Pramāṇa - J. Phys., Vol. 35, No. 5, November 1990, pp. 495–501. © Printed in India.

Stimulated Brillouin scattering of an electromagnetic wave by an acoustic-like mode in multi-ion species plasmas

S GUHA and RUBY SARKAR
Department of Physics, Ravishankar University, Raipur 492 010, India

MS received 24 July 1989; revised 14 August 1990

Abstract. Using an hydrodynamical model, analytical investigation of the stimulated Brillouin scattering of an electromagnetic wave by an acoustic-like mode has been done in a multi-ion species plasma. The acoustic-like mode is a new mode which propagates in a multi-ion plasma only. The non-linear dispersion relation and the growth rate of the excited modes are derived. The non-linearity arises through the motion of ions which is introduced through ponderomotive force. It is found that the growth rate can be controlled by several parameters like charge number, mass, density and temperature of the ions.

Keywords. Stimulated Brillouin scattering; acoustic-like mode; multi-ion species plasma.

PACS No. 52·35

1. Introduction

Multi-ion species plasma exists in space as well as in laboratory plasmas (Dash *et al* 1984; Sharma *et al* 1986; Milic and Krstic 1987). Many aspects of this complex physical system have received significant attention in the literature (Milic and Krstic 1987). In the vast majority of studies of multi-species plasma, attention is focussed to those aspects of their behaviour which are determined by the specificities of the individual atoms of the species present.

Hansen (1976) extended Monto Carlo calculations to the physically important case, where two ionic species are present, e.g. $H^+ - He^{++}$ mixtures, the corresponding model is the two component plasma in which he considered a mixture of N_1 ions of charge $Z_1 e$ and N_2 ions of charge $Z_2 e$. Dash *et al* (1984) considered a two-ion species magnetoplasma where ions are more energetic than the electrons. They applied their result in the plasma sphere where proton is the dominant ion-species, with 2–5% of He^{++} and O^{++} ions (Hoffman *et al* 1974). Sharma *et al* (1986) obtained a new analytical solution for an ion-acoustic soliton in a two-ion species plasma where the multiple ionization of the ion-species was also discussed. For a pure single ion case the amplitude is independent of the multiplicity of ionization. However, in the presence of different types of ions, it depends on Z_1 and Z_2.

Heating of plasmas by using ordinary electromagnetic waves has been studied extensively both theoretically and experimentally (Tripathi and Sharma 1988). In a single species plasma anomalous ion heating as observed by Hendel and Flick (1973) may occur via parametric instabilities excited by relative electron-ion motion. Ions can also be heated directly when one of the parametrical decay waves is a low-frequency wave (Tripathi and Sharma 1988). Ion heating using excitation of low-frequency waves in a multi-species plasma has been suggested by Kitsenko and Stepanov (1973) Kaw and Lee (1973). In such plasmas coherent relative ion–ion motion can produce parametric instabilities and heat the plasma. Parametric instabilities in a multi-species plasma are of interest because the threshold fields in such a plasma can be lower than in a single species plasma (Baikov 1977).

In (1989) Dwivedi *et al* theoretically found a new mode called acoustic-like mode which propagates in a plasma consisting of hot electrons and two-ion species having differet temperatures, number densities, masses and charge-multiplicities when $C_e \gg C_{ih} \gg V_p \gg C_{ic}$. C_e, C_{ih} and C_{ic} are thermal velocities of hot electron and two types of ions. Such a plasma system arises where preferential heating of ion-species take place. This plasma may be heated non-linearly by the electromagnetic wave through this mode. In the present paper we have investigated SBS process by this new mode in multi-ion species plasma which has not been done so far. In §2 the analysis is given to obtain the non-linear dispersion relation of acoustic-like mode. The non-linear growth rate of the excited mode has been obtained in §3. The results are discussed in §4.

