfilled. Therefore Bonner and Vaidya (1970) and Patel and Koppar (1986) have taken A_i as a null vector for the discussion of their electrovac solutions. These facts provide a motivation for assuming A_i to be a null vector.

We now make two basic assumptions, viz the four current vector J^i and the electromagnetic four potential A_i are null vectors.

We choose the electromagnetic four potential A_i as $A_i = A(\alpha, u, t)\omega_i$, $\omega_i\omega^i = 0$. We name the co-ordinates as $x^1 = u$, $x^2 = \alpha$, $x^3 = \beta$, $x^4 = t$. The tetrad components of ω_i are given by

$$\omega_{(a)} = (1, 0, 0, 0). \tag{7}$$

The tensor components of ω_i can be obtained from $\omega_i = e_i^{(a)}\omega_{(a)}$, $\theta^a = e_i^{(a)}\,dx^i$. ω_i is determined as

$$\omega_i = (1, 0, g\sin\alpha, 0).$$

Consequently we have

$$A_i = (A, 0, Ag\sin\alpha, 0) \tag{8}$$

Using (8), one can find the electromagnetic field tensor $F_{ik} = A_{i,k} - A_{k,i}$. These F_{ik} and metric (3) will determine the electromagnetic energy tensor E_{ik}. The expressions for E_{ik} are lengthy and therefore not reported here. Using these E_{ik} in the results $E_{(ab)} = e^i_{(a)}e^k_{(b)}E_{ik}$, $dx^i = e^i_{(a)}\theta^a$ one can determine the tetrad components $E_{(ab)}$ of E_{ik}. The non-zero $E_{(ab)}$ are given below for ready reference

$$E_{(14)} = E_{(22)} = E_{(33)} = \frac{1}{2}\left[A_t^2 + \frac{4f^2A^2}{M^4}\right]$$

$$E_{(11)} = \frac{1}{M^2}[A_\alpha^2 + (gA_u + HA_t)^2]$$

$$E_{(12)} = \frac{1}{M}\left[A_tA_\alpha + \frac{2fA}{M^2}(gA_u + HA_t)\right]$$

$$E_{(13)} = \frac{1}{M}\left[\frac{2fA}{M^2}A_\alpha - A_t(gA_u + HA_t)\right] \tag{9}$$

where the function f is defined by

$$2f = g_\alpha + g\cot\alpha \tag{10}$$

Here and in what follows a suffix denotes partial derivative eg. $g_\alpha = \partial g/\partial\alpha$, $A_t = \partial A/\partial t$, $L_{ut} = \partial^2 L/\partial u\,\partial t$ etc.

3. The field equations and their solutions

We have verified that the Maxwell equations (2) with $J^1 = J^2 = J^3 = 0$ give the following three differential equations for the function A

$$A_{\alpha t} + 2f\left[g\left(\frac{A}{M^2}\right)_u + H\left(\frac{A}{M^2}\right)_t \right] = 0,$$

$$gA_{ut} + HA_{tt} - \left(\frac{2fA}{M^2}\right)_\alpha = 0,$$

$$A_{tt} + 2A_t\left(\frac{M_t}{M}\right) + \frac{4f^2 A}{M^4} = 0. \tag{11}$$

The value of J^4 is given by

$$-4\pi J^4 = \frac{g^2}{M^2}A_{uu} - \frac{1}{M^2}[(M^2 - Hg)A_t]_u$$
$$+ \frac{1}{M^2}\left[A_{\alpha\alpha} + \left(A_\alpha + \frac{2fAH}{M^2}\right)\cot\alpha + \left(\frac{2fAH}{M^2}\right)_\alpha \right]. \tag{12}$$

Here it should be noted that the metric potential L does not appear explicitly in the electromagnetic field quantites.

