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Effect of the three-body force on triton asymptotic normalization constants by the hyperspherical harmonics expansion method

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Abstract. We investigate the change in the calculated value of asymptotic normalization constant (ANC) by the hyperspherical harmonics expansion method with the inclusion of three nucleon force (3BF) in addition to two nucleon force. We see that ANC does not change very much with the inclusion of 3BF indicating that the 3BF does not alter the asymptotic behaviours of HHE wavefunction significantly.

Keywords. Asymptotic normalization constant; hyperspherical harmonics expansion; three body force.

PACS Nos 21·40; 25·10

1. Introduction

Asymptotic normalization constants (ANC) are important properties of the trinucleon system and can be extracted from experimental results (Plattner *et al* 1977; Bornand *et al* 1978; Giruad and Fuda 1979). The experimental value of C_0^2 (where C_0 is the S-wave ANC) varies widely with the method of measurement and ranges from 2·6 to 3·3 with an average error of $\pm 0·3$. The measured value of the ratio C_2/C_0 (where C_2 is the D-wave ANC) ranges from 0·048 to 0·051 with an average error of 0·007. These constants depend on the asymptotic nature of the trinucleon wave function and a knowledge of this quantity gives an idea of the asymptotic behaviour of the wave function. A number of calculations of ANC have been reported (Kim and Muslim 1979; Harper *et al* 1980; Sasakawa *et al* 1980; Benayoun *et al* 1981; Friar *et al* 1982, 1988; Ishikawa and Sasakawa 1986) based on Faddeev method for trinucleon bound system. These results agree fairly well with the experimental numbers, indicating reasonably correct asymptotic behaviour of the Faddeev wave functions. An alternative method for the treatment of the trinucleon bound states is the hyperspherical harmonic expansion (HHE) method (Ballot and Fabre 1980), in which the wave function is expanded in a complete basis of hyperspherical harmonics (HH) functions spanning the hyper angular space. Binding energy (BE), charge radius, charge form factor etc calculated by the HHE method (Ballot and Fabre 1980; Erens *et al* 1971; Demin *et al* 1973; Coelho *et al* 1982; Das and Coelho 1982a, b; Das *et al* 1982c) agree fairly well with those calculated by the Faddeev method. However no calculation of ANC by HHE method has so far been reported by other groups. The HHE method is basically a variational method satisfying the Rayleigh-Ritz principle. However, convergence of the binding energy (BE) does not necessarily guarantee convergence of the asymptotic form of the wave function, since the major contribution to the BE comes from near the minimum of the effective hyperradial potential well, where the wave function has a maximum. Thus a variational calculation guarantees a convergence of the form of the wave function near its maximum, while the convergence of the asymptotic form remains an open question. This motivated us to calculate ANC of the trinucleon system by the HHE method, hoping to shed light on the reliability and convergence behaviour of the asymptotic form of the HHE wave function. We have already calculated ANC for ^{3}H and studied its convergence behaviour (Ghosh and Das 1990a) confirming our contention that the convergence (with respect to the number of hyperspherical partial waves contributing to the wave function) of BE is much faster than that of ANC. This convergence is worse for potentials having relatively longer range.

In this communication, we investigate the change in the calculated value of ANC with the inclusion of three nucleon force (3BF) in addition to two nucleon force for the triton. This can shed light on the effect of 3BF on the asymptotic form of the wave function. We see that ANC does not change very much with the inclusion of the 3BF indicating that the 3BF does not alter the asymptotic behaviours of HHE wave function significantly. A similar calculation for ^{3}He will not be included since it is not expected to reveal additional information in view of the negligible role of 3BF on the triton ANC.

We use Fujita–Miyazawa 3BF (Fujita and Miyazawa 1957) as the three nucleon force in triton wave function (De and Das 1988a; De 1988).

The paper is organized as follows. We discuss the theory in § 2 and the results are presented in § 3. Conclusions have been discussed in § 4.

