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Three-body formalism for deuteron stripping reactions

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Abstract. A three-body formalism for deuteron stripping reactions has been developed. The equations of Alt *et al* (1967) (AGS) for the three particle system (target A, n, p) are reduced to a set of coupled one-dimensional integral equations with the use of (i) angular momentum basis for representation and (ii) separable approximation for the two body t-matrices (which delineate the interactions between the particle pairs). The on-shell solutions of this set of integral equations are then related to the cross sections of the rearrangement processes. The inputs in this calculation, viz., the separable interactions between the particle pairs in the respective channels are simply constructed from the respective two body bound state in accordance with the bound state approximation (BSA) conforming to the 'unitarity' requirement. Using this formalism preliminary calculations for the (d, p) and (d, n) reaction cross sections on ^{16}O have been carried out and they seem to have considerable semblance with the observed cross sections.

Keywords. Deuteron stripping reactions; three-body equations; bound state approximation; unitarity.

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1. Introduction

Single nucleon transfer reactions, initiated by the deuteron, have been extensively studied in the framework of the 'Distorted wave Born approximation' (DWBA) (Tobocman 1960), whose underlying features are (i) n, p and target 'A' act as inert cores (ii) in the total state of the system all channels, other than the elastic scattering channel, are treated as unimportant and (iii) the incident particle (deuteron) remains bound even as it approaches the target. In spite of apparent shortcomings, this approximation is found to work quite satisfactorily in reproducing the angular distribution of outgoing particles, possibly because of considerable flexibility of the input parameters.

Modifications in the DWBA theory from the theoretical standpoint have been made by Rawitscher and Mukherjee (1969, 1971) who included channels other than elastic (viz. the stripping channel) in the total state of the system. Attempts have also been made by various groups of workers to include the deuteron break-up channel and do the 'continuum discretised coupled channel' (CDCC) calculations (For an extensive review see Austern *et al* (1987)). However, it seems that within this framework it is very difficult to include all the rearrangement as well as break-up channels in the total state and to do a coupled channel calculation.

An alternative approach to the study of transfer reactions is the three-body approach wherein one does retain the first assumption of DWBA but does away with others and treats the 3 particles (n, p, A) (or 3 particles b, x, A in a more general transfer reaction, A + a(b + x) = B(A + x) + b), as a three-body system. Among the earliest attempts in this direction were those of Amado (1963) and Mitra (1965). Both these physicists first considered n–d scattering in the three-body framework and later their work was generalized (Shanley and Aaron (1967) and Mitra (1965)) to consider deuteron elastic scattering and deuteron stripping from a nucleus. More realistic 3-body calculations (with full angular momentum algebra) were done by Doleschall (1973) and Charmordic *et al* (1977) for studying n–d scattering and α–d scattering, respectively. They used 3-body equations of Alt *et al* (1967) and realistic separable interactions between pairs of particles. Mathur and Prasad (1981a, b) also used AGS equations to do the 3-body calculations for α transfer (^{6}Li, d) and deuteron-transfer (^{6}Li, α) reactions on ^{40}Ca, ^{16}O and ^{12}C, treating ^{6}Li nucleus to be a bound system of d–α clusters.

Of the transfer reactions, the deuteron stripping reactions have evoked the greatest interest among theorists as well as experimentalists and so we considered it worthwhile to extend our three-body formalism (Mathur and Prasad 1981a, b) to these reactions as well. This investigation, which we started in 1985, was also motivated by the consideration that even in the latest version of coupled channel calculations for deuteron induced reactions, viz. the CDCC calculations (Ferrel *et al* (1976), Rawitscher and Mukherjee (1980), Yahiro *et al* (1982)) one is able to couple only the breakup channels with the elastic channel, leaving aside the rearrangement channels. On the other hand, in the three-body formalism, the coupling of the elastic channel with all the rearrangement channels, as well as the breakup channel, is inherent. From a single three-body calculation one can, in principle, obtain cross sections of reactions leading to alternative excited states of the residual nuclei as well of the deuteron breakup processes (see Frank *et al* 1988) provided a large number of channels are considered.

However presently, in order to reduce the size of computation, we have not embarked on such an ambitious programme as to consider a very large number of channels. On the other hand we have restricted our consideration to such channels which lead only to one excited state of the residual nucleus per partition. For example, in deuteron induced reactions on ^{16}O we considered, besides elastic scattering, (d, p) and (d, n) reactions leading only to $s_{1/2}$ states of ^{17}O and ^{17}F. Even in this case the total number of channels (hence the number of coupled one-dimensional integral equations to be solved) is eleven as compared to three in the case of α-transfer reactions.

In this calculation, the AGS equations for the system of particles (A, n, p) are reduced to a set of coupled one-dimensional equations with the use of (i) angular momentum representation and (ii) separable approximation for the two-body t-matrix delineating the interaction between pairs of particles. (The physical basis of the separable approximation is to be discussed in § 3). The on-shell solutions of this set of integral equations can be related to the cross sections of the various rearrangement processes. The inputs in this calculation, viz., the separable interactions between the particle pairs in the respective channels are simply constructed from the bound states of the pairs in accordance with the bound state approximation (BSA) for the two body t-matrix $T_k(z)$ (Greben and Levin 1979) conforming also to the unitarity requirement (Fuda 1968). Using this formalism we have carried out a computation for the absolute cross sections of (d, p) and (d, n) reactions on ^{16}O leading to $s_{1/2}$ (0·87 MeV) state of ^{17}O and $s_{1/2}$ (0·50 MeV) state of ^{17}F, respectively.

We have chosen reactions leading to $s_{1/2}$ states of residual nuclei instead of the ground ($d_{5/2}$) states for two reasons:

(i) The angular momentum algebra is simpler and number of channels is smaller and (ii) the cross sections of stripping reactions leading to $s_{1/2}$ states are much larger than those leading to $d_{5/2}$ states. In this calculation the deuteron wavefunction, taken to be a mixture of $^3S_1$ and $^3D_1$ states, is the one obtained from the Reid (1968) soft core potential. The radial wavefunctions of $s_{1/2}$ states of ^{17}O and ^{17}F have been obtained by adjusting Wood-Saxon well depths so as to fit the last particle separation energy and reproduce the desired number of nodes. The 'pole' and 'logarithmic' singularities occurring in the kernels of the one-dimensional coupled integral equations are taken care of by using the methods outlined by Sasakawa (1963), Kowalski (1972) and Doleschall (1973). Despite ignoring the Coulomb effects in the present calculation, and using somewhat crude inputs in the form of $s_{1/2}$ state wavefunction of ^{17}O and ^{17}F, the calculated absolute cross sections of (d, p) and (d, n) reactions on ^{16}O show a considerable semblance to observed cross sections.