The field equations (1) can be expressed in the tetrad basis as

$$R_{(ab)} = -8\pi[E_{(ab)} + \sigma\omega_{(a)}\omega_{(a)}], \quad \omega_{(a)}\omega^{(a)} = 0. \tag{13}$$

The field equations (13) along with (7) and (9) give rise to the following relations:

$$R_{(44)} = \frac{2}{M}\left[M_{tt} - \frac{f^2}{M^3} \right] = 0 \tag{14}$$

$$R_{(24)} = \frac{1}{M}\left[\left(\frac{M_t}{M}\right)_\alpha - g\left(\frac{f}{M^2}\right)_u - H\left(\frac{f}{M^2}\right)_t \right] = 0 \tag{15}$$

$$R_{(34)} = -\frac{1}{M}\left[g\left(\frac{M_t}{M}\right)_u + H\left(\frac{M_t}{M}\right)_t + \left(\frac{f}{M^2}\right)_\alpha \right] = 0 \tag{16}$$

$$R_{(22)} = \frac{1}{M^2}\left[\left(\frac{M_\alpha}{M}\right)_\alpha + \left(\frac{M_\alpha}{M}\right)\cot\alpha - 1 - (M^2)_{ut} - \{L(M^2)_t\}_t \right.$$
$$\left. + 4f\frac{fL-h}{M^2} + g^2\left(\frac{M_u}{M}\right)_u + H^2\left(\frac{M_t}{M}\right)_t + 2gH\left(\frac{M_t}{M}\right)_u \right]$$
$$= -8\pi E_{(22)} \tag{17}$$

$$R_{(14)} = L_{tt} + \frac{2}{M}\left[M_{ut} + L_t M_t + LM_{tt} + f\frac{fL-h}{M^3} \right]$$
$$= -8\pi E_{(14)} \tag{18}$$

$$R_{(12)} = LR_{(24)} + \frac{1}{M}\left[\left(L_t - \frac{M_u}{M}\right)_\alpha + g\left\{\frac{2fL-h}{M^2}\right\}_u + H\left\{\frac{2fL-h}{M^2}\right\}_t\right]$$
$$= -8\pi E_{(12)} \tag{19}$$

$$R_{(13)} = LR_{(34)} + \frac{1}{M}\left[\left\{\frac{2fL-h}{M^2}\right\}_\alpha - g\left(L_t + \frac{M_u}{M}\right)_u - H\left(L_t + \frac{M_u}{M}\right)_t\right]$$
$$= -8\pi E_{(13)} \tag{20}$$

$$R_{(11)} = L^2 R_{(44)} + 2h\frac{2fL-h}{M^4} + \frac{1}{M^2}[L_{\alpha\alpha} + L_\alpha \cot\alpha + g^2 L_{uu} + H^2 L_{tt}$$
$$+ 2gHL_{ut} + 2MM_{uu} + 4LMM_{ut} - 2L_t MM_u + 2L_u MM_t]$$
$$= -8\pi\sigma - 8\pi E_{(11)} \tag{21}$$

where $E_{(ab)}$ are given by (9) and the function h is defined by

$$2h = H_\alpha + H\cot\alpha. \tag{22}$$

Let us first consider the field equations (14), (15) and (16). The structure of these three differential equations is similar to that of the corresponding equations discussed by Vaidya *et al* (1976) in connection with radiating Kerr solutions. Adopting their method, these three differential equations can be solved for the function M. The solution can be expressed in the form

$$M^2 = \frac{f(X^2 + Y^2)}{Y} \tag{23}$$

where X is a function of t, u and α and Y is a function of u and α satisfying the relations

$$X_t = -1, \quad X_y = Y_u, \quad X_u = -Y_y + \frac{H}{g}. \tag{24}$$

Here and in what follows the variable y is defined by the differential relation $g\,d\alpha = dy$. We now turn our attention to the Maxwell equations (11). Substituting M^2 given by (23) and (24), the last equation of the system (11) can be integrated to have the solution

$$A = \frac{eX + \mu(X^2 - Y^2)}{X^2 + Y^2} \tag{25}$$

where e and μ are arbitrary functions of u and y. Using the above value of A and M^2 given by (23) and (24) in the remaining two equations of the system, we obtain the relations

$$e_y = -(2\mu Y)_u, \quad e_u = (2\mu Y)_y. \tag{26}$$

It is known that, for the Kerr-Newman solution, $\mu = 0$. Also non-zero e and μ give very lengthy and complicated expressions for $E_{(ab)}$. In the present paper, we wish to derive radiating Kerr-Newman solutions. Therefore, for simplicity, we assume $\mu = 0$.

Hence we have

$$A = \frac{eX}{(X^2 + Y^2)} \tag{27}$$

where e is now a constant.

We know that $E_{(22)} = E_{(14)}$. Therefore $R_{(22)} = R_{(14)}$.

