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# Higher-dimensional white holes

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**Abstract.** White holes (relativistic anticollapsing objects) are considered both in the five-dimensional Kaluza-Klein theory and in the many-dimensional representation of extended space-time manifolds with horizons within the framework of general relativity. In the last model white holes (e.g. Kerr-Newman) appear as anticollapsing from extra (additional) dimensions. These dimensions are connected with the global structure of space-time manifolds.

**Keywords.** White holes; higher dimensions; extended manifold.

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## 1. Introduction

The problem of space-time dimensionality, in general, is a very old one, but it acquired new sense after creation of general relativity (GR). The development of theories of fundamental interactions including gravitation has shown in the last decade that the inclusion of notions on higher-dimensional space-time was rather attractive. The majority of work in this direction belongs to the some development of the Kaluza-Klein (KK) idea about extra dimensions compactified on very small scales (Bailin and Love 1987; Emel'yanov *et al* 1986; Mecklenburg 1984).

Many-dimensional theories with complex space-time occupy a separate place in this field when there is no necessity in any compactification of higher (extra) dimensions, though these versions are not the most representative in a number of publications (Flaherty 1980; Pavšič 1985; Barut and Pavšič 1988; McIntosh and Hickman 1985; Gurin and Trofimenko 1985, 1986, 1988; Chandola *et al* 1986).

We would like to note the usage of complex space-time in order to work out the euclidean quantum theory of gravitation (Hawking 1979), in the procedure of complexification in exact solutions of GR (Newman *et al* 1965), the approach of the formal introduction of complex manifolds in GR (McIntosh and Hickman 1985; Brown 1966; Das 1966), and the application of complex variables in the Penrose twistor theory (Penrose 1976). There are a number of other propositions for higher-dimensional physics and GR (Vladimirov 1987), but it is cumbersome to mention all of them in the present work.

Meanwhile, it is worth studying, in theories with higher dimensions, different objects and solutions which are well investigated within the framework of conventional GR.

Black holes (BH)* are among the class of such objects which are inherent to Einstein's relativity. There exists much work up-to-date (Gibbons 1982, 1985; Gibbons and Wiltshire 1986; Myers 1987; Myers and Perry 1986; Frolov *et al* 1987; Davidson and Owen 1985; Mazur and Bombelli 1987; Dianyan 1988) devoted to the analysis of geometry and physical properties of BH-like objects in KK theories. In other versions of higher-dimensional theories (e.g., Flaherty 1980; Pavšič 1981a, b; McIntosh and Hickman 1985) similar objects have been investigated considerably less.

It is interesting to point out that occurrence of peculiarities in BH space-time such as event horizons leads to the divergence of metrical coefficients of the additional coordinates near these horizons, and these KK dimensions become "visible" (Davidson and Owen 1985). However, such a non-singular (pseudosingular) horizon is obtained with many restrictions (Sokolowski and Carr 1986). In the present work we shall consider objects which are antipodes of black holes–white holes, i.e. BHs reversed in time (Narlikar *et al* 1974; Narlikar and Apparao 1975; Narlikar and Kapoor 1978; Lake and Roedèr 1976, 1978; Novikov and Frolov 1987). In contrast to BH called "windows to higher dimensions" (Davidson and Owen 1985), white holes (WH) can be called, as it will be shown below, "windows from higher dimensions".

In conventional GR, BH and WH appear naturally in the complete space-time manifolds with horizons (e.g. the Kerr-Newman manifold and particular cases), however, the problem of WH stability is a special question and it will not be considered here. The connection between different regions of the global complete manifold (universes) occurs in the process of collapse-anticollapse of a relativistic gravitating object. In the light of the many-dimensional interpretation, such processes, can be considered as the connection between different dimensions of global space-time. We shall generalize the existing WH model known in the four-dimensional theory (Narlikar *et al* 1974; Narlikar and Apparao 1975) to the five-dimensional KK-type theory. The further generalization to higher dimensionality can be easily carried out if additional dimensions are represented as some $S^n$-sphere (it is the most frequent assumption).