2. Theory

The asymptotic normalization constants C_0 and C_2 for S and D partial waves of the triton wave function is obtained by comparing the triton wave function obtained by HHE method with the direct product of asymptotically free neutron with deuteron wave function i.e. in the limit $y \to \infty$ (figure 1). Jacobi coordinates **x** and **y** are shown in figure 1 for the triton; when particle 1 is a proton and the pair (12) forms the deuteron with relative coordinate **x**. We define a function

$$f_\ell(y) = \langle [\Phi_d(\mathbf{x}) \otimes {}^2\mathscr{\ell}_j(\hat{y})]^{(1/2)} | \Psi_{3_\mathrm{H}}(\mathbf{x}, \mathbf{y}) \rangle \tag{1}$$

where $\Phi_d(\mathbf{x})$ is the deuteron wave function including isospin part and ${}^2\ell_j(\hat{y})$ represents the third particle (spectator) spinangular-isospin function (representing the asymptotically free neutron of spin 1/2, orbital angular momentum relative to the centre of mass of the deuteron ℓ and total angular momentum j) and $\Psi_{3_\mathrm{H}}(\mathbf{x}, \mathbf{y})$ is the fully antisymmetric triton wave function, taking into consideration the contributions of both 2BF and 3BF. The integrations in eq. (1) are over **x** and over the orientations of **y**. The asymptotic behaviours of $f_\ell(y)$ are given by (Friar *et al* 1982).

$$f_0(y) \xrightarrow[y \to \infty]{} C_0 N_{ZR} \frac{\exp(-\beta y)}{y} \tag{2}$$

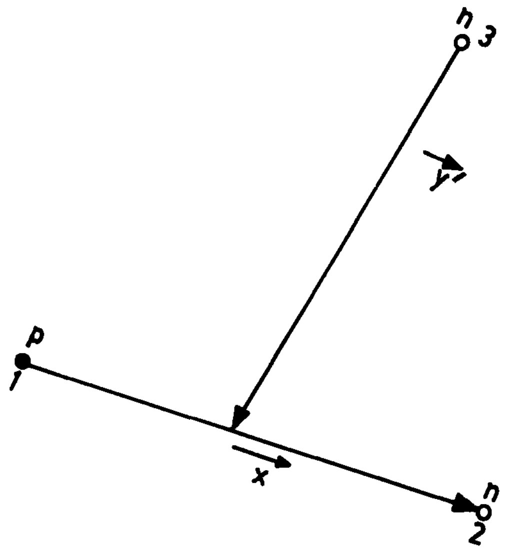

Figure 1. Jacobi coordinates for the triton. The shaded particle is a proton ($\mathbf{y} = (2/\sqrt{3}\mathbf{y}')$.

$$f_2(y) \xrightarrow[y\to\infty]{} C_2 N_{ZR} \frac{\exp(-\beta y)}{y}\left[1 + \frac{3}{\beta y} + \frac{3}{\beta^2 y^2}\right] \tag{3}$$

where

$$N_{ZR} = \sqrt{2\beta}$$

and

$$\beta = [(4M/3\hbar^2)(B_T - B_D)]^{1/2}$$

B_T and B_D are the triton and deuteron binding energies respectively.

The HH expansion of triton wave function $\Psi_{^3\mathrm{H}}(\mathbf{x},\mathbf{y})$ in the optimal subset approximation is given by (Ballot and Fabre 1980)

$$\begin{aligned}\Psi_{^3\mathrm{H}}(\mathbf{x},\mathbf{y}) = {} & \Gamma_{\frac{1}{2}\frac{1}{2}}(\mathrm{A}) \sum_K P^{(0)}_{2K}(\Omega) r^{-5/2} \mathscr{U}^{(s)}_{2K}(r) \\ & + \sum_K \frac{1}{\sqrt{2}} \{\Gamma_{\frac{1}{2}\frac{1}{2}}(M-) P^{(+)}_{2K}(\Omega) - \Gamma_{\frac{1}{2}\frac{1}{2}}(M+) P^{(-)}_{2K}(\Omega)\} r^{-5/2} \mathscr{U}^{(s')}_{2K}(r) \\ & + \sum_K \frac{1}{\sqrt{2}} \{{}^{(+)}D_{2K+2}(\Omega) |(0\tfrac{1}{2})\tfrac{1}{2}, -\tfrac{1}{2}\rangle^{(T)} \\ & + {}^{(-)}D_{2K+2}(\Omega) |(1\tfrac{1}{2})\tfrac{1}{2}, -\tfrac{1}{2}\rangle^{(T)}\} r^{-5/2} \mathscr{U}^{(D)}_{2K+2}(r)\end{aligned} \tag{4}$$

