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A map describing EEG activity of human brain

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Abstract. A model of electrical activity of human brain considered as a complex dynamical system is given based on the EEG time series. The model fits the data remarkably well. The predictive ability of the model is limited to a few time steps as expected for a chaotic time series.

Keywords: Chaotic time series; map; electroencephalogram.

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Recent developments, in understanding dynamics from observed time series of a variable, are especially suited for nonlinear systems, having a broad band power spectrum (Packard *et al* 1980; Broomhead and King 1986 (BK); Abarbanel *et al* 1989 (ABK)). We have applied these ideas to construct a model of electrical activity of human brain, using as input, data obtained from the electroencephalogram. The data we have analyzed consisted of values of electric potential as a function of time from the left frontal lobe of a person who is clinically normal and in the rest state. The total duration of the record was about 5 s. The data set has 439 values of the potential at an interval of 0·01 s.

Our starting point in the present analysis is the integral equation (Parikh and Pratap 1984)

$$\rho(\mathbf{y}(t)) = \int_0^t \mathrm{d}\tau\, G[\rho(\mathbf{y}(t-\tau))]\,\rho(\mathbf{y}(\tau)) \tag{1}$$

for the probability $\rho(\mathbf{y}(t))$ of the system to be in the state $\mathbf{y}(t)$ at time t. Here G is the kernel for evolving the system from time τ to t. Equation (1) is nonlinear as G depends on $\mathbf{y}(t)$ through the function ρ and is non-Markovian through the time dependence $(t-\tau)$. Hence the evolution is path-dependent and has a built-in memory of its past. Continuous observation of a complete set of dynamical variables would determine G, in principle. In the present case we use the electrical activity as recorded by EEG to essentially obtain G. The probability function $\rho(\mathbf{y}(t))$ as defined in (1) determines, in the usual way, the expectation value $f(t)$ of a variable $f(\mathbf{y}(t))$ according to

$$f(t) = \int \mathrm{d}\mathbf{y}\, f(\mathbf{y}(t))\,\rho(\mathbf{y}(t), t). \tag{2}$$

Note that a continuous observation of $f(t)$ generates the time series corresponding to the dynamical variable $f(\mathbf{y}(t))$.

A power spectrum analysis of the EEG data at our disposal showed that the spectrum was band limited. The cut off in frequency being about 70 Hz. Consequently, we attempt to infer the dynamics underlying this EEG time series by using the methods of BK and ABK. It is expected that, after a sufficiently long time interval (no transients left), a non-linear dissipative system will be confined to an attractor (strange) in its phase space. The strange attractor is in general a fractal but it is convenient to embed it in an 'appropriate' Euclidean space and study the dynamics of the system on this manifold. The embedding space is such that the actual geometrical and topological characteristics of the system do not get modified (Packard *et al* 1980; Broomhead and King 1986). Consequently, in this approach, the first step is to determine the dimension of this space. Towards this end, we follow the method developed by Broomhead and King (1986), wherein the time series is written in a delayed matrix form, as first suggested by Packard *et al* (1980). The series is recast as a matrix F where,

$$F = \begin{bmatrix} f_1 & f_2 & \cdots & f_m \\ f_2 & f_3 & \cdots & f_{m+1} \\ \vdots & \vdots & & \\ f_N & f_{N+1} & \cdots & f_{N+m-1} \end{bmatrix} \equiv \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{bmatrix} \tag{3}$$

and $f_j(j = 1,\ N + m - 1)$ are the observed readings of the potential at times $t_j(j = 1..N + m - 1)$. $\mathbf{x}_j s$ are a set of N vectors defined in an m dimensional Euclidean space. F is an $N \times m$ rectangular matrix with $m < N$.

From our data with $N + m - 1 = 439$, points which are equally separated in time, we construct an $m \times m$ ($m = 3, 4, \ldots 15$) dimensional correlation matrix $C = F^T F$ which is real and symmetric. We determine the eigenvalues and eigenvectors of C, for each value of m. The eigenvalues for larger m values break up into two classes, one containing n non-zero values which characterize the deterministic part, while the remaining $m - n$ of near zero value correspond to the noise in the system (see Broomhead and King 1986). In the present case we get $n = 4$. This also implies that the dynamics of the system on the attractor requires at least four variables for its description. The corresponding eigenvectors form the basis vectors which are orthogonal. We next project the time series on to these basis vectors, which is the optimal basis to describe the dynamics on the attractor (Devijver and Kittler 1988). Note that this procedure also filters the noise from the data (Broomhead and King 1986). We make use of this projected and filtered data to determine the explicit form of G, following Abarbanel *et al* (1989).