2. Mathematical analysis

We consider a linearly polarized electromagnetic pump wave $(\omega_0, \mathbf{k}_0)$ of amplitude $\mathbf{E}_0 \approx E_{0x} \exp(-i\omega_0 t + ik_0 \mathbf{z})$ interacts with plasma consisting of two-ion species and electrons. After interaction the pump wave decays into an acoustic-like mode $(\omega, \mathbf{k})$ which interacts with the incoming wave to produce side bands $(\omega_{1,2} \mathbf{k}_{1,2})$ propagating in the backward direction such that

$$\omega_{1,2} = \omega_0 \mp \omega, \quad \mathbf{k}_{1,2} = \mathbf{k}_0 \mp \mathbf{k}.$$

The excited acoustic-like mode is assumed to be of the form $\mathbf{E} = E_x \exp(-i\omega t + ik \cdot \mathbf{x})$. We have neglected $(\omega_0 + \omega, \mathbf{k}_0 + \mathbf{k})$ component as being non-resonant $(D_2 \neq 0)$. The non-linear low frequency force known as ponderomotive force produced by the beating of the two electromagnetic waves (scattered and incident) drives the acoustic-like mode.

The set of self-consistent equations for acoustic-like mode are

$$\frac{\partial \mathbf{v}_{ih,c}}{\partial t} + (\mathbf{v}_{ih0,c0} \cdot \nabla)\mathbf{v}_{ih,c} = Z_{h,c} \frac{e}{M_{ih,c}} (\mathbf{E} + \mathbf{v}_{ih0,c0} \times \mathbf{B}_1) - C_{ih,c}^2 \frac{\nabla n_{ih,c}}{n_{ih0,c0}} \tag{1}$$

$$\frac{\partial n_{ih,c}}{\partial t} + \nabla(n_{ih,c} \mathbf{v}_{ih,c}) = 0 \tag{2}$$

and

$$\nabla \cdot \mathbf{E} = 4\pi e(Z_h n_{ih} + Z_c n_{ic}) \tag{3}$$

where h and c denote hot and cold ions. The electrons are not directly involved in the dynamics of low frequency wave providing simply a neutralizing background. In the above equations $v_{ih,c}$ are respectively the velocities of hot and cold ions, $M_{ih,c}$ are the masses of hot and cold ions, $n_{ih,c}$ are the densities and $Z_{h,c}$ are the charge multiplicities of hot and cold ions.

The quiver velocities of hot and cold ion components and electrons for the incident pump wave and scattered wave are calculated as

$$\mathbf{v}_{ih0,c0} = Z_{h,c}\frac{ie\mathbf{E}_0}{M_{ih,c}\omega_0}, \mathbf{v}_{e0} = \frac{-ie\mathbf{E}_0}{m_e\omega_0}$$

and

$$\mathbf{v}_{ih1,c1} = Z_{h,c}\frac{ie\mathbf{E}_1}{M_{ih,c}\omega_1}, \mathbf{v}_{e1} = \frac{-ie\mathbf{E}_1}{m_e\omega_1}.$$

rewriting equation of motion for hot ion in terms of ponderomotive force, we get

$$\frac{\partial \mathbf{v}_{ih}}{\partial t} = \frac{1}{M_{ih}}(Z_h e\mathbf{E} + \mathbf{F}_{pih}) - C_{ih}^2\frac{\nabla n_{ih}}{n_{ih0}}$$

where $\mathbf{F}_{pih}$ is the ponderomotive force on hot ions. In terms of potential we can write

$$\mathbf{F}_{pih} = -Z_h e\nabla\phi_{pih}$$

and

$$\phi_{pih} = Z_h\frac{e(\mathbf{E}_0\cdot\mathbf{E}_1)}{4M_{ih}\omega_0\omega_1}.$$

Now applying the perturbation technique and assuming the dependence of all variable quantities as $\exp i(\mathbf{k}\cdot\mathbf{x} - \omega t)$, this yields

$$n_{ih} = \frac{k^2\chi_{ih}}{4\pi e}(\phi + \phi_{pih}) \tag{4}$$

where

$$\chi_{ih} = \omega_{p0}^2\frac{n_{ih0}}{n_0}Z_h\frac{m_e}{M_{ih}}\frac{1}{(\omega^2 - k^2C_{ih}^2)}$$

and

$$\omega_{p0}^2 = \frac{4\pi n_0 e^2}{m_e} \text{ is the electron plasma frequency.}$$