Thus the field equations (17) and (18) are equivalent to (17) and $R_{(22)} = R_{(14)}$. The equation $R_{(22)} = R_{(14)}$ can be solved for the function $2L$. The solution can be expressed as

$$2L = qX + 2S + \frac{2(FX + EY)}{X^2 + Y^2} \tag{28}$$

where q, S, F and E are functions of u and y satisfying the relations

$$q = -\frac{Y_u}{Y} \tag{29}$$

and

$$2S = -\frac{g^2 Y}{2f}\left[\left(\frac{Y_y}{Y}\right)_u + \left(\frac{Y_y}{Y}\right)_y - \left(\frac{f_y}{f}\right)_y\right] + Y\left(\frac{f_y}{f}\right) - Y_y + 2X_u - \frac{Y}{f}. \tag{30}$$

Using (28), (29) and (30) in (17) we obtain

$$E + 2SY + \frac{2\pi e^2}{Y} = \left[\frac{h}{f} + \frac{H}{g} - Y_y\right] Y \tag{31}$$

Using the above mentioned results (23)–(29) in the field equations (19) and (20) we obtain the relations

$$F_y = \left[E + \frac{2\pi e^2}{Y}\right]_u, \quad F_u = -\left[E + \frac{2\pi e^2}{Y}\right]_y. \tag{32}$$

The equations (12) and (21) now determine J^4 and the radiation density σ respectively. They are given by

$$4\pi J^4 = -e\left(\frac{Y_u}{Y}\right)(X^2 + Y^2)^{-1} \tag{33}$$

and

$$\begin{aligned} -8\pi\sigma(X^2 + Y^2) = {} & -\frac{8\pi e^2 X Y_u}{Y(X^2 + Y^2)} + \frac{3FY_u}{Y} + YY_{uu} - 2Y_u^2 \\ & + 2E_y + 2YS_y - 4\pi e^2 \frac{Y_y}{Y^2} + g^2\left(\frac{Y}{f}\right) \\ & \times \left[S_{uu} + S_{yy} - Y_{uy}\left(\frac{Y_u}{Y}\right) - Y_{uu}\left(\frac{Y_y}{Y}\right)\right]. \end{aligned} \tag{34}$$

From the above analysis, it is easy to establish the following theorem.

Theorem. *Given a solution* (L_0, H_0, M_0, g_0) *of metric* (3) *representing a radiating vacuum space-time, one can obtain the corresponding radiating electromagnetic space-time* (L, H_0, M_0, g_0, A) *characterized by*

$$A = \frac{eX}{(X^2 + Y^2)}, \quad L = L_0 - \frac{2\pi e^2}{(X^2 - Y^2)}$$

where e is a constant representing the charge parameter.

Here it should be noted that J^4 is, in general, non-zero and it can be determined from (12).

In the next section we shall apply the above mentioned theorem to the three physically meaningful radiating Kerr solutions discussed by Vaidya and Patel (1990).

The relations (24) and (32) imply the following differential equations:

$$\begin{aligned} &Y_{uu} + Y_{yy} = \left(\frac{H}{g}\right)_y, \quad X_{uu} + X_{yy} = 0 \\ &\left[E + \frac{2\pi e^2}{Y}\right]_{uu} + \left[E + \frac{2\pi e^2}{Y}\right]_{yy} = 0 \\ &F_{uu} + F_{yy} = 0 \end{aligned} \tag{35}$$

All the equations of (35) except the first one are Laplace's equations in u and y. The first equation of (35) states that the Laplacian of Y is a function of y only. We shall not try to discuss the general solutions of (35). In the next section, we shall discuss some important particular solutions of (35).

4. Three new radiating Kerr-Newman metrics

It is well known that Y is a separable function of u and y and $E = 0$ for the radiating Kerr metric discussed by Vaidya and Patel (1973). For the usual Kerr-Newman metric Y is a function of y only and $E + 2\pi e^2/Y = 0$. Therefore it seems reasonable to make the following two simplifying assumption:

$$Y = UV, \quad E + \frac{2\pi e^2}{Y} = 0 \tag{36}$$

where U is a function of u and V is a function of y only. From (32) it is clear that we can take, $E + 2\pi e^2/Y = \text{constant} = B$ say. We have verified that the assumptions $Y = UV$ and $E + (2\pi e^2/Y) = B$ are compatible only if $B = 0$. Therefore F is constant.