The method is based on the discrete map theory, viz.,

$$\mathbf{x}_{i+1} = G(\mathbf{x}_i, \boldsymbol{a}) \tag{4}$$

where $\mathbf{x}_i$ have n components and $\boldsymbol{a} = (a_1, \ldots, a_p)$ represents a set of parameters. A specific form suggested by Abarbanel *et al* (1989) is used in the present work and this is given by

$$G(\mathbf{x}, a) = \sum_{j=1}^{n-1} \mathbf{x}_{j+1}\, g_\sigma(\mathbf{x}, \mathbf{x}_j, \boldsymbol{a}) \tag{5}$$

where

$$g(\mathbf{x}, \mathbf{x}_j, a) = \exp\left[-\frac{|\mathbf{x}-\mathbf{x}_j|^2}{\sigma}\right]\left[a_1 + a_2\mathbf{x}_j\cdot(\mathbf{x}-\mathbf{x}_j) + \sum_{k=3}^{5} a_k\left(\frac{|\mathbf{x}-\mathbf{x}_j|^2}{\sigma}\right)^k\right]. \tag{6}$$

Note that all points on the attractor irrespective of the time ordering are used to define the map G. The form of g eq. (6) is such that the function is more sensitive to near neighbours in the phase space. When $\sigma \to 0$, g becomes essentially a Kronecker delta and the point $\mathbf{x}_j$ is mapped to $\mathbf{x}_{j+1}$.

We have taken $\sigma = 10$ following Abarbanel *et al* (1989), and determined the parameters a_is, using the first 380 vectors (obtained after the Broomhead and King (1986) procedure) in a least squares fit procedure. For the EEG series we have considered, the resulting parameters are given in table 1, together with the root mean square deviation. These parameters completely determine the map. The quality of the fit is very striking as can be seen in figure 1 where figure 1a is the filtered data series projected on the lowest energy eigenvector and figure 1b the corresponding series generated by the map. To check the validity of this map further, we used it together with the remaining vectors (about 60) not included in the fit. More precisely, from the pth ($p > 380$) data vector, we predict the $(p+1)$ data vector using the map. Again, we find the predictions to be in remarkable agreement with the data. This can be seen in figure 2a and figure 2b. Note that in this analysis, at each stage the map is used to predict only one time step in the future. Finally, we start from the data vector # 380 and use the map to predict the future time evolution of the system. We are able to predict, within the r.m.s. deviation about six time steps in the future (see table 2) beyond which the map fails. Note that the predictions do not reproduce the fluctuations – only the average behaviour is within the r.m.s. deviation. It should be mentioned that the predictability of this model, is limited by the intrinsic chaotic behaviour of the system, as already pointed out by Lorenz (1963) in his original work (see also Gade and Amritkar (1990)).

It is worth stressing that the excellent fit (figure 1) has been possible since we used the projected and filtered data as against the raw series suggested by Abarbanel *et al* (1989).

Table 1. Values of the parameters $a_1, a_2, \ldots, a_5$ and the r.m.s. deviation obtained by fitting the data to the function $G(\mathbf{x}, a_1, \ldots, a_5)$.

σ	10·0
a_1	$2{\cdot}982 \times 10^{-3}$
a_2	$2{\cdot}215 \times 10^{-4}$
a_3	0·1138
a_4	−0·3279
a_5	0·2300
RMS DEV	0·2003