2. Angular momentum reduction of AGS equations

For the system of three particles (^{16}O, n and p, labelled as 1, 2 and 3) we write the equations of Alt *et al* (1967) in the operator form as

$$U_{ij}(z) = (1-\delta_{ij})(z-H_0) + \sum_{k=1}^{3} (1-\delta_{ik})\, T_k(z) G_0(z) U_{kj}(z), \tag{1}$$

where $T_k(z)$ is the two-body transition operator in three-body space, defined by the Lippman-Schwinger equation,

$$T_k(z) = V_k + V_k G_k(z) V_k = V_k + V_k G_0(z)\, T_k(z). \tag{2}$$

Also

$$G_k(z) = (z-H_k)^{-1} = (z-H_0-V_k)^{-1},$$

H_0 being the free Hamiltonian of the three-particle system.

The matrix element of the AGS operator $U_{ij}(z)$ between the asymptotic states, viz.

$$\left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi^{n_i}_{(L_i S_i) J_i M_i} \middle| U_{ij}(z) \middle| \mathbf{Q}_j d_j \sum_{L_j S_j} \phi^{n_j}_{(L_j S_j) J_j M_j} \right\rangle$$

can be easily related to the cross section of the process $j \to i$ (Mathur and Prasad 1981)

$$\frac{d\sigma_{j\to i}}{d\Omega} = \frac{\hbar^2}{2\mu_j Q_j}(2\pi)^4 \left| \left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi^{n_i}_{(L_i S_i) J_i M_i} \middle| U_{ij}(z) \right.\right.$$
$$\times \left.\left. \middle| \mathbf{Q}_j d_j \sum_{L_j S_j} \phi^{n_j}_{(L_j S_j) J_j M_j} \right\rangle \right|^2_{\mathrm{av}}. \tag{3}$$

Here the suffix 'av' means that the quantity is to be averaged over the initial magnetic quantum numbers and summed over the final magnetic quantum numbers. Also $\Sigma_{L_jS_j}\phi^{n_j}_{(L_jS_j)J_j}$ represents the bound state of jth pair, assumed generally to be a mixed L_jS_j state (for instance deuteron ground state which is a mixture of ${}^3S_1$ and ${}^3D_1$). In case the bound state of the jth pair happens to be a pure L_jS_j state, the summation over L_jS_j may be taken as non-existent. $\mathbf{Q}_j$ and $\mathbf{Q}_i$ are the on-shell momenta of the initial and final particles, $Q_j^2 = (E + \varepsilon^{n_j}_{B_j})$ and $Q_i^2 = (E + \varepsilon^{n_i}_{B_i})$.

With the use (i) of angular momentum basis for representation, viz. $|p_i q_i((L_i S_i) J_i s_i)\cdot K_i l_i : JM\rangle \equiv |p_i q_i (L_i S_i)\beta_i : JM\rangle$, (where p_i denotes the magnitude of the relative momentum of the ith pair, q_i the momentum of the ith particle relative to ith pair in the centre of mass frame (both momenta expressed in units of $E^{1/2}$) and $(L_i S_i)\beta_i JM$ denotes the angular momentum coupling scheme) and (ii) invoking a separable approximation for the two-body t-matrix viz.

$$\langle r_k u_k (L_k S_k)\beta_k : JM|\, T_k(z) | r'_k u'_k (L'_k S'_k)\beta'_k : JM\rangle = \frac{\delta(u_k - u'_k)}{u_k^2}\delta_{\beta_k\beta'_k} g^{n_k}_{(L_k S_k)J_k}(r_k)\tau_k^{n_k}(z - u_k^2) g^{n_k}_{(L'_k S'_k)J_k}(r'_k) \tag{4}$$

one can, after some simplification, reduce the AGS operator eq. (1) to a set of one-dimensional coupled integral equations

$$T_{ij}(q_i q'_j \beta_i \beta_j : J) = K_{ij}(q_i q'_j \beta_i \beta_j : J) + \sum_{k\beta_k}\int u_k^2\, \mathrm{d}u_k K_{ik}(q_i u_k \beta_i \beta_k : J)\tau_k^{n_k}(z - u_k^2)\, T_{kj}(u_k q'_j \beta_k \beta_j : J) \tag{5}$$

where the quantities $T_{ij}(q_i q_j \beta_i \beta_j : J)$ are actually related to the matrix elements of the AGS operator $U_{ij}(z)$, in the angular momentum basis i.e.

$$T_{ij}(q_i q'_j \beta_i \beta_j : J) = \sum_{L_i S_i}\sum_{L_j S_j}\int\!\!\int \frac{g^{n_i}_{(L_i S_i)J_i}(p_i) p_i^2\, \mathrm{d}p_i}{z - p_i^2 - q_i^2} \times \langle p_i q_i (L_i S_i)\beta_i : J | U_{ij}(z) | p'_j q'_j (L_j S_j)\beta_j : J\rangle \times \frac{g^{n_j}_{(L_j S_j)J_j}(p_{j'}) p_j'^2\, dp'_j}{z - p_j'^2 - q_j'^2} \tag{6}$$

In the set of eqs (5), for given $j\beta_j$ there are as many coupled equations as the number of channels, characterized by the suffixes i, β_i, viz. eleven. (For $i = 1, \beta_i$ corresponds to $K_i = 1$, $l_i = J - 1$, J, $J + 1$ (3 channels). For $i = 2$, β_i corresponds to $K_i = 0$, $l_i = J$ and $K_i = 1$, $l = J - 1$, J, $J + 1$ (4 channels). Similarly for $i = 3$ there are four sets of β_i i.e. 4 channels). The Born term K_{ij} (or K_{ik}) is given by

$$K_{ik}(q_i u_k \beta_i \beta_k : J) = (1 - \delta_{ik})\sum_{L_i S_i}\sum_{L_k S_k}\int\!\!\int \frac{g^{n_i}_{(L_i S_i)J_i}(p_i) p_i^2\, \mathrm{d}p_i g^{n_k}_{(L_k S_k)J_k}(r_k) r_k^2\, \mathrm{d}r_k}{z - p_i^2 - q_i^2} \times \langle p_i q_i (L_i S_i)\beta_i : J | r_k u_k (L_k S_k)\beta_k : J\rangle. \tag{7}$$

We have evaluated the overlap $\langle p_i q_i (L_i S_i)\beta_i : J | r_k u_k (L_k S_k)\beta_k : J\rangle$ by using graphical methods of spin algebras (Elbaz and Castle (1972)) and our results are consistent with Doleschall's (1973).

