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Scaling violations of multiplicity distributions of *pn* interactions at 400 GeV

D K BHATTACHARJEE, P J CHERIAN and D K MAITY
High Energy Physics Division, Department of Physics, Jadavpur University, Calcutta 700 032, India

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Abstract. Results on charged multiplicity distributions in high energy proton-neutron interactions derived from *pd* data at 400 GeV are presented. These are compared with other available data on *pn* distributions at and above 100 GeV. The study reveals that the proton-neutron distribution at 400 GeV do not obey KNO scaling. It has been shown that the negative binomial distribution gives a fair description of the multiplicity distribution and that the early onset of KNO scaling below 400 GeV is approximate.

Keywords. High energy *pn* interaction; bubble chamber; experimental data; multiplicity distribution; scaling violation.

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1. Introduction

Scaling of charged multiplicity distributions in high energy interactions has attracted recent attention due to its violation reported by UA5 group (Alner *et al* 1984, 1985, 1986) from studies on data from $\bar{p}p$ interactions between 200 and 900 GeV centre of mass energy and due to its consequent effect on the theoretical models (Pancheri 1984; Subramanian 1986; Van Hove 1987; Giovannini and Van Hove 1988). It has also been previously pointed out (Thome *et al* 1977; Guettler *et al* 1976) that Feynman scaling which served as the ingredient of KNO scaling (Koba *et al* 1972) was also violated at ISR energies ($\sqrt{s} = 30$–60 GeV) (Breakstone *et al* 1984). These results were obtained from studies with data from experiments with doubly charged or neutral initial states, leaving enough scope to analyze data from singly charged initial state.

Consequence of the scaling violations has found expression in the development of theoretical models which favoured non-single diffractive part of the charged multiplicity distribution to be fitted to a negative binomial (NB) distribution (Carruthers and Shih 1987; Van Hove 1987; Lo and Schreiber 1986; Burgers *et al* 1987; Subramanian 1986). It was pointed out sometime ago (Giovannini *et al* 1974; Knox 1974) that the NB distribution should be successful representation of multiplicity distributions for full phase space inelastic data. Data from several experiments on $\bar{p}p$ and pp in the energy range $\sqrt{s} = 200$–900 GeV (Ansorge *et al* 1989 and 800 GeV (Ammar *et al* 1986) respectively have confirmed this. It is therefore worthwhile to analyze charged multiplicity distributions in proton-neutron interactions at high energy to look into their behaviour.

In this paper we report our experimental data on *pn* interactions at 400 GeV and other available data at energies at and above 100 GeV. In §2 we briefly report the methodologies adopted to extract *pn* distributions from *pd* data considered in this paper. Section 3 gives the method and analysis of the *pn* data samples to look for their scaling behaviour. In §4 the fit of data with negative binomial distribution is presented and discussed. Section 5 outlines the conclusions arrived at from this study.

2. Extraction of *pn* multiplicity distribution from *pd* data

According to a naive spectator model, an effective proton-free-neutron distribution may be obtained from either odd-prong events or odd-prong plus backward spectator events. But this simple methodology should not work as the difference between the number of odd-prong and proton-free-neutron events also result due to contributions from (i) coherent deuteron interactions, (ii) symmetry requirements on the final-state wave functions and (iii) rescattering or double scattering within deuterium.

The contribution of inelastic coherent processes populating the even-prong sample is limited to low multiplicity events as caused by deuteron form factor. Moreover the calculation of this contribution and its subtraction results to an increase of average multiplicity by an amount almost equal to the reduction due to the correction applied for proton visibility factor. Combined contribution to the multiplicity distribution due to these two corrections, therefore, is small [Lys *et al* (1977) (100 GeV); Dombeck *et al* (1978) (200 GeV); Sheng *et al* 1975 (300 GeV), Bhattacharjee *et al* (1983a, b, c) (400 GeV)].

Dean's (1972) formula for the spectator momentum distribution in the framework of Glauber theory may be evaluated by using Hulthen Wave function for elastic rescattering and a multi-Gaussian fit to the Reid soft core wave function for no-rescattering. By omitting both the *D*-wave component and elastic rescattering, the value of the forward to backward spectator ratio is evaluated by using appropriate Moller flux factor. The value used by Lys *et al* (1977) is 1.23 at 100 GeV. Dombeck *et al* (1978) used a value of 1·19 at 200 GeV and Sheng *et al* (1975) have not applied any correction for wave function symmetry effects on their *pd* data at 300 GeV. The present authors used a value of $1{\cdot}26 \pm 0{\cdot}03$ independent of multiplicity (Bhattacharjee *et al* 1983a, b, c) on our *pd* data at 400 GeV.