Substitution of $Y = UV$ in the first eq. of (35) yields the differential equation

$$VU_{uu} + UV_{yy} = \left(\frac{H}{g}\right)_y. \tag{37}$$

Vaidya and Patel (1990) have discussed three particular solutions of (37). We shall now briefly discuss three electromagnetic generalizations of these solutions with the aid of the theorem cited in the previous section.

In all the generalizations we shall give the forms of the line-elements along with the corresponding values of X, Y, $g\sin\alpha$, $H\sin\alpha$, J^4 and the radiation density σ.

Case 1. $V_{yy}=0$, and $U_{uu}=\text{constant}$.

In this case we have

$$X=\int(1+au^2)\mathrm{d}u+(\bar{k}-a^2y^2)u-ay^2-t,$$

$$Y=-y(1+au^2),$$

$$y=-b\cos\alpha+C[1+\cos\alpha\log(\tan(\alpha/2))]$$

$$g\sin\alpha=b\sin^2\alpha+C[\cos\alpha-\sin^2\alpha\log(\tan(\alpha/2)]$$

$$H=g(\bar{k}-a^2y^2) \tag{38}$$

where a, b, c and $\bar{k}$ are arbitrary constants. We shall set $\bar{k}=a^2b^2$ when $C=0$ and $\bar{k}=a^2c^2$ when $b=0$.

The values of J^4 and σ are given by

$$J^4=-\frac{2ae}{(1+au)(X^2+Y^2)},\quad -8\pi\sigma(X^2+Y^2)=\frac{6aF}{1+au}$$

$$+2a^2c^2(1+au)^2\operatorname{cosec}^4\alpha-\frac{16\pi ae^2X}{(1+au)(X^2+Y^2)} \tag{39}$$

where X and Y are given by (38).

When $C=0$ we get a new radiating Kerr-Newman metric. This metric can be expressed in the final form as

$$\begin{aligned}\mathrm{d}s^2={}&2(\mathrm{d}u+b\sin^2\alpha\mathrm{d}\beta)(\mathrm{d}t+a^2b^3\sin^4\alpha\mathrm{d}\beta)-(X^2+Y^2)(1+au)^{-2}\\&\times(\mathrm{d}\alpha^2+\sin^2\alpha\mathrm{d}\beta^2)-[(1+au)^2+3a^2b^2\sin^2\alpha-aX(1+au)^{-1}\\&+(2FX-4\pi e^2)(X^2+Y^2)^{-1}](\mathrm{d}u+b\sin^2\alpha\mathrm{d}\beta)^2\end{aligned} \tag{40}$$

where X and Y are given by (38) and F is a constant. When $e=0$, the electromagnetic field disappears and we get the radiating Kerr metric discussed by Vaidya and Patel (1973), with slight change of notations.

When $b=0$, our solution reduces to a new associated radiating Kerr solution discussed by Vaidya and Patel (1990). Here it should be mentioned that the solution when $b=0$ has a singularity along the axis of symmetry $\alpha=\pi$.

We believe that the solution discussed in this case is a new one. The constants F and e in the solution are the mass and the charge parameters respectively. The constants b and c are related to the angular momentum of the body and a is the radiation parameter. The choice $a=0$, $C=0$ gives us the usual Kerr-Newman metric.

Case 2. $U_{uu}-k^2U=\text{constant}$, $V_{yy}+k^2V=0$.

In this case we have

$$U=\left[\left(\frac{2a}{k}\right)\sinh(ku/2)+\cosh(ku/2)\right]^2$$

$$V=\left(-\frac{1}{k}\right)\sin ky,\quad X=U_u\int V\mathrm{d}y-t,\quad Y=UV$$

$$(H/g)=\bar{k}+\left[\frac{2a^2}{k}-\frac{1}{2}\right]\cos ky$$

$$g\sin\alpha=l^2-\frac{4c^2}{k^2}\sin^2(ky/2) \tag{41}$$

where $a, k, \bar{k}, l^2$ and c are arbitrary constants. In this case the values of J^4 and radiation density σ are given by

$$4\pi J^4 U^{1/2}(X^2+Y^2)=-e[2a\cosh(ku/2)+K\sinh(ku/2)],$$

$$-8\pi\sigma(X^2+Y^2)=[2a\cosh(ku/2)+K\sinh(ku/2)]$$
$$\times[3FU^{-1/2}-8\pi eXU^{-1/2}(X^2+Y^2)^{-1}] \tag{42}$$

where U, X and Y are given by (41).