Explicitly we have, for $i=1$, $k=2$ (or 3):

$$K_{ik}(q_i u_k \beta_i \beta_k : J) = \sum_L \sum_{\Lambda_k}^{L_k} f_{LL_k\Lambda_k}(q_i, u_k) \int_{-1}^{+1} \frac{P_L(x)\,\mathrm{d}x\, g^{n_i}_{{}^3S_1}(r_i) g^{n_k}_{J_k}(p_k)}{(D_{ik} - x) p_k^{L_k}}$$
$$+ \sum_L \sum_{\Lambda_i}^{L_i=2} \sum_{\Lambda_k}^{L_k} f'_{LL_i\Lambda_i L_k\Lambda_k}(q_i, u_k) \int_{-1}^{+1} \frac{P_L(x)\,\mathrm{d}x\, g^{n_i}_{{}^3D_1}(r_1) g^{n_k}_{J_k}(p_k)}{(D_{ik} - x) r_i^{L_i} p_k^{L_k}} \tag{8}$$

For $i=2$ (or 3) and $k=1$ we have

$$K_{ik}(q_i u_k \beta_i \beta_k : J) = \sum_L \sum_{\Lambda_i} f_{LL_i\Lambda_i}(u_k, q_i) \int_{-1}^{+1} \frac{P_L(x)\,\mathrm{d}x\, g^{n_i}_{J_i}(r_i) g^{n_k}_{{}^3S_1}(p_k)}{(D_{ik} - x) r_i^{L_i}}$$
$$+ \sum_L \sum_{\Lambda_i}^{L_i} \sum_{\Lambda_k}^{L_k=2} f'_{LL_k\Lambda_k L_i\Lambda_i}(u_k, q_i) \int_{-1}^{+1} \frac{P_L(x)\,\mathrm{d}x\, g^{n_i}_{J_i}(r_i) g^{n_k}_{{}^3D_1}(p_k)}{(D_{ik} - x) r_i^{L_i} p_k^{L_k}} \tag{9}$$

For $i=2$ and $k=3$ (or vice versa) we have

$$K_{ik}(q_i u_k \beta_i \beta_k : J) = \sum_L \sum_{\Lambda_i}^{L_i} \sum_{\Lambda_k}^{L_k} f'_{LL_i\Lambda_i L_k\Lambda_k}(q_i, u_k) \int_{-1}^{1} \frac{P_L(x)\,\mathrm{d}x\, g^{n_i}_{J_i}(r_i) g^{n_k}_{J_k}(p_k)}{(D_{ik} - x) r_i^{L_i} p_k^{L_k}} \tag{10}$$

(On the rhs of (8), (9) and (10), 'summation' over $L_i S_i$ and $L_k S_k$ (see (7)) has been carried out. In the first case $L_i = 0, 2$ and $S_i = 1$ while $L_k S_k$ have single values; in the second case $L_k = 0, 2$ and $S_k = 1$ while $L_i S_i$ have single values and in the third case both $L_i S_i$ and $L_k S_k$ have single values).

Here

$$f_{LL_k\Lambda_k}(q_i, u_k) = C_{ik} E_{ik} \frac{B_{ik}^2 u_k^{\Lambda_k} q_i^{L_k - \Lambda_k}}{2A_{ik} q_i u_k}$$

and

$$f'_{LL_i\Lambda_i L_k\Lambda_k}(q_i u_k) = G_{ik} C'_{ik} \frac{B_{ik}^2}{2} \frac{u_k^{L_i - \Lambda_i + \Lambda_k} q_i^{\Lambda_i + L_k - \Lambda_k}}{2A_{ik} q_i u_k} \sum_{L'_1} \sum_{L'_2} \sum_X \sum_K F'_{ik}.$$

Also

$$r_i = p_i = \frac{1}{B_{ik}} |\mathbf{q}_i A_{ik} + \mathbf{u}_k|,$$

$$r_k = p_k = \frac{1}{B_{ik}} |\mathbf{q}_i + A_{ik} \mathbf{u}_k|,$$

$$x = \hat{\mathbf{q}}_i \cdot \hat{\mathbf{u}}_k,$$

and

$$D_{ik} = (zB_{ik}^2 - q_i^2 - u_k^2)/(2A_{ik} q_i u_k).$$

The coefficients C_{ik}, E_{ik} are expressed as follows:

$$C_{ik} = \left(\frac{1}{B_{ik}}\right)^{L_k+3} \sqrt{\pi} (\varepsilon_{ik})^{S_i + S_k - 2s_j - s_k - s_i - L_k} \frac{[K_k]^2}{[J]} [J_k][s_i][l_i][l_k]$$
$$\times (-1)^{2S_j + 2J_k} \begin{Bmatrix} l_i & L_k & l_k \\ K_k & J & S_i \end{Bmatrix} \begin{Bmatrix} s_i & s_j & s_k \\ J_k & K_k & L_k \end{Bmatrix}$$

and

$$E_{ik} = (A_{ik})^{\Lambda_k} \begin{bmatrix} 2L_k + 1 \\ 2\Lambda_k \end{bmatrix}^{1/2} [L]^2 [L_k - \Lambda_k] \begin{bmatrix} L & L_k - S_k & l_i \\ 0 & 0 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} L & l_k & \Lambda_k \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} l_i & L_k - \Lambda_k & L \\ k & l_k & L_k \end{Bmatrix}.$$

Here $0 \leqslant \Lambda \leqslant L_k$ and $|l_i + l_k - L_k| \leqslant L \leqslant (l_i + l_k + L_k)$. The coefficients G_{ik}, C'_{ik}, F'_{ik} are expressed as follows:

$$G_{ik} = (\varepsilon_{ik})^{S_i + S_k + L_i + L_k - 2s_j - s_i - s_k} (-)^{L_i + S_i + 2J_i + s_k - s_j}$$

$$\times (B_{ik})^{-3 - L_i - L_k} [L_i][L_k][S_i][S_k][J_i][J_k][K_i][K_k][l_i][l_k]$$

$$C'_{ik} = (A_{ik})^{\Lambda_i + \Lambda_k} \begin{bmatrix} 2L_i + 1 \\ 2\Lambda_i \end{bmatrix}^{1/2} \begin{bmatrix} 2L_k + 1 \\ 2_k \end{bmatrix} [L]^2 [L_k - \Lambda_k][L_i - \Lambda_i]$$

and

$$F'_{ik} = \begin{bmatrix} l_i & L'_1 & \Lambda_i \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L'_1 & L & L_k - \Lambda_k \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} L & L_i - \Lambda_i & L'_2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_k & \Lambda_k & L'_2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\times [L'_1]^2 [L'_2]^2 [X]^2 [K]^2 \begin{Bmatrix} \Lambda_i & L_i - \Lambda_i & L_i \\ X & l_i & L'_1 \end{Bmatrix} \begin{Bmatrix} L_i - \Lambda_i & L'_2 & L \\ L_k - \Lambda_k & L'_1 & X \end{Bmatrix}$$

$$\times \begin{Bmatrix} J_i & S_i & L_i \\ K & K_i & s_i \end{Bmatrix} \begin{Bmatrix} s_i & S_k & s_j \\ s_k & S_i & K \end{Bmatrix} \begin{Bmatrix} L_k & J_k & S_k \\ s_k & K & K_k \end{Bmatrix} \begin{Bmatrix} L_k - \Lambda_k & \Lambda_k & L_k \\ l_k & X & L'_2 \end{Bmatrix}$$

$$\times \begin{Bmatrix} l_i & K_i & J \\ K & X & L_i \end{Bmatrix} \begin{Bmatrix} J & l_k & K_k \\ L_k & K & X \end{Bmatrix}.$$