The rescattering need to be considered because the final states recorded by the experiment cannot be considered as due to a single interaction in deuterium. In this case, the interactions within deuterium may be due to some intermediate processes and their complexity depends to a large extent on the particular kinematical configuration of the final state. Each of these intermediate processes must be fully accounted for if we want to isolate the basic interaction between the incident proton and one of the nucleons. In order to do so we need to know the complete *pn* scattering amplitudes. Unfortunately, this is just what the experiment is supposed to determine. Moreover, we are to take into account considerable ($\sim 20\%$) rescattering in deuterium events which cause the depletion of the odd-prong sample and hence a modellistic approach has to be adopted to extract proton-free-neutron distributions from *pd* data. The distinction between the approaches depend on the assumption whether the rescattering probability is dependent on or independent of multiplicity of the first interaction. A no-cascade model assumes the rescattering probability as independent of first interaction multiplicity or constant at all multiplicities. A cascade model assumes a complete dependence of the rescattering probability on the first interaction multiplicity. Lys *et al* (1977) at 100 GeV and Dombeck *et al* (1978) at 200 GeV analyzed their data according to both the models and could not detect any substantial difference between the two. The *pn* multiplicity distributions at 300 GeV (Sheng *et al* 1975) and 400 GeV (Bhattacharjee *et al* 1983a, b, c) are extracted from *pd* data through a cascade model. The details of the calculations and assumptions are given in the references cited. The size of the data samples used in this analysis is given in table 1. Our extracted *pn* and *pp* distributions from *pd* data at 400 GeV are given in table 2.

Table 1. Size of *pd* and *pn* data samples at energies from 100 to 400 GeV.

Energy	No. of pictures exposed	No. of corrected *pd* events	No. of *pn* events $n \geqslant 3$	Reference
100	26000	6643 ± 86	2083 ± 51	Lys *et al* (1977)
200	60000	14040	3931	Dombeck *et al* (1978)
300	52500	13385 ± 116	5741 ± 181	Sheng *et al* (1975)
400	44000	11579 ± 193	5472 ± 209	Present work

(*n* indicates charged multiplicity)

Table 2. Multiplicity distributions of *pn* and *pp* interactions derived from deuterium bubble chamber data at 400 GeV.

Multiplicity	*pn* events	*pp* events
1	283 ± 37	
2		527 ± 62
3	749 ± 68	
4		751 ± 72
5	947 ± 92	
6		959 ± 89
7	945 ± 104	
8		1110 ± 90
9	860 ± 93	
10		897 ± 85
11	627 ± 69	
12		741 ± 72
13	430 ± 47	
14		516 ± 50
15	292 ± 34	
16		259 ± 37
17	171 ± 37	
18		176 ± 23
19	85 ± 14	
20		106 ± 14
21	42 ± 12	
22		43 ± 9
23	23 ± 5	
24		16 ± 8
25	9 ± 2	
26		4 ± 1
27	9 ± 8	
28		2 ± 2

3. Scaling problem and *pn* data

Asymptotic scaling of hadronic multiplicity distribution at high energies has been predicted by Koba *et al* (1972) (KNO scaling hypothesis) and Mueller (1971) (Mueller-Regge hypothesis). The KNO scaling predictions include the energy independence of C-moments and curvilinear relations between the average multiplicity $\langle n \rangle$ and each of the Mueller correlation parameters f_2, f_3 and f_4. The Mueller-Regge hypothesis, however, predicts a linear dependence of each of these correlation parameters on the average multiplicity.

We have analyzed our *pn* data at 400 GeV to look for their scaling behaviour. In table 3 we show the average multiplicities and C-moments calculated from the experimental *pn* data within the energy range 100 to 400 GeV. It can be seen that the values of the successive C moments show their energy independence quite fairly thereby affirming the validity of KNO scaling on this characteristic of data. Figures 1 (a, b, c) respectively show the variation of the correlation parameters f_2, f_3 and f_4 with $\langle n \rangle$ for *pn* interaction data from 100–400 GeV. It can be seen from figure 1a that the parameter f_2 gives a linear relationship with $\langle n \rangle$ thereby showing

Table 3. The average multiplicities $\langle n \rangle$ and the C-moments defined by $C_q = \langle n^q \rangle / \langle n \rangle^q$ calculated from *pn* data at energies from 100 to 400 GeV.