where $\Gamma_{TS}(R)$ is a three body isospin-spin function of total isospin, spin and symmetry component T, S and R respectively (Ballot and Fabre 1980) and $|(t\frac{1}{2})\ T, M_T\rangle^{(T)}$ is the three-body isospin wave function (with total isospin and its projection are T and M_T respectively), t being the isospin of (12) pair. The hyperangular functions $P^{(\varepsilon)}_{2K}(\Omega)$ with $\varepsilon = +, -$ or 0 and ${}^{(\varepsilon)}D_{2K+2}(\Omega)$ with $\varepsilon = +$ or $-$ having specified symmetry under particle exchange are given in Ballot and Fabre (1980) and will not be reproduced here. The symbol Ω represents a collection of five hyperangles constituted by the polar angles (θ_x, ϕ_x) and (θ_y, ϕ_y) of x and y respectively and a hyperangle ϕ defined through $\phi = \tan^{-1}(x/y)$. The hyperradius $r = (x^2 + y^2)^{1/2}$ is invariant under all permutations and three dimensional rotations. The hyperradial partial waves for the S, S' and D state of triton are $\mathscr{U}^{(S)}_{2K}(r)$, $\mathscr{U}^{(S')}_{2K}(r)$ and $\mathscr{U}^{(D)}_{2K+2}(r)$ respectively.

Substitution of the form (4) in the Schrödinger equation and projection on a particular HH gives rise to a system of coupled differential equation (CDE) of the form (Das *et al* 1982c; De and Das 1988a; De 1988)

$$\left[-\frac{\hbar^2}{m}\frac{d^2}{dr^2} + \frac{\hbar^2}{m}\frac{\mathscr{L}_K(\mathscr{L}_K+1)}{r^2} - E\right]\mathscr{U}^{(\mathscr{R})}_{2K+\ell}(r)$$

$$+ \sum_{K',K'',\mathscr{R}'} [V_{2K''}(r) + W_{2K''}(r)] \langle K, \mathscr{R} | K'' | K', \mathscr{R}' \rangle \mathscr{U}^{(\mathscr{R}')}_{2K'+\ell}(r) = 0 \qquad (5)$$

where $\mathscr{L}_K = 2K + \ell + \frac{3}{2}$ and $V_{2K''}(r)$ and $W_{2K''}(r)$ are the 2BF and 3BF potential multipoles respectivley and $\langle K\mathscr{R}|K''|K'\mathscr{R}'\rangle$ is the geometrical structure coefficient (De and Das 1987; Das and De 1987).

With the expression (4) for $\Psi_{^3\mathrm{H}}(\mathbf{x}, \mathbf{y})$ we calculate $f_\ell(y)$ from eq. (1) in a straightforward manner and is given by

$$f_0(y) = \int_0^\infty \left[\frac{\omega_0(x)}{\sqrt{2}}\left\{\sum_K\left(\frac{1}{\sqrt{2}}\mathscr{U}^{(S')}_{2K}(r)^{(+)}F^{00}_{2K}(\pi/2)^{(+)}N_{2K}\right.\right.\right.$$

$$\left.\left. - \mathscr{U}^{(S)}_{2K}(r)^{(0)}N_{2K}{}^{(0)}F^{00}_{2K}(\pi/2)\right)^{(2)}P^{0,0}_{2K}(\phi)\right\}$$

$$\left. + \frac{\omega_2(x)}{\sqrt{2}}\left\{\sum_K \mathscr{U}^{(D)}_{2K+2}(r)^{(+)}N_{2K+2}{}^{(+)}F^{0,2}_{2K+2}(\pi/2)^{(2)}P^{0,2}_{2K+2}(\phi)\right\}\right]$$

$$\times r^{-5/2}\mathrm{d}\mathbf{x} \qquad (6)$$

$$f_2(y) = \int_0^\infty \left[\frac{\omega_0(x)}{\sqrt{2}}\sum_K\left\{\mathscr{U}^{(D)}_{2K+2}(r)^{(+)}N_{2K+2}{}^{(+)}F^{2,0}_{2K+2}(\pi/2)^{(2)}P^{2,0}_{2K+2}(\phi)\right\}\right.$$

$$+ \frac{\omega_2(x)}{2}\sum_K\left\{-\sqrt{2}\,\mathscr{U}^{(S)}_{2K}(r)^{(0)}N_{2K}{}^{(0)}F^{2,2}_{2K}(\pi/2)^{(2)}P^{2,2}_{2K}(\phi)\right.$$

$$+ \mathscr{U}^{(S')}_{2K}(r)^{(+)}N_{2K}{}^{(+)}F^{2,2}_{2K}(\pi/2)^{(2)}P^{2,2}_{2K}(\phi)$$

