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Singlet N-N scattering with exchange of confined gluons among relativistic quark clusters

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Abstract. Confined-gluon-exchange among relativistically confined quark clusters is used to obtain singlet S and P wave N-N scattering phase-shifts. A good agreement is obtained with the experimental results.

Keywords. Quark-model; harmonic model; resonating group; confined gluon; current confinement.

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In this paper we have investigated the effect of exchange of confined gluons among relativistically confined quarks in nucleon-nucleon (N-N) scattering calculations. Earlier quantum chromo dynamics (QCD) based models such as the MIT bag model (Detar 1978) or the non-relativistic-quark-model (NRQM) (Warke and Shanker 1980) using the resonating group method (RGM) (Oka and Yazaki 1981; Faessler *et al* 1982, 1983; Shimizu 1989), have successfully explained short range repulsion, which arises from the colour magnetic part of Fermi-Breit unconfined one-gluon-exchange potential (OGEP) (De Rujula *et al* 1975). In MIT and NRQM it is necessary to introduce σ and π mesons for obtaining the intermediate and long range attraction. Though these models have incorporated the confinement of quark, the effect of confinement of gluons has not been taken into account. We present here the results of calculation of singlet S and P wave N-N scattering phase-shifts. For the quarks, we use the relativistic harmonic oscillator. Lorentz scalar + vector confinement model (RHM) (Khadkikar and Gupta 1983) which explains the properties of light hadrons. A similar confinement scheme for gluons, the so called current confinement model (CCM) (Khadkikar and Gupta 1983; Khadkikar and Vinod Kumar 1987) which describes glueballs, is employed to obtain confined-one-gluon-exchange potential (COGEP), which is used in these calculations.

In RHM (Khadkikar and Gupta 1983) quarks in a hadron are confined through the action of a Lorentz scalar plus vector harmonic oscillator potential

$$\tfrac{1}{2}(1+\gamma_0)\alpha^2 r^2 + M, \tag{1}$$

where $\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and M is a constant mass and α^2 is the confinement strength. In

RHM (Khadkikar and Gupta 1983) the quark wave function (ψ) is given by:

$$\psi = \mathrm{N}\begin{bmatrix} \chi \\ \dfrac{\sigma\cdot\mathbf{P}}{E+M} & \chi \end{bmatrix}, \tag{2}$$

where,

$$\mathrm{N} = \left(\frac{2(E+M)}{3E+M}\right)^{1/2}. \tag{3}$$

Here it should be noted that E is an eigen-value of the single particle Dirac equation with the interaction given in (1). We perform a similarity transformation to eliminate the small component such that,

$$U\psi = \begin{pmatrix} \chi \\ 0 \end{pmatrix}, \tag{4}$$

where U is given by

$$\frac{1}{\mathrm{N}\left[1+\dfrac{\mathbf{P}^2}{(E+M)^2}\right]}\begin{bmatrix} 1 & \dfrac{\sigma\cdot\mathbf{P}}{E+M} \\ \dfrac{-\sigma\cdot\mathbf{P}}{E+M} & 1 \end{bmatrix}. \tag{5}$$

here U is a momentum and state (E) dependent transformation operator. With this χ satisfies the 'harmonic oscillator' wave equation

$$\left(\frac{\mathbf{P}^2}{E+M} + \alpha^2 r^2\right)\chi = (E-M)\chi. \tag{6}$$

Any operator (O) acting on ψ is then transformed to $\tilde{O}$ acting on χ

$$\tilde{O} = (U^+)^{-1} O U^{-1}. \tag{7}$$

Generalization to many quarks is trivial. The Hamiltonian is transformed to that acting on χ and then extended to six quark systems inclusive of centre of mass (CM) correction:

$$\tilde{H}_1 = \sum_{i=1}^{6} \frac{\mathbf{P}_i^2}{E+M} - \mathbf{K}_G - \sum_{i<j} \alpha'^2 \mathbf{r}_{ij}^2 \lambda_i\cdot\lambda_j + \sum_{i<j} \tilde{V}_{ij}^{\mathrm{COGEP}}, \tag{8}$$

where $(E+M)/2$ is the dynamic effective mass of the quark, $\mathbf{K}_G (= \mathbf{P}_c^2/6(E+M))$ is the kinetic energy for the CM motion of the total system and $\mathbf{r}_{ij}$ is the distance between quarks i and j. The α'^2 is the confinement strength. The RHM (Khadkikar and Gupta 1983) would be equivalent to the NRQM where the constituent masses for the various operators have to be defined according to the transformation eq. (7). So, the 'constituent mass' for kinetic energy is $(E+M)/2$. Thus we have an exactly equivalent non-relativistic Hamiltonian which we can use as in NRQM (Shimizu 1989). However it should be noted that the Hamiltonian given in (8) will be evaluated in static approximation only. In this approximation the momentum dependent terms in