Here $0 \leqslant \Lambda_k \leqslant L_k$; $0 \leqslant \Lambda_i \leqslant L_i$ and $|l_i + l_k - L_i - L_k| < L < (l_i + l_k + L_i + L_k)$; $\varepsilon_{ij} = +1$ if $ij = 12, 23, 21$; -1 if $ij = 21, 32, 13$ and 0 if $i = j$. Also

$$A_{ik}^2 = \frac{m_i m_k}{(M - m_i)(M - m_k)}; \quad B_{ik} = (1 - A_{ik}^2)^{1/2}; \quad \begin{bmatrix} N \\ R \end{bmatrix} = \frac{N!}{(N-R)!\,R!};$$

$[L]^2 \equiv L(L+1)$, $\begin{bmatrix} J_1 & J_2 & J_3 \\ 0 & 0 & 0 \end{bmatrix}$ is the Wigner $3jm$ coefficient and

$\begin{Bmatrix} J_1 & J_2 & J_3 \\ l_1 & l_2 & l_3 \end{Bmatrix}$ is the 6-j symbol.

Now, the transition matrix element occurring in (3) viz.

$$\left\langle \mathbf{Q}_i d_i \sum_{L_i S_i} \phi^{n_i}_{(L_i S_i) J_i M_i} | U_{ij}(z) | \mathbf{Q}_j d_j \sum_{L_j S_j} \phi^{n_j}_{(L_j S_j) J_j M_j} \right\rangle$$

may be expressed in terms of the on-shell solutions,

$$T_{ij}(Q_i Q_j \beta_i \beta_j : J) \equiv T_{i\beta_i, j\beta_j}(J) \equiv T^R_{i\beta_i, j\beta_j}(J) + i T^{(Im)}_{i\beta_i, j\beta_j}(J)$$

of (5). Using graphical methods of spin algebras (Prasad (1981); Elbaz and Castle

Table 1.

Particle pair	Quantum state	C	C_1 (F)	C_2 (F^2)	C_3 (F^3)	μ (F)	ν (F)	a (F)	b	Separation energy ε_{Bk}^{n} (MeV)
$n+p$	$^3S_1$	$0{\cdot}5886\,F^{-5/2}$	1·1400	−0·0560	−0·147			1·725		2·226
$n+p$	$^3D_1$	$0{\cdot}2718\,F^{-3/2}$	0·0150	0·0440		1·005	2·0170		3·8320	
$^{16}O+p$	$S_{\frac{1}{2}}$	$0{\cdot}9146\,F^{-5/2}$	1·7400	−0·6800		0·4624		4·4980		3·58
$^{16}O+n$	$S_{\frac{1}{2}}$	$0{\cdot}7300\,F^{-5/2}$	1·9436	−0·7420		−0·3700		3·7800		3·27

(1972)), we get,

$$\left|\left\langle \mathbf{Q}_i d_i \sum_{L_iS_i} \phi^{n_i}_{(L_iS_i)J_iM_i} \middle| U_{ij}(z) \middle| \mathbf{Q}_j d_j \sum_{L_jS_j} \phi^{n_j}_{(L_jS_j)J_jM_j} \right\rangle\right|^2_{\mathrm{av}}$$

$$= \frac{(1/4)Q_iQ_j(N_iN_j)^2}{(2s_j+1)(2J_j+1)} \frac{1}{3} \sum_J \sum_{J'} (2J+1)(2J'+1) \sum_{l_i=|J-1|}^{J+1} \sum_{l'_i=|J'-1|}^{J'+1}$$

$$\times \frac{P_{l_i}(\cos\theta)P_{l'_i}(\cos\theta)}{16\pi^2} \left\{ T^{(R)}_{i\beta_i,j\beta_j}(J)\, T^{(R)}_{i\beta'_i,j\beta'_j}(J') + T^{(I_m)}_{i\beta_i,j\beta_j}(J)\, T^{(I_m)}_{i\beta'_i,j\beta'_j}(J') \right\} \quad (11)$$

where

$$\beta_i = (J_i, s_i)K_i, l_i; \quad \beta_j = (J_j s_j)K_i l_i; \quad \beta'_i = (J_i s_i)K_i l'_i$$

and

$$\beta'_j = (J_j s_j)K_i l'_{i'} \,(\because K_i = K_j = K'_i = K'_j; l_i = l_j \text{ and } l'_i = l'_j).$$

Here N_i, N_j are normalization constants defined in §3.

So we have to find the on-shell solutions (i.e. for $q_i = Q_i$ and $q'_j = Q_j$) of the set of one-dimensional coupled equations (5). We shall return to this problem in §4 when we discuss the methods of tackling the singularities in the kernels of (5). In the next sections we discuss the logic of the separable approximation (4).