$$\left.\left. - \mathscr{U}^{(D)}_{2K+2}(r)^{(+)}N_{2K+2}{}^{(+)}F^{2,2}_{2K+2}(\pi/2)^{(2)}P^{2,2}_{2K+2}(\phi)\right\}\right] r^{-5/2}\mathrm{d}\mathbf{x} \qquad (7)$$

where $\omega_\ell(x)$ is the radial deuteron S and D partial waves for $\ell = 0$ and 2 respectively; hyperspherical function ${}^{(2)}P^{\ell_1,\ell_2}_L(\phi)$, normalization constants ${}^{(\varepsilon)}N_L$ and symmetrizing factors ${}^{(\varepsilon)}F^{\ell_1,\ell_2}_L$(with $\varepsilon = +, -$ or 0) are given in Ballot and Fabre (1980) and will not be reproduced here for brevity.

3. Results

We have chosen the Afnan-Tang S3 potential (Afnan and Tang 1968) as well as the 'EXP I' and 'EXP II' potential (Dzyuba *et al* 1971) for the two body interaction (2BF).

Table 1a. Parameters of S3 potential.

	A_1(MeV)	B_1(fm)	A_2(MeV)	B_2(fm)	A_3(MeV)	B_3(fm)
Triplet (T)	1000·0	0·5774	−326·7	0·9759	−43·0	1·291
Singlet (S)	1000·0	0·5774	−166·0	1·118	−23·0	1·5811

Table 1b. Parameters of EXP I and EXP II potentials.

Potential	V_{13}^0(MeV)	r_{13}(fm)	V_{31}^0(MeV)	r_{31}(fm)	V_1^t(MeV)	r_1^t(fm)
EXP I	107·97	0·710	150·93	0·598	74·705	0·890
EXP II	107·97	0·710	179·13	0·578	56·570	0·973

The three nucleon interaction (3BF) is the Fujita-Miyazawa (FM) force (Fujita and Miyazawa 1957). This 3BF is extremely singular ($\sim r^{-6}$) for $r \to 0$ and is attractive for the triangular configuration of the trinucleon system. Hence we regularize this 3BF by introducing a cut-off parameter x_0, below which 3BF takes its value at x_0. The S3 potential (Afnan and Tang 1968) is a superposition of Gaussians:

$$V^{(S,T)}(r) = \sum_{i=1}^{3} A_i^{(S,T)} \exp(-(r/B_i^{(S,T)})^2) \tag{8}$$

when S and T stand for singlet and triplet interactions respectively. The parameters $A_i^{(S,T)}$ and $B_i^{(S,T)}$ are given in table 1a.

EXP I and EXP II potentials (Dzyuba *et al* 1971) are sum of exponential terms and are given by

$$V^c(12) = \sum_{S,T=0,1} V^c_{2S+1,2T+1}(r) \frac{1-(-1)^T P_\tau(12)}{2} \cdot \frac{1-(-1)^S P_\sigma(12)}{2},$$

$$r = |\mathbf{r}_1 - \mathbf{r}_2| \tag{9a}$$

$$V^t(12) = \sum_{T=0,1} V^t_{2T+1}(r) \frac{1-(-1)^T P_\tau(12)}{2} S_{12} \tag{9b}$$

$$S_{12} = [3(\boldsymbol{\sigma}_1 \cdot \hat{n})(\boldsymbol{\sigma}_2 \cdot \hat{n}) - (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2)], \hat{n} = \frac{\mathbf{r}_1 - \mathbf{r}_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \tag{9c}$$

$$V_{ij}^{c,t}(r) = -V_{ij}^0 \exp(-r/r_{ij}). \tag{9d}$$

Here $i,j = 1$ or 3 corresponding to S(or T) = 0 or 1 where S and T are the total spin and total isospin of the two nucleon system. The quantities V^c and V^t are the central and tensor potentials. Parameters of these potentials are given in table 1b.

Since S3 potential has no tensor part, there are no D-state components in the ground state of either deuteron or triton. Consequently C_2 is identically zero for this interaction. However for the 'EXP I' and 'EXP II' potentials all the components of the ground state wave function exist. The radial deuteron wave functions are obtained by solving the two body Schrödinger equation by Runge-Kutta (RKGS) method (Ghosh and Das 1987) for S-projected S3 potential and renormalized Numerov method