Energy (GeV)	$\langle n \rangle$	C_2	C_3	C_4	C_5
100	$6{\cdot}2 \pm 0{\cdot}11$	$1{\cdot}3 \pm 0{\cdot}02$	$2{\cdot}0 \pm 0{\cdot}06$	$3{\cdot}53 \pm 0.21$	$6{\cdot}97 \pm 0{\cdot}54$
200	$7{\cdot}11 \pm 0{\cdot}15$	$1{\cdot}32 \pm 0{\cdot}02$	$2{\cdot}06 \pm 0{\cdot}08$	$3{\cdot}65 \pm 0{\cdot}21$	$7{\cdot}15 \pm 0{\cdot}57$
300	$7{\cdot}84 \pm 0{\cdot}17$	$1{\cdot}32 \pm 0{\cdot}02$	$2{\cdot}06 \pm 0{\cdot}09$	$3.67 \pm 0{\cdot}33$	$7{\cdot}27 \pm 1{\cdot}13$
400	$8{\cdot}20 \pm 0{\cdot}14$	$1{\cdot}32 \pm 0{\cdot}01$	$2{\cdot}08 \pm 0{\cdot}06$	$3{\cdot}76 \pm 0{\cdot}2$	$7{\cdot}6 \pm 0{\cdot}64$

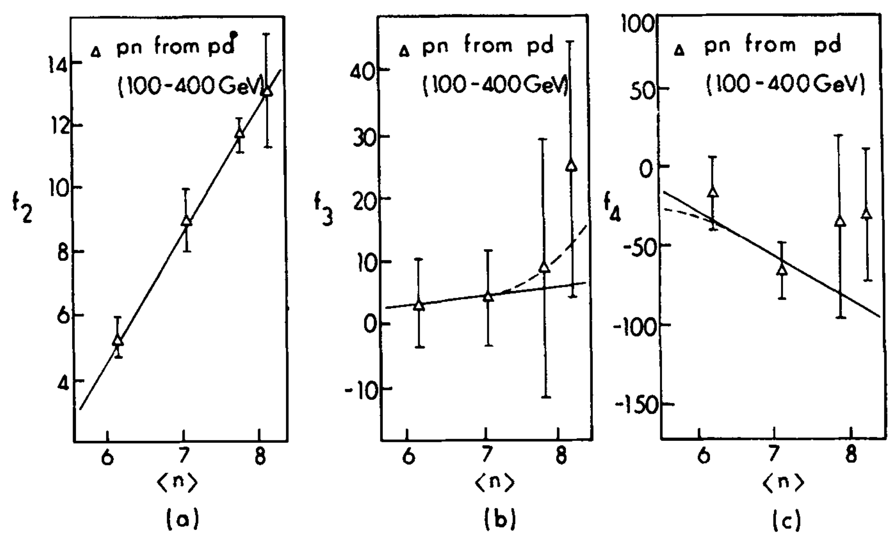

Figure 1. Average multiplicity $\langle n \rangle$ dependence of the Mueller correlation parameters for *pn* distributions from deuterium bubble chamber data. The solid lines show the predictions of Muller-Regge and the broken curves show the predictions of KNO scaling hypotheses.

compatibility of data Mueller-Regge view point only. The variation of the parameter f_3 with $\langle n \rangle$ as shown in figure 1b does not show any compatibility with Mueller-Regge view point (solid line). The 400 GeV point seems to be much away from fitting the prediction of KNO scaling (curve) (Bhattacharjee *et al* 1989a). The striking feature shown by the third correlation parameter f_4 for 400 GeV *pn* interaction, as shown in figure 1c, still confirms the trend shown by f_3 in figure 1b. The value of f_4 in this case does not satisfy either a linear fit (Mueller-Regge view point) or a curvilinear fit (KNO scaling) (Bhattacharjee *et al* 1990a). This behaviour of the correlation parameters, therefore significantly point towards some other mechanism at play. It also signifies that the asymptoticity has not been reached at this energy and that the data do not support scaling. Similar scaling violations has been reported by Thome *et al* (1977) at ISR energies and by Alner *et al* (1984, 1985, 1986) at $\bar{p}p$ collider energy. In our recent studies (Bhattacharjee *et al* 1989a, b, 1990a) we have shown that *pp* interaction data at 400 GeV from hydrogen bubble chamber also show scaling violations.