$\tilde{V}^{\text{COGEP}}$ are dropped and the norm of the wave function (χ) is unity when they belong to the same nucleon. The one-body kinetic energy term is related to the original one-body Dirac operator (eq. (6)) by removal of 'center of mass' of oscillator and supplying the $-\lambda_i \cdot \lambda_j$ factor with appropriate redefinition of α'^2 so the eq. (6) and one body part of eq. (8) give identical expression for intrinsic average nucleon-delta energy. However α'^2 is varied later to take account of $\tilde{V}^{\text{COGEP}}$ on the nucleon. This results in a very small change in α'^2. We obtain below central part of the COGEP ($\tilde{V}^{\text{COGEP}}$).

The COGEP is obtained from the relativistic expression for the scattering amplitude for the quarks

$$M_{fi} = \frac{g_s^2}{4\pi} \bar{\psi}_i \gamma^\mu \frac{\lambda_i^a}{2} \psi_i \mathbf{D}_{\mu\nu}^{ab}(q) \bar{\psi}_j \gamma^\nu \frac{\lambda_j^b}{2} \psi_j, \tag{9}$$

here $\psi_{i/j}$ are the wave functions of the quarks, $D_{\mu\nu}^{ab} = \delta_{ab} D_{\mu\nu}$ are the CCM gluon propagators in momentum representation, $g_s^2/4\pi (= \alpha_s)$ is the quark-gluon coupling constant and λ_i is the colour SU(3)$_c$ generator of the ith quark.

For completeness we give a short description of the gluon propagators. In CCM (Khadkikar 1985; Khadkikar and Vinod Kumar 1987) propagators are obtained very simply using the properties of harmonic oscillator wave-functions as follows

$$D_1(\mathbf{r}, \mathbf{r}', E = 0) \equiv \left\langle \mathbf{r} \left| \frac{c}{2\mathbf{a}\cdot\mathbf{a}^+ + 3} \right| \mathbf{r}' \right\rangle = c \sum_{\{N\}} \frac{\psi_N^*(\mathbf{r}) \psi_N(\mathbf{r}')}{2N + 3}. \tag{10}$$

Here c corresponds to the CCM parameter. Transferring the source point ($\mathbf{r}'$) to origin we obtain ($\mathbf{r} - \mathbf{r}' \Rightarrow \mathbf{r}$)

$$D_1(\mathbf{r}, 0, E = 0) \equiv D_1 = c \sum_{\{N\}} \frac{\psi_N(\mathbf{r}) \psi_N(0)}{2N + 3} = \frac{\Gamma_{3/4}}{(4\pi)^{3/2}} c(cr)^{-3/2} W_{0;-1/4}(c^2 r^2). \tag{11}$$

Similarly

$$D_0(\mathbf{r}, 0, E = 0) \equiv D_0 = c \sum_{\{N\}} \frac{\psi_N(\mathbf{r}) \psi_N(0)}{2N + 1} = \frac{\Gamma_{1/4}}{(4\pi)^{3/2}} c(cr)^{-3/2} W_{1/2;-1/4}(c^2 r^2), \tag{12}$$

where W's are Whittaker functions $\left(\sim \frac{\bar{e}^{(rc)^2/2}}{r} \right)$.

The complete propagators are given by

$$D_{00}(r) = 4\pi D_0(r), \tag{13}$$

where $D_0(r)$ is given by (12). The $D_{ik}(r)$ is given by,

$$D_{ik}(r) = -4\pi \left\{ \delta_{ik} - \frac{a_i^+ a_k}{\mathbf{a}\cdot\mathbf{a}^+} \right\} D_1(r), \tag{14}$$

where D_1 is given by (11).

It should be noted that these propagators are similar to those given by (Feynmann *et al* 1976) apart from the time coordinate which is suppressed here.

Using (7) the scattering amplitude (De Rujula *et al* 1975) becomes,

$$M_{fi} = \frac{\alpha_s}{2} N^4 \chi_i^+ \chi_j^+ \tilde{U}(\mathbf{P}_i, \mathbf{P}_j, \mathbf{q}) \chi_i \chi_j \lambda_i \cdot \lambda_j, \tag{15}$$

where N is given by (3) and $\tilde{U}(\mathbf{P}_i, \mathbf{P}_j, \mathbf{q})$ is the particle interaction operator in the momentum representation. By taking the Fourier transform of each term we get the potential operator in the coordinate space. The central part of the COGEP in the static limit is

$$\tilde{V}_{i,j}^{\text{COGEP}} = \frac{\alpha_s N^4}{2} \lambda_i \cdot \lambda_j \Big[D_0(r) + \frac{1}{(E+M)^2} (4\pi\delta^3(r) - c^4 r^2 D_1(r))(1 - 2/3\sigma_i \cdot \sigma_j) \Big]. \tag{16}$$

Here, we have ignored higher order terms in $\mathbf{a}$ and $\mathbf{a}^+$ in the spirit of Fermi-Breit interaction. In addition to the above interaction, full COGEP has the usual tensor, Darwin and orbit-orbit terms.