(Johnson 1988; Ghosh and Das 1990b) for interactions including tensor terms (i.e. for EXP I and EXP II potentials). Plots of deuteron S and D radial wave functions, $\omega_0(x)$ and $\omega_2(x)$, against x have been given in figures 2 and 3. Deuteron binding energies (B_D) for 'S3', 'EXP I' and 'EXP II' potentials are 2·224, 2·2253 and 2·2079 MeV respectively. We have solved eq. (5) numerically for the chosen 2BF and 3BF to obtain the energy (E) and partial wave functions $\mathscr{U}^{(\mathscr{R})}_{2K+\ell}(r)$, retaining 13 partial waves each for the S,S' and D components. This gives a set of 39 coupled differential eigen value equation (CDEE) to be solved. A numerical solution of such a large number of CDEE is unstable and difficult numerically. Hence we have adopted the uncoupled adiabatic approximation (Das *et al* 1982d) to decouple the set of CDEE and solved the resulting differential equation by the RKGS method. The uncoupled adiabatic approximation (UAA) has been shown to be quite reliable (to within a few per cent for most potentials) for the calculation of binding energy and electric charge form factor of trinucleon systems (Das *et al* 1982c,d). Although no specific test has been carried out on the reliability of UAA for the calculated ANC, we can argue that it is expected to be sufficiently reliable for this purpose. Following the argument of Das *et al* 1982d, one sees that the system CDEE will be nearly completely decoupled in the asymptotic region (i.e. for large value of r), where the coupling terms arising out of the nuclear interaction are vanishingly small. Thus for the asymptotic trinucleon wave function, the UAA is expected to be sufficiently reliable. This in turn makes the calculation of ANC by UAA reliable.

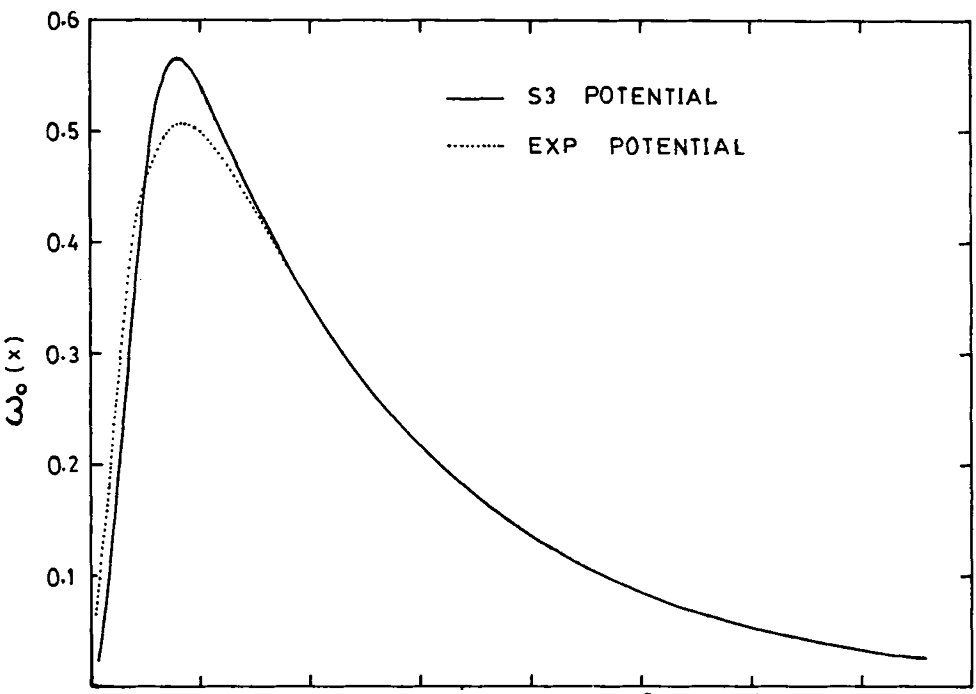

Figure 2. Plot of $\omega_0(x)$ against x for the S3 potential (continuous line) and exponential potentials (dotted line). Note that EXP I and EXP II waves are practically overlapping.