Similarly for cold ion component, the perturbed number density is given by

$$n_{ic} = \frac{k^2\chi_{ic}}{4\pi e}(\phi - \phi_{pic}) \tag{5}$$

where

$$\chi_{ic} = \omega_{p0}^2\frac{n_{ic0}}{n_0}Z_c\frac{m_e}{M_{ic}}\frac{1}{(\omega^2 - k^2C_{ic}^2)}.$$

From Poisson's equation (3), we can obtain the wave equation for the low-frequency acoustic-like mode as

$$\varepsilon\phi = A\mathbf{E}_0\cdot\mathbf{E}_1 \tag{6}$$

where

$$\varepsilon = 1 - Z_h\chi_{ih} - Z_c\chi_{ic}$$

is the dielectric function of acoustic-like mode, and

$$A = \frac{e}{4\omega_0\omega_1}\left(Z_h\frac{\chi_{ih}}{M_{ih}} + Z_c\frac{\chi_{ic}}{M_{ic}}\right).$$

Using Maxwell's equation, the wave equation of the scattered electromagnetic wave $(\omega_1, \mathbf{k}_1)$ is given by

$$(\omega_1^2 - k_1^2 c^2)\mathbf{E}_1 = -4\pi i \omega_1 \mathbf{J}_1 \tag{7}$$

where $\mathbf{J}_1$, the current density is given by

$$\mathbf{J}_1 = (n_{ih0} + n_{ic0})e(\mathbf{v}_{ih,1} + \mathbf{v}_{ic,1}) + \tfrac{1}{2}(n_{ih,1} + n_{ic,1})e(\mathbf{v}_{ih0} + \mathbf{v}_{ic0}) - n_{e0}e\mathbf{v}_{e1} - \tfrac{1}{2}n_e e\mathbf{v}_{e0}. \tag{8}$$

Substituting all the values in (7), neglecting ponderomotive force on ions and assuming that the electrons obey Boltzmann relation

$$\frac{n_e}{n_{e0}} \approx \frac{e\phi}{T_e}$$

one can obtain

$$\mathbf{J}_1 = n_0 e\left[Z_c \frac{ie\mathbf{E}_1}{M_{ic}\omega_1} + Z_h \frac{ie\mathbf{E}_1}{M_{ih}\omega_1}\right] + \frac{1}{2}(\mathbf{v}_{ih0} + \mathbf{v}_{ic0})e \times \left[\frac{k^2\chi_{ih}}{4\pi e}\phi + \frac{k^2\chi_{ic}}{4\pi e}\phi\right] + n_0 e\frac{ie\mathbf{E}_1}{m_e\omega_1} - \frac{1}{2}v_{e0}e\left[\frac{e\phi}{T_e}n_{e0}\right].$$

Using the above equation in (7), the wave equation of the scattered e.m.w. is given by

$$D_1\mathbf{E}_1 = B\mathbf{E}_0\phi \tag{9}$$

where

$$D_1 = \omega_1^2 - k_1^2c^2 - \omega_{pe}^2 - Z_h\omega_{pih}^2 - Z_c\omega_{pic}^2$$

is the dispersion relation of the scattered e.m.w. and

$$B = \frac{e\omega_1}{2\omega_0}\left(\frac{\omega_{pe}^2}{T_e} + Z_h\frac{k^2\chi_{ih}}{M_{ih}} + Z_c\frac{k^2\chi_{ic}}{M_{ic}}\right).$$