3. The origin of separable approximation

The separable form of the matrix element of $T_k(z)$ in the angular momentum basis (4), which we use in our 3-body calculation, has its origin in the bound state approximation (BSA) invoked by Greben and Levin (1979). Accordingly, in the spectral resolution of $G_k(z)$, one retains only the two body bound state contribution, ignoring the contribution of the two body continuum states (see also Mathur and Prasad 1981a, b). Thus

$$G_k(z) = (z - H_k)^{-1} \approx \sum_{n_k} \int \frac{\mathrm{d}^3 q_k |\mathbf{q}_k \phi^{n_k}_k\rangle\langle \mathbf{q}_k \phi^{n_k}_k|}{(z - q_k^2 + \varepsilon^{n_k}_{B_k})}$$

which implies that

$$T_k(z) = V_k + V_k G_k(z) V_k \approx \sum_{n_k} \int \mathrm{d}^3 q_k\, V_k |\mathbf{q}_k \phi^{n_k}_k\rangle \tau^{n_k}_k(z - u_k^2) \langle \mathbf{q}_k \phi^{n_k}_k| V_k \quad (12)$$

or

$$\langle \mathbf{r}_k \mathbf{u}_k | T_k(z) | \mathbf{r}'_k \mathbf{u}'_k \rangle = \delta(\mathbf{u}_k - \mathbf{u}'_k) \langle \mathbf{r}_k | t_k(z - u_k^2) | \mathbf{r}'_k \rangle \quad (13)$$

where

$$t_k(z - u_k^2) \approx \sum_{n_k} v_k |\phi^{n_k}_k\rangle \tau^{n_k}_k(z - u_k^2) \langle \phi^{n_k}_k | v_k \quad (14)$$

and

$$\tau^{n_k}_k(z - u_k^2) \equiv \frac{1}{z - u_k^2 + \varepsilon^{n_k}_{B_k}}. \quad (15)$$

In (13) the delta function takes care of the fact that kth particle in a spectator, while the argument of the two body t-matrix t_k in two body space indicates that the energy associated with the interacting (kth) pair equals the total energy z minus the energy u_k^2 carried by the spectator. $\varepsilon_{B_k}^{(n_k)}$ is the energy of the n_k bound state of the kth pair. The BSA (set of eqs (13), (14) and (15)) was put to test by several authors (Greben and Levin 1979; Eyre and Osborn 1979; Adhikari 1981) in solvable 3-body models but was found to be somewhat unsatisfactory obviously due to the neglect of the continuum contribution.

One may recall that long ago Lovelace (1964a, b) had derived a separable expansion of the two body transition operator in two body space on the basis of the idea of 'pole dominance'. He obtained

$$t_k(z) \approx \sum_{n_k} v_k |\phi_k^{n_k}\rangle \tau_k^{n_k}(z) \langle \phi_k^{n_k}| v \tag{16}$$

where

$$\tau_k^{n_k}(z) = \frac{1}{z + \varepsilon_k^{n_k}}. \tag{17}$$

Comparing Lovelace pole approximation (eqs (16) and (17)) with the bound state approximation (eqs (13), (14), (15)) we note that the latter is just the generalization of the former to two-body t-matrix $T_k(z)$ in 3-body space.

Fuda (1968) observed that Lovelace (1964a, b) approximation for $t_k(z)$ though conforms to the pole behaviour, it does not satisfy Lippmann-Schwinger equation (hence 'unitarity') because of the truncation of the continuum contribution. Now, if the two body interactions in a particular channel admits only one bound state (which assumption could be justified since a 'channel' is characterized not only by a partition (k) but also by a set of quantum numbers $\beta_k = (J_k, K_k, l_k)$, J_k being the total angular momentum of kth pair) then according to Fuda (1968) one can make the separable approximation ((16) without summation sign) satisfy the twin condition of 'pole dominance' and 'unitarity' by just modifying the expression for the 'propagator' $\tau_k(z)$. Thus

$$t_k(z) = v_k |\phi_k^{n_k}\rangle \tau_k^{n_k}(z) \langle \phi_k^{n_k}| v_k \tag{18}$$

where

$$\tau_k^{n_k}(z) = \left[\frac{1}{\lambda'} - \langle \phi_k^{n_k} | v_k g_0(z) v_k | \phi_k^{n_k} \rangle \right]^{-1} \tag{19}$$

and

$$\frac{1}{\lambda'} = \langle \phi_k^{n_k} | v_k g_0(-\varepsilon_{B_k}^{n_k}) v_k | \phi_k^{n_k} \rangle. \tag{20}$$

Fuda's separable approximation, referred to as the 'unitary pole approximation' is certainly valid in the domain of negative values of energy z because of the proximity of the latter to the bound state pole at $z = -\varepsilon_{B_k}^{n_k}$. However, surprisingly, UPA is found to be a fairly good approximation for positive energies as well ($z > 0$) according to test calculations done for some test potentials (Lagu *et al* 1975; Vyas and Mathur 1978). In these calculations one finds that the agreement of exact and UPA phase shifts and half shell function is quite satisfactory. This is due to the imposition of the 'unitarity' condition onto the separable approximation.

In view of Fuda's work, the bound state approximation ((13), (14), (15)) could also be modified to conform to the 'unitarity' requirement for $T_k(z)$. Accordingly for $T_k(z)$ we can write, in addition to (13), the set of eqs (18), (19) and (20) with z replaced by $(z-u_k)^2$.

For our three body calculations we need the matrix element of $T_k(z)$ in the angular momentum basis. We can write

$$\langle r_k u_k (L_k S_k)\beta_k : JM | T_k(z) | r'_k u'_k (L'_k S'_k)\beta'_k : JM\rangle$$
$$= \frac{\delta(u_k - u'_k)}{u_k^2}\delta_{\beta_k\beta'_k}\langle r_k(L_k S_k)J_k | t_k(z-u_k^2) | r'_k(L'_k S'_k)J_k\rangle \tag{21}$$

where the delta function and kronecker delta take care of the fact that kth particle is a 'spectator'. Using (18) we get, in BSA, conforming to the unitarity requirement

$$\langle r_k(L_k S_k)J_k | t_k(z-u_k^2) | r'_k(L'_k S'_k)J_k\rangle \approx g^{n_k}_{(L_k S_k)J_k}(r_k)\tau^{n_k}_k(z-u_k^2)g^{n_k^*}_{(L'_k S'_k)J_k}(r_k) \tag{22}$$

where

$$g^{n_k}_{(L_k S_k)J_k}(r_k) = \langle r_k(L_k S_k)J_k | v_k | \phi^{n_k}_k\rangle = -(r_k^2 + \varepsilon^{n_k}_{B_k})\phi^{n_k}_{(L_k S_k)J_k}(r_k) \tag{23}$$

Here $\phi^{n_k}_{(L_k S_k)J_k}(r_k)$ is the momentum space wave-function representing the projection of the bound state of the kth pair viz. $|\phi^{n_k}_k\rangle$ onto $|(L_k S_k)J_k\rangle$ state. In particular, if $|\phi^{n_k}_k\rangle$ is a pure $(L_k S_k)J_k$ state, then $\phi^{n_k}_{(L_k S_k)J_k}(r_k)$ is just the momentum space wavefunction representing this state. In (22) the propagator $\tau^{n_k}_k(z-u_k^2)$ is given, in accordance with (19) by

$$\tau^{n_k}_k(z-u_k^2) = \left[\frac{1}{\lambda'} - \sum_{L_k S_k}\int \frac{(g^{n_k}_{(L_k S_k)J_k}(r_k))^2 r_k^2\,\mathrm{d}r_k}{z-u_k^2-r_k^2}\right]^{-1}$$
$$\underset{u_k\to Q_k}{\approx} N_k^2(z-u_k^2+\varepsilon^{n_k}_{B_k})^{-1} \tag{24}$$

where

$$\frac{1}{\lambda'} = \sum_{L_k S_k}\int \frac{(g^{n_k}_{(L_k S_k)J_k}(r_k))^2 r_k^2\,\mathrm{d}r_k}{-\varepsilon^{n_k}_{B_k} - r_k^2}$$

and

$$N_k^{-2} = \sum_{L_k S_k}\int \frac{(g^{n_k}_{(L_k S_k)J_k}(r_k))^2}{(r_k^2+\varepsilon^{n_k}_{B_k})^2} r_k^2\,\mathrm{d}r_k. \tag{25}$$

Equation (4) (or eqs (21), (22)), together with eqs (23), (24) and (25) furnish the separable approximation for $T_k(z)$ that we make use of in our 3-body calculations.