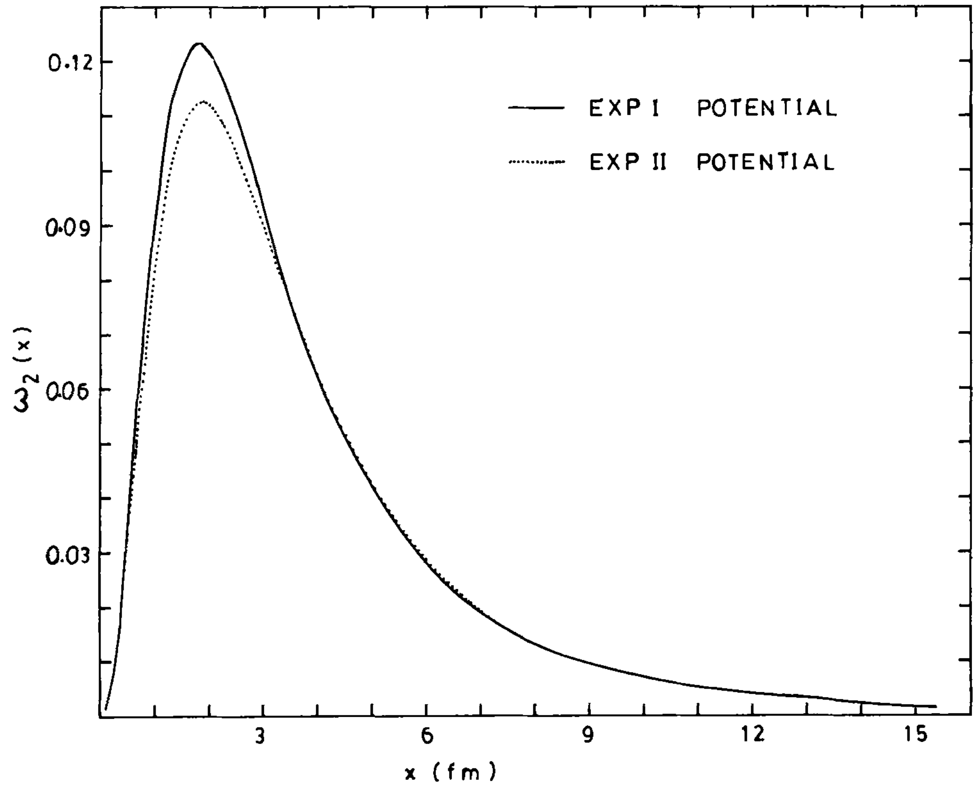

Figure 3. Plot of $\omega_2(x)$ against x for EXP I and EXP II potentials.

Then $f_0(y)$ and $f_2(y)$ are calculated from (6) and (7) by a Gaussian quadrature for the x-integration. To calculate C_0 and C_2 we compare the asymptotic forms of (6) and (7) with (2) and (3) respectively. This is done graphically by plotting $\log_{10}[yf_0(y)]$ and $\log_{10}[yf_2(y)\{(1+3/\beta y)+(3/\beta^2 y^2)\}^{-1}]$ against y, and we obtain straight line for $y > 50$ fm (figures 4 and 5). The straightness of the lines in figures 4 and 5 infers that the asymptotic nature represented by (2) and (3) are reached by (6) and (7) for $y > 50$ fm. The intercepts of the straight lines give the values of C_0 and C_2. We have calculated C_0 and C_2 by least square fit of calculated $f_\ell(y)$ for $y > 50$ fm and these are presented in table 2. For comparison we also present in table 2 the values of C_0 and C_2 obtained considering the contribution of 2BF alone.

It is seen that C_2 values increase by about 5–7% when 3BF is included. The C_0 value for S3 potential also increases by about 9% over the calculation with 2BF alone. This can be understood by the fact that the 3BF is very short ranged and affects the asymptotic part of the trinucleon wave function by a small amount only. Such small relative *increase* in the values of C_0 and C_2 also agree with Faddeev calculations (Friar *et al* 1988). However an interesting observation is that the value of C_0 for EXP I and EXP II potentials increase very little (by less than 2% for EXP II and practically unchanged for EXP I) when 3BF is included. This may be due to the following. The exponential potentials (EXP I and EXP II) have a much slower decrease with increase of r, as compared to S3 potential. Hence this 2BF influences the