From (6) and (9), we obtain the non-linear dispersion relation as

$$\varepsilon D_1 = \mu \tag{10}$$

where

$$\mu = \frac{e^2E_{0x}^2}{8\omega_0^2}\left(Z_h\frac{\chi_{ih}}{M_{ih}} + Z_c\frac{\chi_{ic}}{M_{ic}}\right)\left(\frac{\omega_{pe}}{T_e} + \frac{k^2\chi_{ih}}{M_{ih}}Z_h + \frac{k^2\chi_{ic}}{M_{ic}}Z_c\right)$$

is the non-linear coupling term. If there is no pump wave, i.e., $E_{0x} = 0$, we get the linear dispersion relation ($\varepsilon = 0$) of the acoustic-like mode as obtained by Dwivedi *et al* (1989), viz.

$$\frac{k^2}{\omega_{p0}^2} + \frac{n_{ih0}}{n_0}Z_h^2\frac{m_e}{M_{ih}}\frac{1}{(-v_p^2 + C_{ih}^2)} + \frac{n_{ic0}}{n_0}Z_c^2\frac{m_e}{M_{ic}}\frac{1}{(-v_p^2 + C_{ic}^2)} = 0.$$

3. Non-linear growth rate

In the absence of linear damping, the non-linear growth rate is given by

$$\gamma_0^2 = \frac{-\mu}{\left(\frac{\partial \varepsilon_r}{\partial \omega_r}\right)\left(\frac{\partial D_{1r}}{\partial \omega_{1r}}\right)}.$$

Under the approximation

$$C_{ih} \gg v_p \gg C_{ic}$$

and substituting

$$\omega = \omega_r + i\gamma_L$$

and

$$\omega_1 = \omega_{1r} + i\gamma_{1L}$$

we get

$$\frac{\partial \varepsilon_r}{\partial \omega_r} = 2\omega_r \omega_{p0}^2 Z_c^2 \frac{n_{ic0}}{n_0} \frac{m_e}{M_{ic}} \frac{1}{(\omega_r^2)^2}\left[\frac{Z_h^2}{Z_c^2}\frac{n_{ih0}}{n_{ic0}}\frac{M_{ic}}{M_{ih}}\frac{(\omega_r^2)^2}{-k^2 C_{ih}^2} + 1\right]$$

and

$$\frac{\partial D_{1r}}{\partial \omega_{1r}} = -2\omega_{1r}$$

hence the non-linear growth rate is

$$\gamma^2 = \frac{e^2 E_{0x}^2 k^2 \omega_r Z_c}{32 M_{ic}^2 \omega_0^2 \omega_{1r}}\left[\frac{M_{ic}}{Z_c}\frac{\omega_{pe}^2}{k^2 T_e} + \omega_{p0}^2 \frac{1}{\{1 + (Z_h n_{ih0})/(Z_c n_{ic0})\}}\frac{m_e}{M_{ic}}\frac{1}{\omega^2}\right]. \quad (12)$$

4. Results and discussion

From the expression of the growth rate one can conclude that the growth rate of acoustic-like mode depends linearly upon the intensity of the incident electromagnetic wave. As the concentration of the hotter ion increases the growth rate decreases. The increase in the growth rate is associated with the increase in Z_c and n_{ic0}, the charge multiplicity and concentration of cold ions, whereas it is inversely proportional to Z_h and M_{ic}, charge multiplicity and mass of hot and cold ions respectively. The growth rate is found to depend only on temperature of hot electrons. Thus one can control growth rate injecting hot electrons. Rockets and satellites have made possible artificial perturbations in the ionosphere (addition of chemicals, injection of charged electrons from accelerators, etc.). A particularly interesting idea was the possibility of temporarily altering the properties of the ionosphere by radiation from ground radio transmitters, whose power can now be made large enough to produce non-linear effects, in particular the development of parametric instability. Hence the numerical calculations for the non-linear growth rate in two-ion species plasma have been done for typical parameters of ionosphere in order to investigate the relative possibility of the growth of this acoustic-like mode in the two-ion species plasma. The planetary ionosphere