4. Fitting of *pn* data with negative binomial distribution and its scaling

The negative binomial (NB) distribution considered here is of the form:

$$P_n = \begin{bmatrix} n+k-1 \\ n \end{bmatrix} \left[\frac{\langle n \rangle / k}{1 + \langle n \rangle / k} \right]^n \frac{1}{(1 + \langle n \rangle / k)^k}$$

where P_n is the probability of observing a charged multiplicity n, $\langle n \rangle$ is the average multiplicity and k is a parameter affecting its shape. We recall that $k = 1$ means a geometric (exponential) distribution and $k \to \infty$ a Poisson distribution. We have fitted our experimental *pn* distribution data at 400 GeV with the NB distribution until a best fit was achieved using the minimum χ^2 method. Similar fittings with NB distribution of *pn* data at 100, 200 and 300 GeV have also been made. The range of multiplicities fitted at each energy together with fitted parameters and goodness of fit are given in table 4. Figure 2 shows the result of 400 GeV *pn* interaction data with NB distribution.

It can be seen from figure 2 that the fitted distribution deviates from the experimental one at 400 GeV for multiplicities $3 \leqslant n \leqslant 11$. We recall that diffractive events, which dominate lower multiplicity events, may cause such deviation. Since the diffractive

Table 4. Fitted multiplicity ranges and parameters of NB distribution fitted to the *pn* data at different energies and the resulting χ^2_{min} and $y = (2\chi^2_{min})^{1/2} - (2\,\mathrm{DF} - 1)^{1/2}$ where DF is the number of degrees of freedom.

Energy E_{lab} (GeV)	Fitted multiplicity range	Parameters of fit $\langle n \rangle$	k	Goodness of fit χ^2_{min}/DF	y	$1/\langle n \rangle + 1/k$
100	9–23	6.8 ± 0.13	12.4 ± 2.06	2.4/6	−0.8	0.228
200	9–23	8.2 ± 0.08	11.4 ± 0.90	2.43/6	−0.8	0.21
300	11–27	8.9 ± 0.08	11.0 ± 0.57	7.45/7	0.54	0.203
400	11–23	9.6 ± 0.09	9.3 ± 0.63	1.4/5	−0.97	0.212

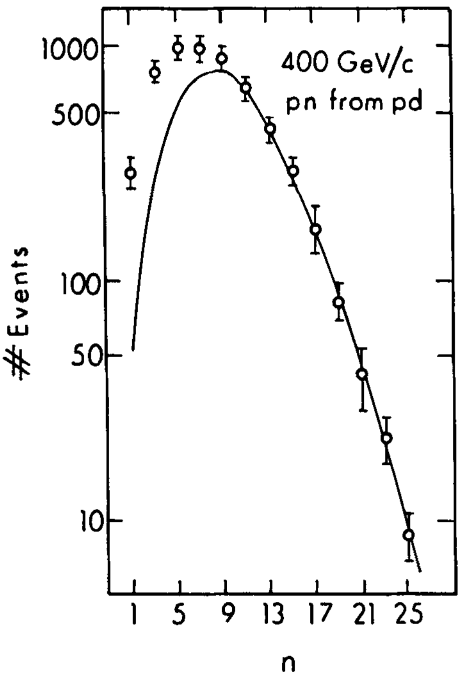

Figure 2. Multiplicity distribution in 400 GeV *pn* interactions. The solid curve shows the result of the fit with NB distribution.

Table 5. Normalized multiplicity distribution of diffractive events in *pn* interactions derived from DBC data (see text).

n \ Energy (GeV)	100	200	300	400
1	552 ± 154	710 ± 237	603 ± 177	422 ± 66
3	605 ± 82	975 ± 71	784 ± 58	892 ± 121
5	317 ± 58	430 ± 66	289 ± 50	724 ± 163
7	38 ± 38	130 ± 59		373 ± 183
9		20 ± 48		203 ± 163
11				13 ± 117
Total	1512 ± 188	2265 ± 267	1676 ± 193	2627 ± 345

events in this experiment were not either identified or available, we have determined the magnitude of this deviation both energywise and multiplicitywise. Also since the number of events at energies considered here is different, we have normalized total events at each energy by setting it at 10000 (inelastic events). The deviations after the normalization are given in table 5. In one of our recent studies (Bhattacharjee *et al* 1990b) we have shown that the energy dependence of these deviations is in agreement with that of the single diffractive events in *pp* interactions (Goulianos 1976, 1983, 1987). In this context it is quite appropriate to surmise that the deviations so arrived at in *pn* interactions may be quite reasonably accounted for as being equivalent to those due to single diffractive events and that the fitting of *pn* interaction data with NB distribution is in order.