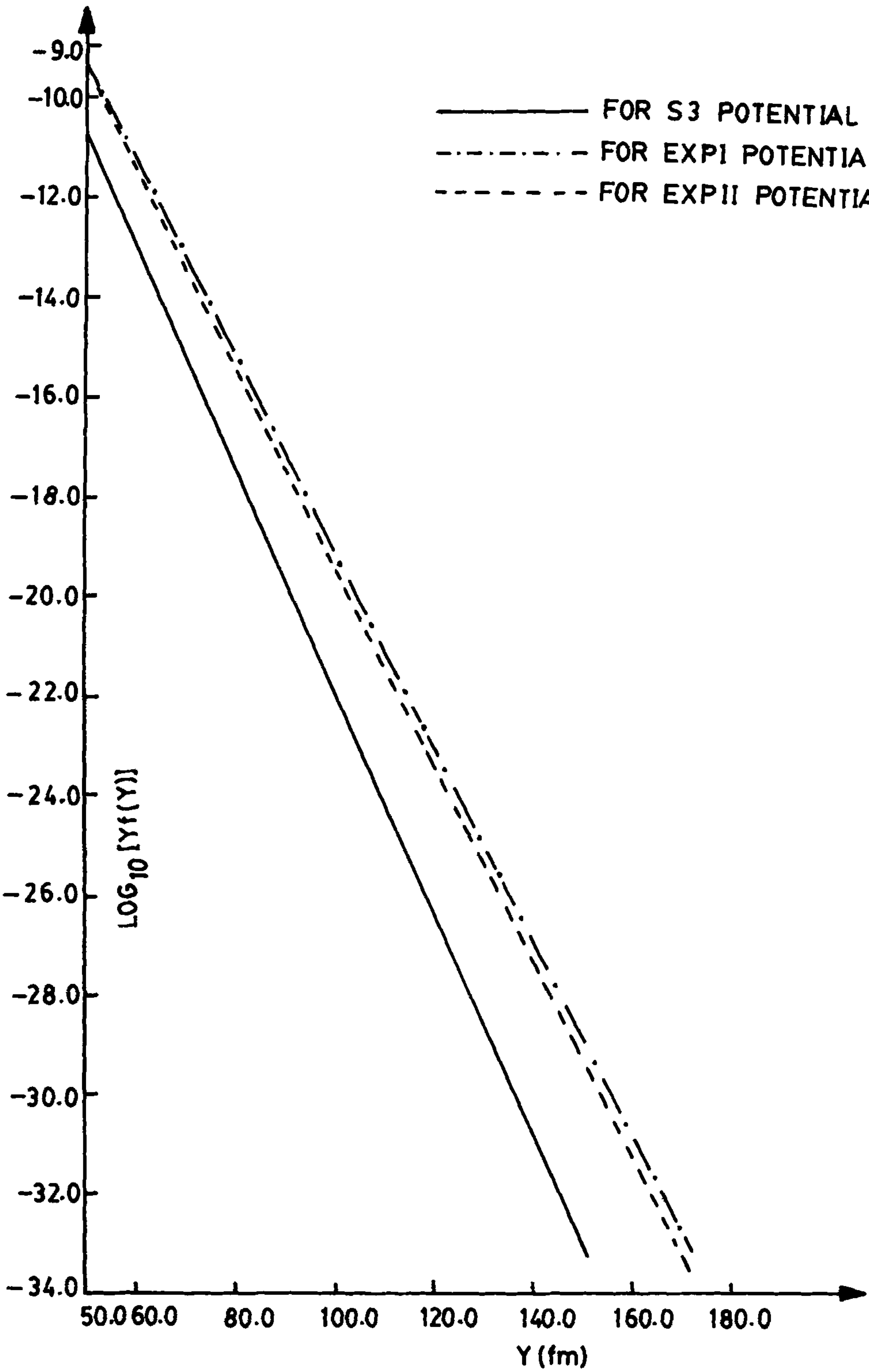

Figure 4. Plot of $\log_{10}[yf_0(y)]$ against y for calculation of C_0.