## 2. Kaluza-Klein white hole

The model of a non-rotating uncharged WH is constructed on the basis of matching an anticollapsing matter metric with an exterior region, which, according to the Birkhoff theorem, must be the Schwarzschild spacetime. In the five-dimensional theory the Schwarzschild analogue (Davidson and Owen 1985) in the isotropic form is described as

$$\mathrm{d}s^2 = -A^2\mathrm{d}T^2 + B^2\mathrm{d}X^i\mathrm{d}X^i + C^2\mathrm{d}Y^2, \tag{1}$$

where

$$A = A(r) = \left(\frac{ar-1}{ar+1}\right)^{k\varepsilon}, \tag{2}$$

* Black holes (BH) belong to the more wide class of relativistic objects with event horizons called "otons" from the Russian abbreviation of "general theory of relativity" – "OTO" (Zeldovich and Novikov 1971; Trofimenko and Gurin 1985).

$$B = B(r) = \frac{(ar+1)^{\varepsilon(k-1)+1}}{a^2 r^2 (ar-1)^{\varepsilon(k-1)-1}}, \tag{3}$$

$$C = C(r) = C_0 \left(\frac{ar+1}{ar-1}\right)^{\varepsilon}, \tag{4}$$

$a$, $\varepsilon$, and $k$ are constants connected by the relation

$$\varepsilon^2(k^2 - k + 1) = 1. \tag{5}$$

We choose the following particular case $\varepsilon = 1 \Rightarrow k = 1$, that satisfies the restriction on $\varepsilon : |\varepsilon| \leqslant 2/\sqrt{3}$ on physical grounds. Then the metric coefficients of the KK additional dimensions are given by

$$C = C_0(ar+1)/(ar-1) = C_0(r/r_g + 1)(r/r_g - 1)^{-1}, \tag{6}$$

where the gravitational radius in the isotropic coordinates is $r_g = a^{-1}$. Transforming $r \to l : l = (1 + ar)^2 r$, we come to the usual form of the Schwarzschild-like metric coefficient

$$C = (1 - 2M/l)^{-1}, \quad r_g = 2M \tag{7}$$

and omit $C_0$ without a loss of generality.

Thus the metric coefficient of the additional coordinate coincides with that of the radial coordinate, but the $Y$ coordinate itself does not appear in it, and its peculiarity at $r_g$ provides the peculiarity of the additional dimension.

As the interior metric, similar to those in the canonical WH model (Narlikar *et al* 1974; Narlikar and Apparao 1975), we use the Friedmann–Robertson–Walker metric, and its five-dimensional generalization (Gibbons and Wiltshire 1986)

$$ds^2 = -dt^2 + R^2(t)(1 + \tfrac{1}{4}qx^2)^{-2} dx^i dx^i + b(t)^2 dy^2. \tag{8}$$

In such a geometry the following solution of the Einstein equations is obtained for dust-like matter (Davidson and Owen 1985) with $\Lambda = 0$:

$$R^2(t) = -qt^2 + ct + d \tag{9}$$

$$b(t) = dR/dt = \tfrac{1}{2}(c - 2q)(-qt^2 + ct + d)^{-1/2}, \tag{10}$$

where $c$ and $d$ are constants determined by the initial data.

Evidently, the matching of the four-dimensional part in this case does not distinguish it at all from the usual one in the canonical model of a WH, which is performed for the Schwarzschild and Friedmann space-times at the boundary of expanding (or contracting) matter: $l = l_b = x_b S(t)$, $0 \leqslant S(t) \leqslant 1$. The matching of the five dimensions, evidently, can be carried out according to the relation

$$(ar+1)(ar-1)^{-1} dY = b(t) dy \tag{11}$$

which leads to the following connection of the fifth coordinates:

$$\frac{dY}{dy} = \frac{(c-2q)(ar-1)}{2(-qt^2 + ct + \mathrm{d})^{1/2}(ar+1)} \tag{12}$$

or, by the transformation to the Schwarzschild analogue with $l$

$$\frac{\mathrm{d}Y}{\mathrm{d}y}=\frac{(c-2q)(1-2M/l)^{1/2}}{2(-qt^2+ct+\mathrm{d})^{1/2}}. \tag{13}$$

At the horizon $l=2M$ (13) shows that $Y$ does not depend on $y$.

Let us analyze now the space-time structure of such a five-dimensional WH model by means of the Penrose diagram. Since for this purpose one uses the two-dimensional section of a metric including peculiarities of the space-time structure under consideration and every point on the diagram corresponds to $S^2$, we omit the spatial dimensions of the usual 4-dimensional sector and take into account only the temporal and the fifth (additional) coordinates. An immediate generalization to four dimensions in the form of a similar diagrammatic representation is rather difficult (Gurin 1986). The two-dimensional approach permit us to analyze the influence of additional dimensions on the causal structure, though the coefficient at the spatial coordinate is also singular at the horizon. Thus, our initial metric

$$\mathrm{d}s^2=-(1-2M/l)\mathrm{d}T^2+(1-2M/l)^{-1}\mathrm{d}Y^2, \tag{14}$$

where $l$ is the spatial coordinate of 4-dimensional sector. Its appearance almost coincides with the usual 2-dimensional Schwarzschild case, differing only in that the metric coefficients are independent of the $Y$-coordinate; $l$ appears in it as a parameter.