The immediate consequence of fitting of *pn* data with NB distribution opens up

Table 6. *C*-moments calculated from the two parameters $\langle n \rangle$ and k of the fitted NB distributions of *pn* data at energies from 100 to 400 GeV.

Energy (GeV)	C_2	C_3	C_4	C_5
100	$1{\cdot}23 \pm 0{\cdot}01$	$1{\cdot}75 \pm 0{\cdot}05$	$2{\cdot}83 \pm 0{\cdot}15$	$5{\cdot}08 \pm 0{\cdot}41$
200	$1{\cdot}21 \pm 0{\cdot}01$	$1{\cdot}69 \pm 0{\cdot}03$	$2{\cdot}67 \pm 0{\cdot}07$	$4{\cdot}66 \pm 0{\cdot}20$
300	$1{\cdot}20 \pm 0{\cdot}005$	$1{\cdot}67 \pm 0{\cdot}02$	$2{\cdot}61 \pm 0{\cdot}05$	$4{\cdot}51 \pm 0{\cdot}13$
400	$1{\cdot}21 \pm 0{\cdot}01$	$1{\cdot}70 \pm 0{\cdot}03$	$2{\cdot}70 \pm 0{\cdot}08$	$4{\cdot}78 \pm 0{\cdot}21$

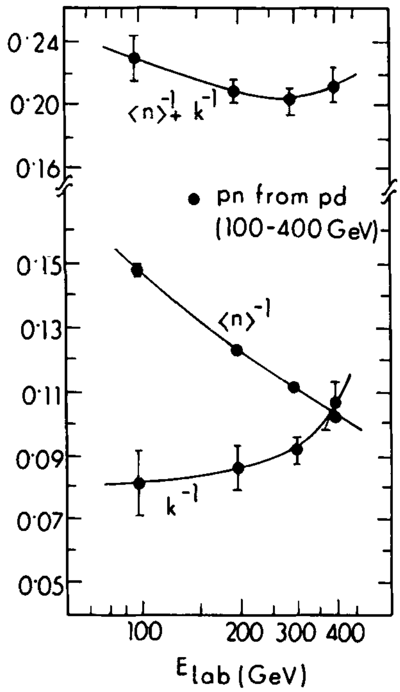

Figure 3. Apparent scaling up to 300 GeV *pn* data shown with the help of $\langle n \rangle^{-1}$, k^{-1} of NB distribution and their sum which determine the magnitude of *C*-moments in the NB description.

avenues for further studies of energy independence of *C*-moments and apparent scaling observed so far at non-asymptotic energies. In table 6 are given the values of *C*-moments for *pn* data at different energies calculated from the parameters $\langle n \rangle$ and k of the fitted NB distributions. It can be seen that the values show the energy independence fairly but a closer look will reveal an enhancement at 400 GeV. At this point, we recall that at asymptotic energies (i.e. $\langle n \rangle \gg k$) scaling would need $k = \text{constant}$. In figure 3 we have shown the energy variation of k^{-1} and $\langle n \rangle^{-1}$ in the lower part of the figure. It can be seen that while the $\langle n \rangle^{-1}$ shows a decrease, the k^{-1} shows a rising trend, the slope being higher from 300 to 400 GeV than at lower energies where it is more linear. Again, scaling requires that the moments of the well-known scaling parameter $Z = n/\langle n \rangle$ are energy independent. For a NB distribution it can be shown that the sum $1/\langle n \rangle + 1/k$ dominates the expressions for the C-moments C_2 through C_5. The sum $1/\langle n\rangle + 1/k$ plotted against energy in the upper part of the figure 3 shows that there is a minimum up to and around 300 GeV after which it rises again. This shows that within experimental errors the C-moments can be said to be constant up to about 300 GeV data. This explains the apparent scaling of *pn* data at non-asymptotic energies and its violation at and above 400 GeV

5. Conclusion

We have analyzed our *pn* data at 400 GeV together with those available from 100 GeV and above. The data show no scaling above 300 GeV. The distributions follow negative binomial description. The apparent scaling observed in this energy range is shown to be suitably accounted for by the nature of variation of the parameters of fitted negative binomial distributions over the energy range considered in this paper. The scaling violation in *pn* interactions shows compatibility with similar characteristics of *pp* and $\bar{p}p$ data at high energy.

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