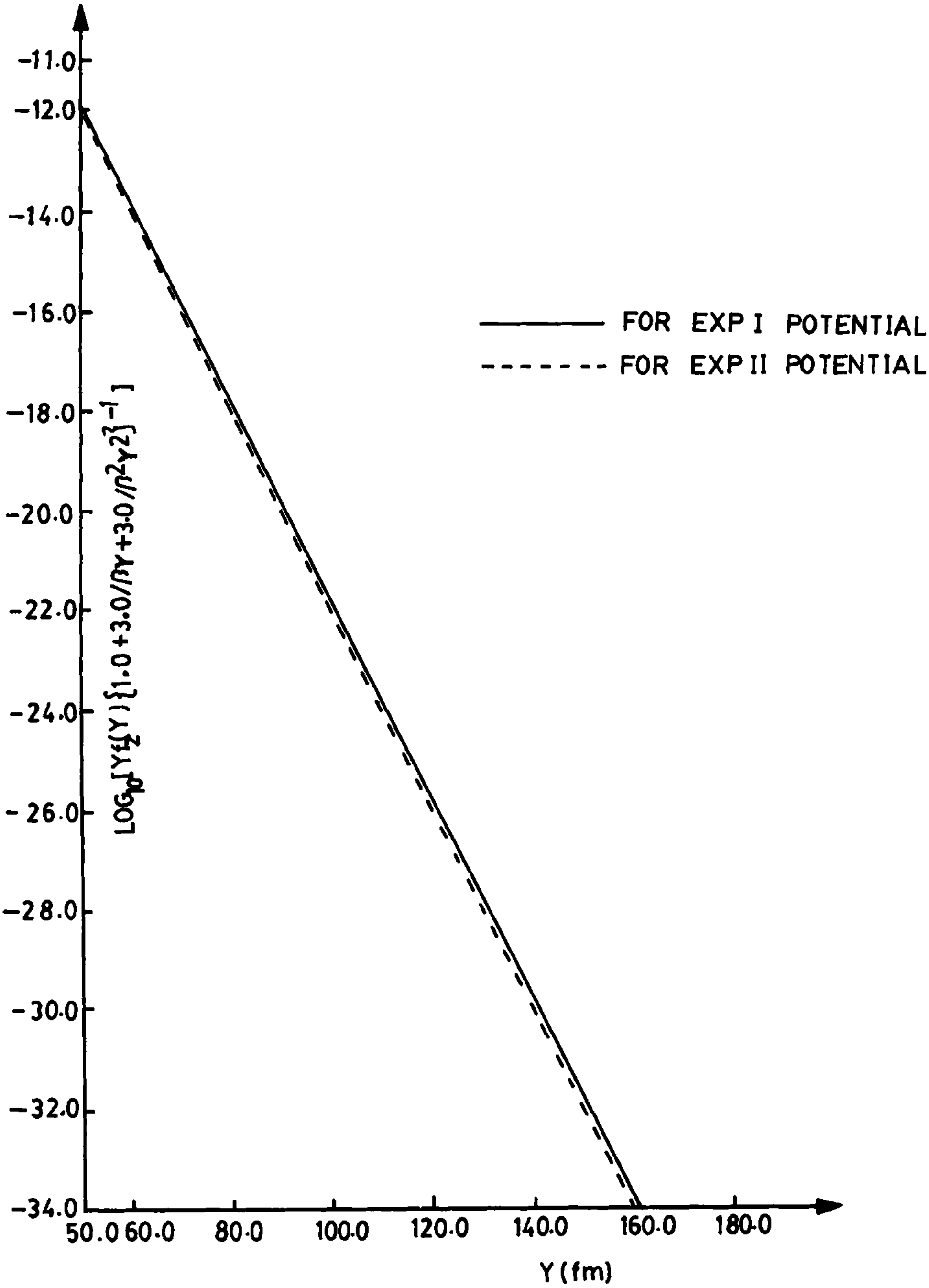

Figure 5. Plot of $\log_{10}[yf_2(y)\{1+(3{\cdot}0/\beta y)+(3{\cdot}0/\beta^2 y^2)\}^{-1}]$ against y for calculation of C_2.

asymptotic part of trinucleon wave function considerably, which also explains why the value of C_0 is considerably larger than that for the S3 potential which is close to the experimental values ($\simeq 1{\cdot}7$). Since the 3BF is exteremely short ranged, it can hardly affect the relatively long (as compared to S3) asymptotic tail of the exponential

Table 2. Calculated values of C_0 and C_2 for the triton wave function.

Potential	x_0 (fm)	BE(MeV) (B_T)	$P_S(\%)$	$P_{S'}(\%)$	$P_D(\%)$	C_0	C_2
S3		9·2374	95·89	4·11	—	1·8600	—
EXP I		7·5235	89·68	5·44	4·88	2·4476	0·0097
EXP II		7·6217	91·96	4·48	3·56	2·3642	0·0092
S3 + FM	0·385	10·562	93·30	6·70	—	2·0274	—
EXP I + FM	0·50	8·0992	90·44	4·88	4·68	2·4450	0·010i
EXP II + FM	0·50	8·2900	92·50	4·10	3·40	2·4071	0·0096

2BF. This means that inclusion of 3BF hardly influences the asymptotic part of the trinucleon wave function, resulting in practically the same value of C_0. However, the trinucleon D-state wave function is strongly influenced by the 3BF, which results in a somewhat larger increase in C_2 value.

4. Conclusion

Our calculation by hyperspherical harmonics expansion method shows that the S and D-wave asymptotic normalization constants increase by a small percentage, due to the inclusion of three nucleon force (3BF) in addition to the two nucleon force. This can be understood from the fact that 3BF is extremely short ranged and does not influence the asymptotic wave function greatly.

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