4. Treatment of singularities

Problems arise in the solution of the set of one-dimensional integral equations (5) due to two kinds of singularities in the kernel $K_{ik}\tau_k u_k^2$:

(i) The propagator $\tau^{n_k}_k(z-u_k^2)$ has a 'pole' at $u_k = Q_k = (z+\varepsilon^{n_k}_{B_k})^{1/2}$, and (ii) the term $K_{ik}(q_i u_k \beta_i \beta_k : J)$ has a 'logarithmic singularity', since it involves, in the integral over x with limits -1 to $+1$ (see (8), (9), (10)), a term $(D_{ik}-x)^{-1}$.

The 'pole' singularity is dealt with according to a method by Sasakawa (1963) and

Kowalski (1972) wherein one subtracts out the pole behaviour from the kernel $K_{ik}\tau_k^{n_k}u_k^2$ to define a new (subtracted) kernel Λ_{ik} i.e.

$$\Lambda_{ik}(q_i u_k \beta_i \beta_k : J) = u_k^2 \Bigg[K_{ik}(q_i u_k \beta_i \beta_k : J)\tau_k^{n_k}(z - u_k^2) - N_k^2 \frac{(Q_k^2 + 1)K_{ik}(q_i Q_k \beta_i \beta_k : J)}{(u_k^2 + 1)(E - u_k^2 + \varepsilon_{B_k}^{n_k})} \Bigg], \tag{26}$$

(so that Λ_{ik} vanishes for $u_k \to Q_k$), and sets up a new set of one-dimensional coupled integral equations, similar to (5) but using the subtracted kernel, viz.

$$\Gamma_{ij}(q_i q_j, \beta_i \beta_j : J) = K_{ij}(q_i q_j' \beta_i \beta_j : J) + \sum_{k,\beta_k} \int \mathrm{d}u_k \Lambda_{ik}(q_i u_k \beta_i \beta_k : J)\Gamma_{kj}(u_k q_j' \beta_k \beta_j : J) \tag{27}$$

The subtracted kernel Λ_{ik} still involves 'logarithmic singularities' which are tackled by Doleschall's technique (1973, 1978), wherein the singular behaviour is subtracted from Λ_{ik} and these very terms are subsequently added, so that logarithmic singularity appears only in a few terms which contain Legendre function of the second kind:

$$Q_L(D_{ik}) = \frac{1}{2}\int_{-1}^{1} \frac{P_L(x)\mathrm{d}x}{D_{ik} - x}.$$

For latter terms one has to use 'special quadratures' and 'special weights*' for u_k-integration. For all other terms in Λ_{ik}, multiplied by unknown function Γ_{kj}, one can use ordinary Gauss Legendre's quadratures and weights for u_k-integration.

For $i = 1$, $k = 2$ (or 3), $\Lambda_{ik}(q_i u_k \beta_i \beta_k : J)$ may be rewritten as†

$$\Lambda_{ik}(q_i u_k \beta_i \beta_k : J) = u_k^2 \Bigg[\sum_{L\Lambda_k} f_{LL_k\Lambda_k}(q_i, u_k) \Bigg\{ \int_{-1}^{1} \frac{P_L(x)\mathrm{d}x}{D_{ik} - x} \Bigg(\frac{g_{s_i S_1}^{n_i}(r_i) g_{J_k}^{n_k}(p_k)}{p_k^{L_k}} - \frac{g_{s_i S_1}^{n_i}(\tilde{r}_i) g_{J_k}^{n_k}(\tilde{p}_k)}{\tilde{p}_k^{L_k}} \Bigg) + \frac{2 g_{s_i S_1}^{n_i}(\tilde{r}_i) g_{J_k}^{n_k}(\tilde{p}_k)}{\tilde{p}_k^{L_k}} Q_L(D_{ik}) \Bigg\} \tau_k^{n_k}(z - u_k^2)$$

---

* The method of determining the 'special weights' is outlined by Doleschall (1971, 1978) and Mathur and Prasad (1981). It may be remarked in this context that, for the integral $I_m = \int Q_L(D_{ik}(z)) P_m(z)\mathrm{d}z$, where $z = z(y_k)$ and $y_k = u_k/\sqrt{E}$, an analytic expression can be obtained by circumventing the logarithmic singularities. The special weights are then given by the matrix equation

$$\sum_l p_{ml} h_l^{(s)} = I_m,$$

where

$$P_{ml} = P_m(z).$$

† From eq. (23) we see that in $\Lambda_{ik}(q_i u_k \beta_i \beta_k : J)$, the singular behaviour (pertaining to logarithmic singularity) is removed by subtracting, from the integrands of x-integration, terms in which r_i and p_k are, respectively, replaced by $\tilde{r}_i = (E - q_i^2)^{1/2}$ and $\tilde{p}_k = (E - u_k^2)^{1/2}$. Since r_i, p_k join smooth with $\tilde{r}_i$ and $\tilde{p}_k$ as $x \to D_{ik}$, the integrand for x-integration vanishes as $x \to D_{ik}$ and this subtracted integral is devoid of logarithmic singularity. This subtracted term is subsequently added and it contains the singular behaviour in the form of the function $Q_L(D_{ik})$.