We apply the standard transformation in order to obtain the conformal diagram

$$u=T-Y^*, \quad v=T+Y^*, \tag{15}$$

$$\mathrm{d}Y^*=(1-2M/l)^{-1}\mathrm{d}Y \tag{16}$$

leading to

$$u=\tfrac{1}{2}(T-2Y/(1-2M/l)), \quad v=\tfrac{1}{2}(T+2Y/(1-2M/l)), \tag{17}$$

and we get the Kruskal-like metric in the isotropic (null) $u$ and $v$ coordinates

$$\mathrm{d}s^2=-f(u,v)\mathrm{d}u\,\mathrm{d}v. \tag{18}$$

Further, the Penrose-like diagram can be constructed in the usual way (Hawking and Ellis 1973), and its appearance coincides with that of the Schwarzschild case apart from the sense of null lines (figure 1). The horizon surface $l=2M$, as can be concluded, is the time limit ($T=\pm\infty$) of the 4-dimensional Schwarzschild space-time,

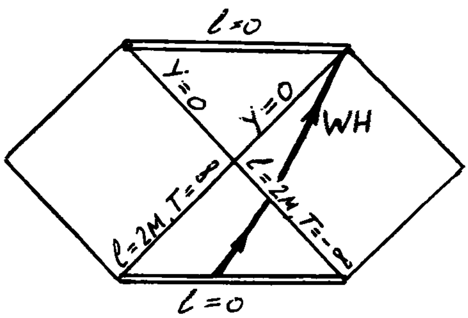


**Figure 1.** The conformal Penrose diagram for the spherically symmetric space-time in five-dimensional KK theory. $Y$ is the additional coordinate, $T$ is time (from the 4-dimensional space-time).

since at $l = 2M$ we have $Y = 0$, and the metric (14) degenerates into the usual Schwarzschild form after taking into account radial and angular parts.

Thus, the event horizon in the extended manifold appears as the place of beginning of additional coordinates, that can be intrinsic not only for this rather particular model. Anticollapsing objects (WH) crossing the event horizon in this process move from the region of the additional coordinate variation. For this movement we may indicate time-like geodesics as some model (figure 1).

## 3. Some comments on higher dimensions

This approach to the introduction of higher dimensions of the KK type is not the only possible one. Although KK-type models have been popular in recent years we note some others which are of some interest from the point of view of investigating space-time structure for black and white holes with event horizons. Event horizon is known to be an important property of space-time manifolds (Hawking and Ellis 1973).

We would like to mention the representation of complex space-time structure developed by the authors in a series of papers (Gurin 1984, 1985; Trofimenko and Gurin 1986b, 1987; Gurin and Trofimenko 1985, 1986, 1988). In such an approach the global space-time manifold can be considered as the product of Riemannian four-dimensional spaces $R^8 = R^4_{\mathrm{Re}} \otimes R^4_{\mathrm{Im}}$. Then, $R^4_{\mathrm{Re}}$ for example, is just a hypersurface in the global (complex) 8-dimensional manifold. This falls in line with similar conclusions of a generalized special relativity known as extended relativity (Recami 1986; Recami and Mignani 1975; Recami and Maccarrone 1980, 1984; Recami and Rodrigues 1982). We notice that such 'additional' dimensions are not "extra" ones in contrast to the KK schemes (see references in the introduction). In their appearance within the framework of extended relativity, especially in extended GR, we notice the genuine sense of the extended manifold theory; such 'additional' dimensions do not associate with any auxiliary fields.

Anticollapsing objects (white and grey holes) in such a representation can be described by the introduction of an 8-dimensional matching procedure of metric with matter and an exterior one (complex Schwarzschild, Reissner-Nordstrom, Kerr-Newman, etc.). For example, we can write for anticollapsing matter in the complex manifold the following form in comoving coordinates

$$g_{\mathrm{AB}} = \begin{pmatrix} -1 & & & \\ & g_{\mu\nu} R^2_{\mathrm{Re}} & & \\ & & -g_{ik} R^2_{\mathrm{Im}} & \\ & & & +1 \end{pmatrix} \tag{19}$$

where $R_{\mathrm{Re}}$ and $R_{\mathrm{Im}}$ have sense of expansion factors in the corresponding subspaces, $g_{\mu\nu}$ and $g_{ik}$ being space-like parts of the submanifolds. Anticollapsing bodies in such a context can be interpreted as windows from higher dimensions, since the matter emerges from inside their event horizon. Such anticollapsars are simulated by geodesics (figure 2) crossing different ($M_i$, $M_{-i}$, $M_{\pm}$) regions and horizons. The complete matching is attainable only numerically.