The parameters in our model b, E, M, α'^2, where b is the oscillator size parameter, are chosen in RHM (Khadkikar and Gupta 1983) to give reasonable values for the root mean square charge radius and the magnetic moments of the nucleon. The parameters E and M are fitted from hadron spectroscopy. The confinement strength α'^2 is fixed by the stability condition for the nucleon mass against the variation of the size parameter b.

$$\frac{\partial}{\partial b} \langle N|H|N \rangle = 0. \tag{17}$$

In (16) $c(\text{fm}^{-1})$ gives the range of propagation of gluons and is fitted in CCM (Khadkikar and Vinod Kumar 1987) to obtain the glueball spectra. The parameters used in our calculation are listed in table 1.

We can now employ the exactly analogous formulation of NRQM (Oka and Yazaki 1981; Faessler *et al* 1982) and calculate the scattering phase-shifts as functions of energy using Kohn-Hulten-Kato variational principle (Kamimura 1977).

Using the parameters in table 1 we searched for the single parameter α_s consistent with other parameters for both $^1S_0$ and $^1P_1$ to obtain agreement with experimental phase shifts. We obtain $\alpha_s = 2{\cdot}80$. The energy dependence of the phase shifts is remarkably reproduced in both the cases. The plots for the $^1S_0$ and $^1P_1$ phase shifts are given in figures 1 and 2 in comparison with the experiment (Arndt *et al* 1983). The

Table 1. Parameters used in our calculation.

$b = 0{\cdot}86$ fm
$M = 160{\cdot}6$ meV
$\alpha_s = 2{\cdot}8$
$\alpha'^2 = 15{\cdot}01$ meV fm^{-2}
$E = 428{\cdot}65$ meV
$c = 1{\cdot}80$ fm^{-1}

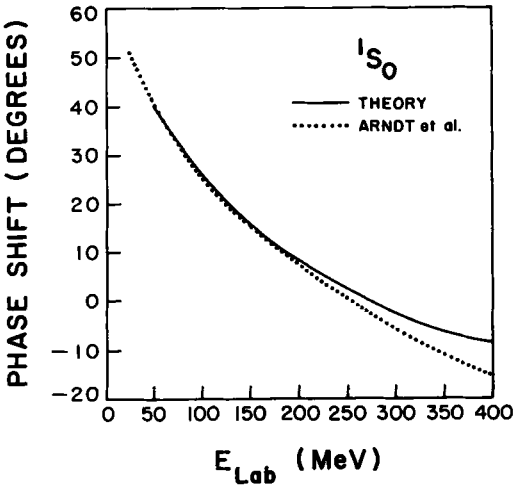

Figure 1. $^1S_0$ N-N phase-shifts as a function of the laboratory energy. The solid curve is the theoretical calculation and the dotted curve represents the experimental phase-shifts (Arndt *et al* 1983).

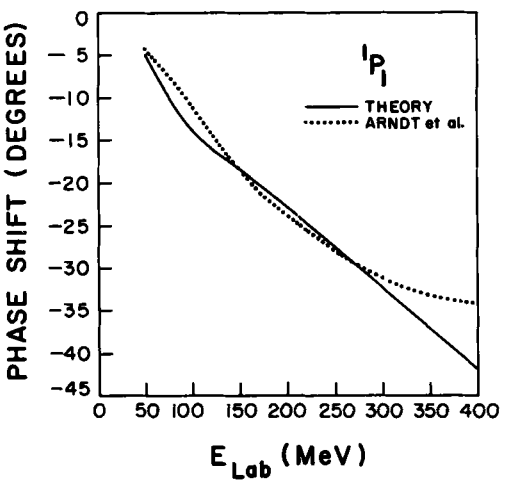

Figure 2. $^1P_1$ N-N phase-shifts as a function of the laboratory energy. The solid curve is the theoretical calculation and the dotted curve represents the experimental phase-shifts (Arndt *et al* 1983).

hard core radius (r_c) for $^1S_0$ is given by: $r_c = -\partial\delta(k)/\partial k|_{k=0}$, and is found to be 0·621 fm.