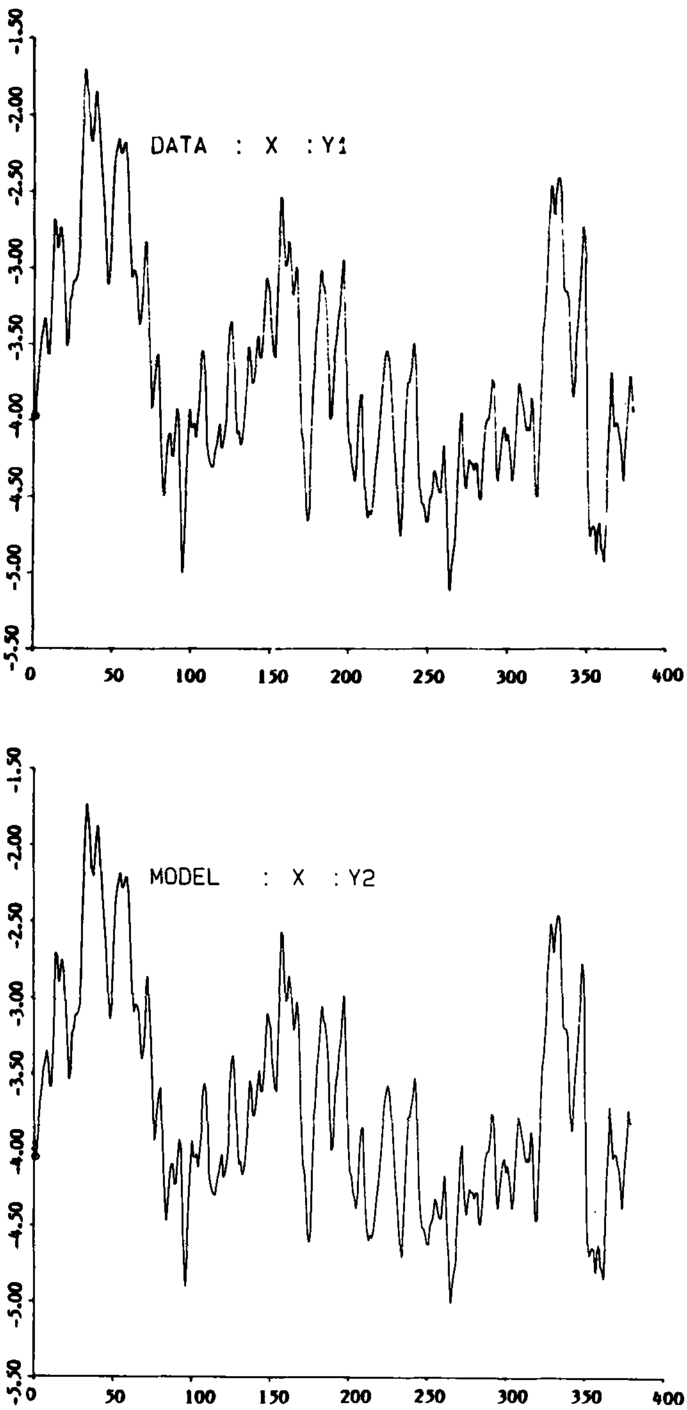

Figure 1. (**a**) The observed data after noise filtering and projection on the optimal basis and (**b**) the data generated by the model.

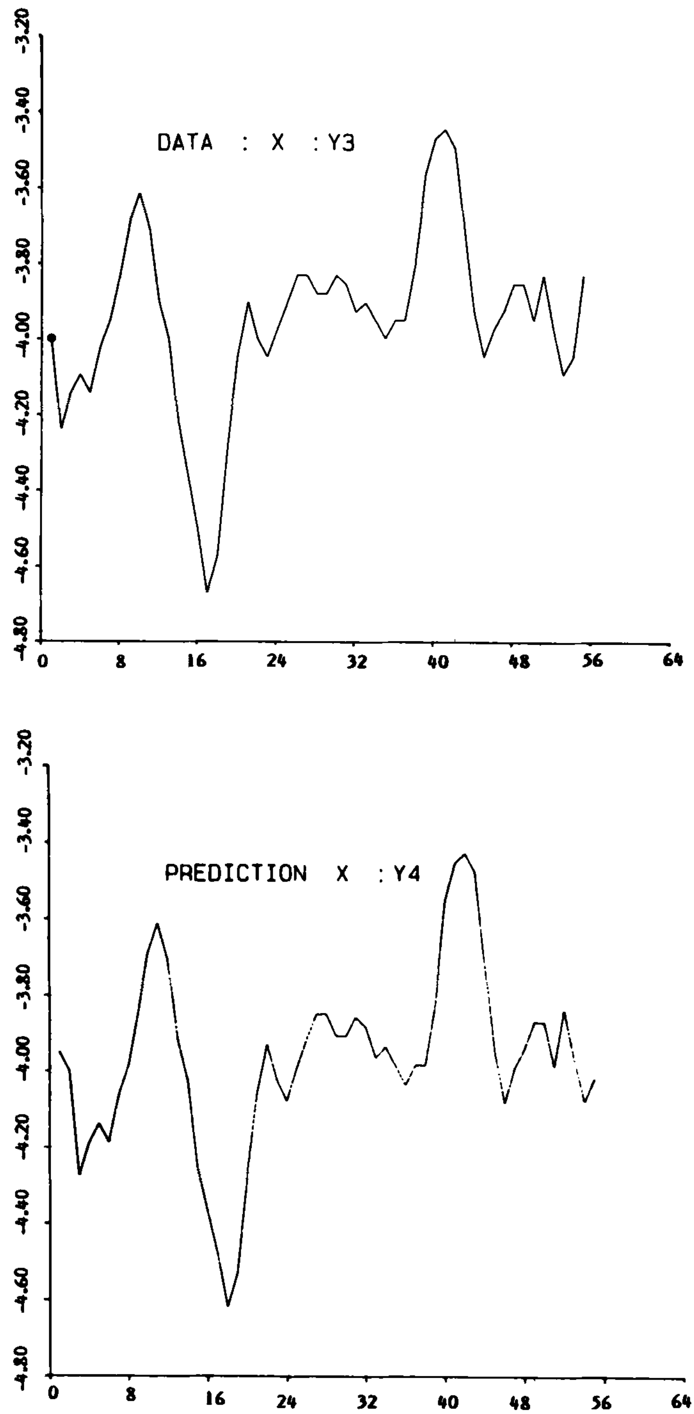

Figure 2. (a) The observed data (not used in the fit) after noise filtering and projection on the optimal basis and (b) the prediction (only one time step in the future) made by the model.

Table 2. The predictions of the map for a few steps beyond the 380th time instant compared with the filtered data not used in the fit.

Time instant	Filtered data	Prediction of the map
381	−3·999	−3·949
382	−4·238	−3·989
383	−4·143	−4·032
384	−4·095	−4·078
385	−4·142	−4·126
386	−4·023	−4·175
387	−3·951	−4·224

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