We would like to point out that the minimal dimensionality of the global manifold (in this interpretation) must be determined by the embedding class of the manifold

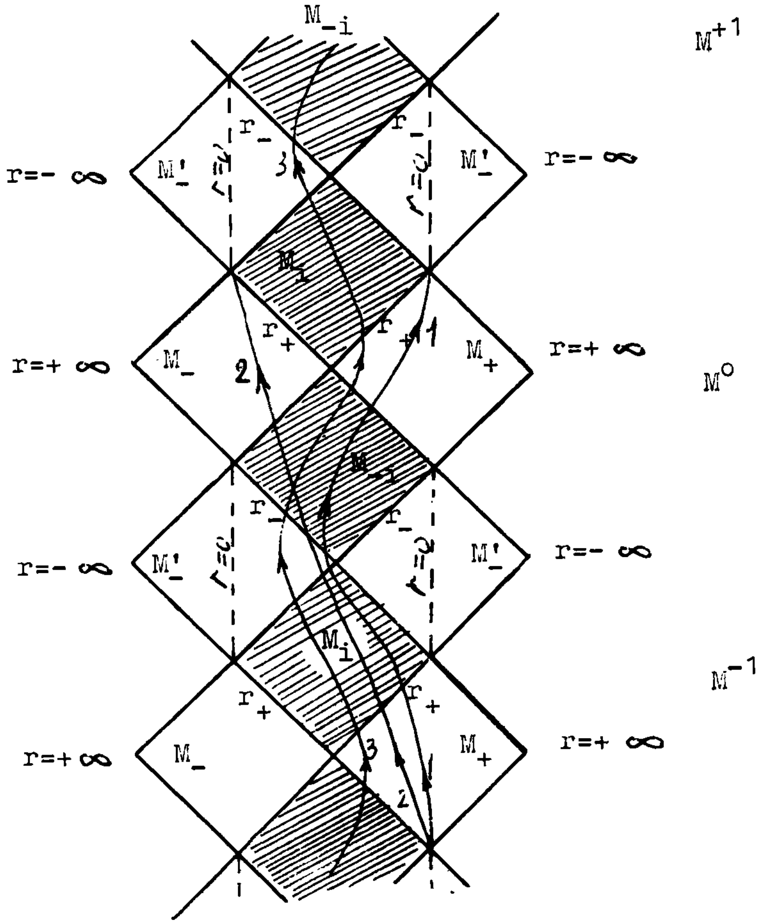


**Figure 2.** The conformal Penrose diagram for the Kerr-Newman extended manifold (along the symmetry axis). Curves show possible (time-like) geodesics corresponding to collapsing and anticollapsing motions. Lines passing horizons between different regions (subluminal and superluminal-dashed) can be interpreted as connection between higher dimensions (see text).

under consideration. For the Schwarzschild one it is equaled to six (Rosen 1965; Plazowski 1973), and both subluminal and superluminal versions of this metric can be embedded into the pseudoeuclidean 6-dimensional space (Gurin 1984). Taking into account the pseudoeuclidity of this space-time we ought to consider the possibility of solving the familiar problem of energy-momentum conservation in such many-dimensional representations of GR. $T_{AB}$ can be conserved in the global sense though in local four-dimensional space-time the conservation will be absent.

The Kerr-Newman space-time is embeddable in the 9-dimensional pseudoeuclidean space (Kuzeev 1980). The increase of the embedding class allows many pseudosingularities and multiplicity of different regions in the complex manifold. In the light of the present many-dimensional (complex) interpretation, the global Kerr-Newman (Kerr) manifold possesses the following structure

$$R^8 = R^4_{j=1} \otimes R^4_{j=2} \otimes \dots \otimes R^4_{j=n}, \tag{20}$$

where $j$ is the number of subspace, and Im-coordinates are varied in the region between horizons ($r_{\rm Im}$) or between infinite shift surfaces ($t_{\rm Im}$).

Before closing, we would like to point out a number of possible astrophysical manifestations of anticollapsars (Trofimenko and Gurin 1989), which must have certain features due to higher-dimensionality. In the light of the present model and in the model of 'retarded cores' (Novikov 1964; Ne'eman 1965) the main features of WH are rather similar: localization in small spatial regions, transientness, gigantic energy extractions, big blue shift, etc. (Novikov and Frolov 1987; Dadhich 1977; Narlikar *et at* 1974; Narlikar and Apparao 1975; Narlikar and Kapoor 1978; Trofimenko and Gurin 1989). Specific manifestations and consequence of a many-dimensional interpretation of anticollapsing objects are the subject of further studies by the authors and will be published elsewhere.

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