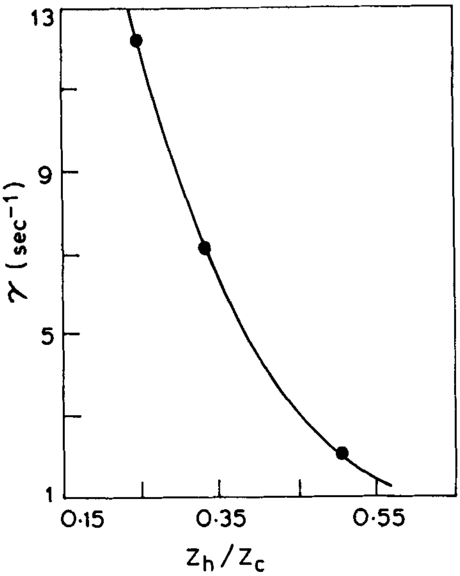

Figure 1. Variation of growth rate with different ratio of charge multiplicities.

consists of many kinds of ions and the ion composition ratio changes with height.

The main investigations of the ionosphere have been conducted in the F-layer ($h = 150$–$400\,\mathrm{km}$). Experiments on the artificial heating of the ionosphere have been carried out at Boulder (Utlaut and Cohen 1971; Utlaut 1970; Cohen and Whitehead 1970) with radio waves corresponding to $\omega_0 \simeq \omega_{pe}$. It was expected that high-power radio waves being efficiently absorbed, would heat the ionosphere in the range $\omega_{pe} \approx \omega_0$ and would reflect at this point called critical density. In later experiments (Wong and Taylor 1971; Wong *et al* 1971), the ionosphere was exposed during the day time at height $h = 200\,\mathrm{km}$ from the earth.

We suggest that the parametrically excited acoustic-like mode can be associated with the low-frequency (LF) wave emission during controlled wave-generation experiments. We assume the frequency of the incident e.m.w. as $10^7\,\mathrm{s}^{-1}$ and $k_0 \approx 10^{-3}\,\mathrm{cm}^{-1}$ and take a typical set of values from the ionospheric plasma: (Saleem 1987)

$$\omega_0 \approx \omega_{pe}; \quad \omega_{pe} = 1{\cdot}78 \times 10^7\,\mathrm{s}^{-1}$$

$$n_e = 10^5\,\mathrm{cm}^{-3}; \quad n_{ih0} = 10^4\,\mathrm{cm}^{-3}; \quad n_{ic0} = 5 \times 10^5\,\mathrm{cm}^{-3}$$

$$M_{ih} = 1{\cdot}5 \times 10^{-24}\,\mathrm{g}; \quad M_{ic} = 2 \times 10^{-24}\,\mathrm{g}$$

$$Z_h = 1; \quad Z_c = 2; \quad \omega = 1{\cdot}64 \times 10^3\,\mathrm{s}^{-1}$$

$$C_{ih} = 6{\cdot}2 \times 10^5\,\mathrm{cm\,s}^{-1}; \quad \omega_1 = 0{\cdot}5 \times 10^7$$

$$C_{ic} = 2{\cdot}25 \times 10^5\,\mathrm{cm\,s}^{-1}.$$

We get $k = 0{\cdot}2 \times 10^{-3}\,\mathrm{cm}^{-1}$ and the value of $k_1 = 0{\cdot}6 \times 10^{-3}\,\mathrm{cm}^{-3}$. Thus from the k-matching condition ($k_{1,2} = k_0 \mp k$, where we have neglected $(k_0 + k)$ component as being nonresonant), one can see that k is conserved, and the growth rate for this instability is given by

$$\gamma = 0{\cdot}0206 \times 10^2\,\mathrm{s}^{-1}.$$

Thus we see that for any finite value of pump wave intensity, the instability grows. Considering different ratio of charge-multiplicity, Z_h/Z_c, results have been plotted (figure 1) which show that increasing the ratio of charge multiplicities, the growth rate of the instability decreases and may thus lead to stabilization of the system.

Acknowledgements

We acknowledge thankfully the financial support from CSIR, New Delhi.

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