$$+\sum_{L\Lambda_i\Lambda_k} f'_{LL_i\Lambda_iL_k\Lambda_k}(q_i,u_k)\left\{\int_{-1}^{1}\frac{P_L(x)\mathrm{d}x}{D_{ik}-x}\left(\frac{g^{n_i}_{^3D_1}(r_i)g^{n_k}_{J_k}(p_k)}{r_i^{L_i}p_k^{L_k}}\right.\right.$$
$$\left.\left.-\frac{g^{n_i}_{^3D_1}(\tilde{r}_i)g^{n_k}_{J_k}(\tilde{p}_k)}{\tilde{r}_i^{L_i}\tilde{p}_k^{L_k}}\right)+\frac{2g^{n_i}_{^3D_1}(\tilde{r}_i)g^{n_k}_{k}(\tilde{p}_k)}{\tilde{r}_i^{L_i}\tilde{p}_k^{L_k}}Q_L(D_{ik})\right\}\tau^{n_k}_k(z-u_k^2)$$
$$-\frac{N_k^2(Q_k^2+1)}{(u_k^2+1)(Q_k^2-u_k^2)}\left\{\sum_{L\Lambda_k} f_{LL_k\Lambda_k}(q_i,Q_k)\int_{-1}^{1}\frac{P_L(x)\mathrm{d}x}{\bar{D}_{ik}-x}\right.$$
$$\times\frac{g^{n_i}_{^3S_1}(\bar{r}_i)g^{n_k}_{J_k}(\bar{p}_k)}{\bar{p}_k^{L_k}}+\sum_{L\Lambda_i\Lambda_k} f'_{LL_i\Lambda_iL_k\Lambda_k}(q_i,Q_k)\int_{-1}^{1}\frac{P_L(x)\mathrm{d}x}{\bar{D}_{ik}-x}$$
$$\left.\left.\times\frac{g^{n_i}_{^3D_1}(\bar{r}_i)g^{n_k}_{J_k}(\bar{p}_k)}{\bar{r}_i^{L_i}\bar{p}_k^{L_k}}\right\}\right] \tag{28}$$

Here $\tilde{r}_i=(E-q_i^2)^{1/2}, \tilde{p}_k=(E-u_k^2)^{1/2}, \bar{r}_i=r_i(\mathbf{u}_k=\mathbf{Q}_k), \bar{p}_k=p_k(\mathbf{u}_k=\mathbf{Q}_k)$ and $\bar{D}_{ik}=D_{ik}(\mathbf{u}_k=\mathbf{Q}_k)$.

One can write similar expressions for Λ_{ik} when $i=2$ (or 3), $k=1$ or when $i=2$, $k=3$ (or vice versa). When either q_i or u_k or both are greater than $E^{1/2}$, then the subtracted and added terms (involving $\tilde{r}_i$ and $\tilde{p}_k$) may be dropped since, in this case, $|D_{ik}|>1$.

The set of one-dimensional equations (with subtracted kernel) i.e. (27) have been solved by using 15-point quadrature for u_k-integration (6-points for $0\leqslant u_k\leqslant\sqrt{E}$, 3-points for $\sqrt{E}\leqslant u_k\leqslant(E+2\varepsilon_k^{n_k})^{1/2}$ and 6-points for $(E+2\varepsilon^{n_k}_{B_k})^{1/2}\leqslant u_k<\infty$) and converting this set of integral equations into a matrix equation and inverting this $(15n\times 15n)$ matrix, n being the number of channels.

Now, to express the on-shell amplitude $T_{ij}(Q_iQ_j\beta_i\beta_j:J)\equiv T_{i\beta_i,j\beta_j}(J)$ occurring in (5), in terms of the solutions $\Gamma_{ij}(Q_iQ_j\beta_i\beta_j:J)\equiv\Gamma_{i\beta_i,j\beta_j}(J)$ of (22), we follow Kowalski (1972) and Sasakawa (1963) and write

$$T_{ij}(q_iQ_j\beta_i\beta_j:J)=\Gamma_{ij}(q_iQ_j\beta_i\beta_j:J)+\sum_{k\beta_k}\Gamma_{ik}(q_iQ_k\beta_i\beta_k:J)\mathscr{I}_{k\beta_k,j\beta_j} \tag{29}$$

where $\mathscr{I}_{k\beta_k,j\beta_j}$ is an (11×11) matrix with complex elements (as we shall see later).

Substituting (29) into (5), and using (27), we get

$$\mathscr{I}_{k\beta_k,j\beta_j}=d_{k\beta_k,j\beta_j}+\sum_{p\beta_p}d_{k\beta_k,p\beta_p}\mathscr{I}_{p\beta_p,j\beta_j} \tag{30}$$

where

$$d_{k\beta_k,j\beta_j}\equiv N_k^2(Q_k^2+1)\int_0^\infty\frac{\Gamma_{kj}(u_kQ_j\beta_k\beta_j:J)}{(u_k^2+1)(E-u_k^2+\varepsilon^{n_k}_{B_k}+i0)}u_k^2\,\mathrm{d}u_k$$
$$\equiv d^{(R)}_{k\beta_k,j\beta_j}+id^{(I_m)}_{k\beta_k,j\beta_j} \tag{31}$$

and

$$d_{k\beta_k,j\beta_j}\equiv N_k^2(Q_k^2+1)\mathscr{P}\int_0^\infty\frac{\Gamma_{kj}(u_kQ_j\beta_k\beta_j:J)}{(u_k^2+1)(E-u_k^2+\varepsilon^{n_k}_{B_k})}u_k^2\,\mathrm{d}u_k \tag{32}$$

and

$$d_{k\beta_k,j\beta_j}=-\frac{\pi}{2}N_k^2Q_k\Gamma_{kj}(Q_kQ_j\beta_k\beta_j:J). \tag{33}$$

If, in (29) we take the channel $j\beta_j$ as fixed, then we can regard all the terms involving the suffix $j\beta_j$ in (30) as elements of column matrices and we can write

$$\mathscr{I} = d + D.\mathscr{I} \tag{34}$$

where D is a square matrix whose elements $D_{k\beta_k,p\beta_p} \equiv d_{k\beta_k,p\beta_p}$ are complex. The elements of the column matrix $d_{k\beta_k,j\beta_j}$ are also complex. Naturally the column matrix $\mathscr{I}$ has also complex elements and we can split the column matrices $\mathscr{I}$, d and square matrix D into real and imaginary parts. Making these substitutions into the matrix eq. (34) we get,

$$\begin{pmatrix} \mathscr{I}^{(R)} \\ \mathscr{I}^{(I_m)} \end{pmatrix} = - \begin{pmatrix} D^{(R)} - I & -D^{(I_m)} \\ D^{(I_m)} & D^{(R)} - I \end{pmatrix}^{-1} \begin{pmatrix} d^R \\ d^{I_m} \end{pmatrix}. \tag{35}$$

Equation (35) enables us to determine the column vector $(\mathscr{I}^{(R)}/\mathscr{I}^{I_m})$ for each channel j, β_j. Thus we know $\mathscr{I}^{(R)}_{k\beta_k,j\beta_j}$ and $\mathscr{I}^{(I_m)}_{k\beta_k,j\beta_j}$ so that

$$T^{(R)}_{i\beta_i,j\beta_j}(J) = \Gamma_{i\beta_i,j\beta_j}(J) + \sum_{k\beta_k} \Gamma_{i\beta_i,k\beta_k}(J).\mathscr{I}^{(R)}_{k\beta_k,j\beta_j}$$

and

$$T^{(I_m)}_{i\beta_i,j\beta_j} = \sum_{k\beta_k} \Gamma_{i\beta_i,k\beta_k}(J).\mathscr{I}^{(I_m)}_{k\beta_k,j\beta_j}.$$

The cross sections can now be calculated using (11) and (3). The details of computer program will appear elsewhere.