In this case the function q and $2S$ are given by

$$-q=k\left[\frac{2a}{k}\cosh(ku/2)+\sinh(ku/2)\right]U^{-1/2}$$

$$2S=2\bar{k}+2\left[\frac{2a^2}{k}-\frac{1}{2}\right]\cos ky+U\cos ky+k\left[\frac{2a^2}{k}-\frac{1}{2}\right]\sin ky$$
$$\times\left[l^2-\frac{4c}{k^2}\sin^2(ky/2)\right]\left[\frac{4c}{k}\sin(ky/2)\cos(ky/2)\right]^{-1}. \tag{43}$$

Using the relations (41) and $g\,\mathrm{d}\alpha=\mathrm{d}y$ and choosing the constants as $l^2c=1$, $l^2=b$ we get the following relation between the variables α and y:

$$\tan\left(\frac{ky}{2}\right)=\frac{kb}{A}\left[\frac{\tan^A(\alpha/2)-1}{\tan^A(\alpha/2)+1}\right] \tag{44}$$

where $A=\sqrt{4-k^2b^2}$.

The metric of the solution of this case can be expressed in terms of the co-ordinates u, y, β and t. But this explicit form of the metric is quite lengthy and so we shall not give it here.

An important property of our above solution is worth mentioning. When $k\to 0$, it is easy to see that $y\to -l^2\cos\alpha, g\sin\alpha\to l^2\sin^2\alpha$. We have verified that when $k\to 0$, our solution reduces to the radiating Kerr-Newman metric discussed in case (1). The choice $a=0$ and $k\to 0$ gives us the usual Kerr-Newman solution. But if we set only $a=0$, then we get a radiating Kerr-Newman metric because in this case $\sigma\neq 0$ and $J^4\neq 0$.

When $e=0$, the electromagnetic field disappears and our solution becomes the new radiating Kerr solution.

Case 3. A particular case $k=\pm 2a$ of case (2) is interesting. In this case, the metric function H vanishes. Here the explicit form of the line element is relatively simple. It can be expressed in terms of the co-ordinates u, t, y and β as

$$ds^2 = 2[du + bQ\,d\beta]\,dt - \left(\frac{1}{b}\right)\exp(ku)(X^2 + Y^2)[(bQ)^{-1}dy^2 + bQ\,d\beta^2]$$
$$- [1 + kt + (2FX - 4\pi e^2)(X^2 + Y^2)^{-1}](du + bQ\,d\beta)^2 \tag{45}$$

where

$$X = \left(\frac{1}{k}\right)[\exp(ku)\cos ky - 1] - t, \quad Y = -\left(\frac{1}{k}\right)\sin ky \exp(ku),$$

$$Q = 1 - \frac{4}{b^2k^2}\sin^2(ky/2). \tag{46}$$

The values of σ and J^4 for this simple subcase are given by

$$4\pi J^4 = -ek(X^2 + Y^2)^{-1}$$
$$8\pi\sigma(X^2 + Y^2) = -3kF + 8\pi e^2 kX(X^2 + Y^2)^{-1} \tag{47}$$

where X and Y are given by (46).

When $e = 0$, our above solution reduces to the radiating Kerr metric reported by Vaidya and Patel (1990).

Here one interesting feature of our solution is noteworthy. When $k \to 0$ it can be seen that $J^4 \to 0$, $\sigma \to 0$, $y \to -b\cos\alpha$ and $X \to r = u - t$. In the case $k \to 0$, we can easily check that our solution reduces to the well-known Kerr-Newman solution.

5. Concluding remarks

In this paper, we have obtained some exact solutions of the Einstein-Maxwell equations, which represent radiating Kerr-Newman solutions, with the help of the general scheme developed in § 3. By using the theorem of § 3 we have also obtained some exact solutions of the scheme of § 3 for the cases (1) $U_{uu} + k^2 U = \text{constant}$, $V_{yy} - k^2 V = 0$, $k = \text{constant}$ and (2) $Y = Y(y)$, i.e. Y is a function of y only. But these solutions are only of mathematical interest. We have not been able to attach any physical significance to them. Therefore they are not reported here.

In the above analysis, we have assumed that the function μ vanishes in solution (25) of Maxwell equations. The explicit solutions of Einstein-Maxwell equations in terms of the metric given by (3) and (4) with non-vanishing μ are at present under investigation. The results of this investigation will be discussed later.

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