To understand the nature of the potential we have plotted the diagonal kernels of the N-N potential ($V_{\alpha\alpha}$) for both $^1S_0$ and $^1P_1$ against relative distance (**R**) between the nucleons (see figures 3 and 4). The adiabatic potential calculated in Born-Oppenheimer approximation is defined as,

$$V_{\alpha\beta} = \langle\psi_\alpha(\mathbf{R})|H|\psi_\beta(\mathbf{R})\rangle - \langle\psi_\alpha(\infty)|H|\psi_\beta(\infty)\rangle \tag{18}$$

where H is the Hamiltonian in (8) and $\psi_\alpha(\mathbf{R})$ is the normalized wavefunction of two

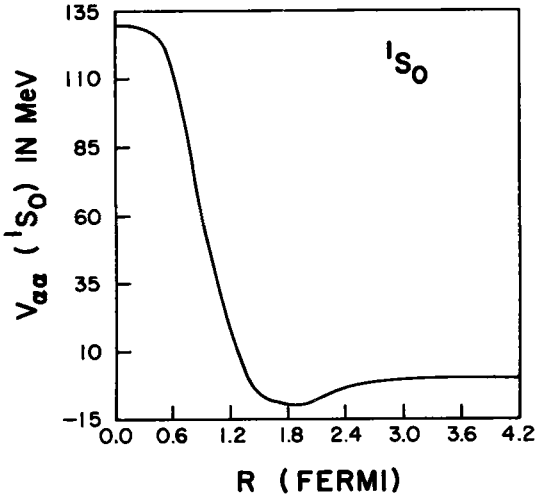

Figure 3. The adiabatic potential defined in (18) for the $^1S_0$ N-N potential as a function of R(fm).

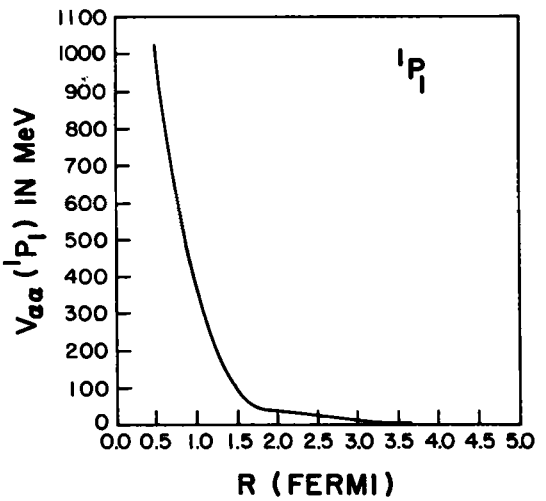

Figure 4. The adiabatic potential defined in (18) for the $^1P_1$ N-N potential as a function of R(fm).

nucleon system in a state α separated by a distance **R**.

$$\psi_\alpha(R) = \mathbf{A}N[\phi_A(1,2,3,\mathbf{R}/2)\phi_B(4,5,6,-\mathbf{R}/2)]_\alpha, \tag{19}$$

where **A** is the totally anti-symmetrizing operator for the six-quark system (Shimizu 1989), and $\mathbf{R}/2$ and $-\mathbf{R}/2$ are the origin of two clusters A and B respectively.

At short distances the exchange kernels of $\delta(r)$ dominate over the exchange kernels of $c^4r^2D_1(r)$ thus providing the short range repulsion. But in the intermediate and long ranges the exchange kernels of $c^4r^2D_1(r)$ dominate over the exchange kernels

of $\delta(r)$ thus providing the intermediate and long range attraction. The color electric terms in COGEP and the confinement potential of the quarks do not play any important role in N-N scattering (Shimizu 1989). It should be noted that due to dependence on $\lambda_i \cdot \lambda_j$, only the exchange term of the matrix elements contribute to N-N interaction (Shimizu 1989). The exchange kernels being non-local contribute significantly to the N-N interaction. Though the height of the adiabatic potential for $^1S_0$ is only about 130 MeV, the non-local kernels which are taken into account in our calculation are able to explain the N-N scattering data satisfactorily (see figures 1 and 3). Figure 4 is a plot of the diagonal kernels of N-N potential *vs* **R** for $^1P_1$ case. The potential is totally repulsive, as expected the phase-shifts are negative (see figures 2 and 4). The significant repulsion in the potential at short distance arises from the centrifugal barrier.

In conclusion, we have investigated the central part of N-N interaction within the COGEP. The COGEP is obtained starting from the relativistic model for quarks (RHM) making use of the confined gluon (CCM) propagators. With a single free parameter α_s we are able to obtain good agreement with the experimental phase-shifts in both $^1S_0$ and $^1P_1$ channels. In our calculation the term in COGEP arising out of confinement of gluons is responsible for the required attraction in the N-N interaction. This clearly shows that the confinement of gluons plays an important role in deriving N-N interaction starting from the quark and the gluon degrees of freedom.

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