5. Conclusions

We see that the three-body calculation of the deuteron stripping reactions, with the use of the bound state approximation (BSA) for two body t matrix conforming to the unitarity requirement, is mathematically feasible. The results of our preliminary calculations for (d, p) reactions cross section on ^{16}O enable us to see whether these have some semblance with the observed cross section. In figures 1 and 2 a comparison of the calculated absolute cross section of (d, p) and (d, n) reaction on ^{16}O with the deuteron laboratory energy of 14·5 MeV leading to $S_{1/2}$ states of ^{17}O (0·87 MeV) and ^{17}F (0·50 MeV) is made with the corresponding experimental values due to Alty *et al* (1967) and Oliver *et al* (1969) for incident deuteron energy of 12 MeV. Though the finer details of angular distribution are not reproduced, the calculated cross section has some semblance to observed one in the sense that peaks and dips of the angular distribution are more or less at the right positions. The reasons for not getting a good agreement are as follows:

(i) Our inputs in the form of the nucleon–nucleus bound states are somewhat crude. A better method of determining the nucleon nucleus bound state wavefunction would have been through the generation coordinate method (Hill and Wheeler 1953).
(ii) We have considered fewer channels. In principle one can within the 3-body framework, include several channels so as to get cross sections of transfer reactions leading to alternative excited states of the residual nuclei. In this attempt however,

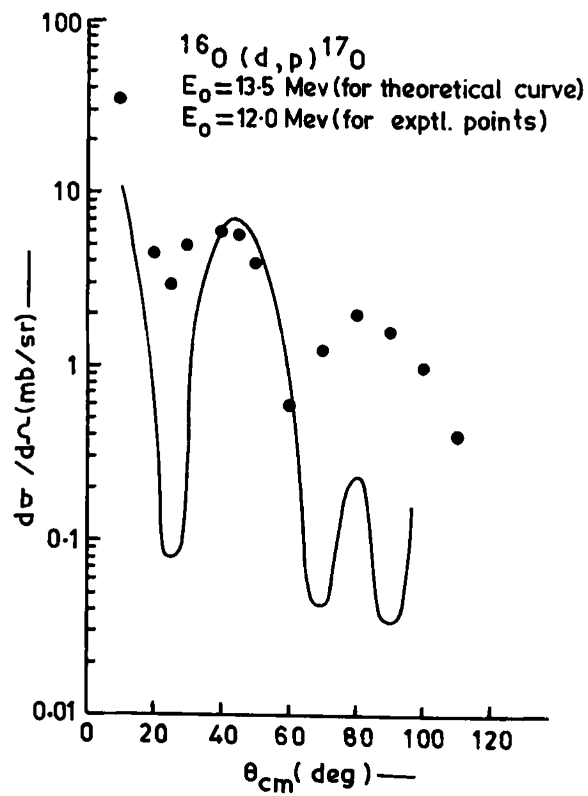

Figure 1. The continuous curve shows the absolute values of differential cross section (in mb/s) of (d, p) reaction on ^{16}O leading to $^{17}O(s_{\frac{1}{2}})$ state as function of θ_{cm} for deuteron lab. energy of 13·5 MeV. The corresponding experimental points for deuteron lab. energy of 12 MeV are due to Alty *et al* (1967).

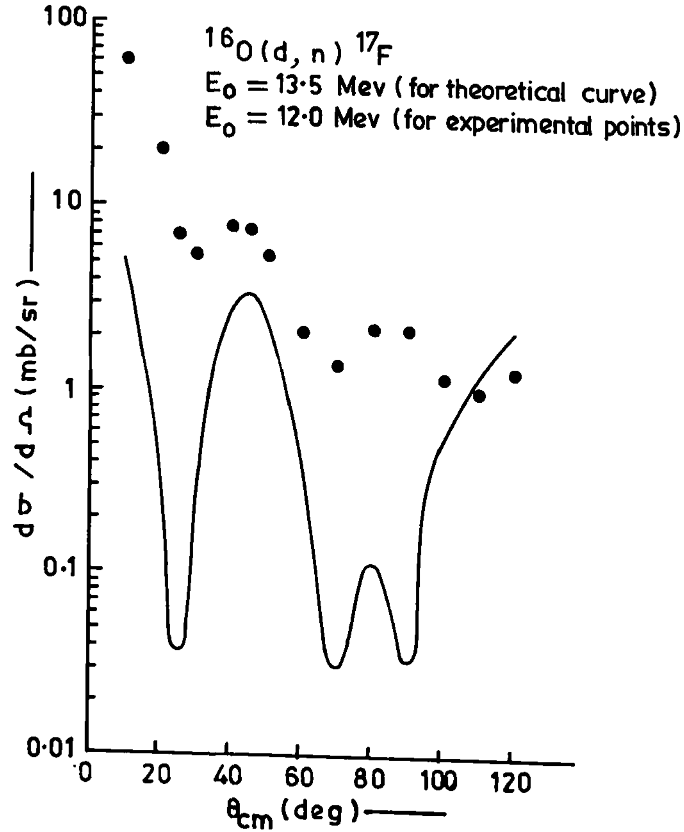

Figure 2. The continuous curve shows the absolute values of differential cross section (in mb/sr) of (d, n) reaction on ^{16}O leading to $^{17}F(s_{\frac{1}{2}})$ state as a function of θ_{cm} for deuteron lab. energy of 13·5 MeV. The corresponding experimental points for deuteron lab. energy of 12·0 MeV are due to Oliver *et al* (1969).

we have not been able to do this as it would need a computer of higher memory and much more computing time. Limitation on the number of channels does effect our results since we are not using complex interactions between particle pairs to account for alternative channels.
(iii) We have not been able to consider Coulomb interactions in these calculations whose effect in transfer reaction, though second to nuclear interaction, is not insignificant. The elastic scattering cross sections resulting from these calculations are not compared with the observed values because Coulomb effects dominate elastic scattering. Similar calculation on ^{40}Ca are in progress and will be reported shortly.

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