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Density of quarks in heavy spherical nuclei using NRQSM

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Abstract. A nonrelativistic quark shell model (NRQSM) is used to derive an expression for the density of quarks in heavy spherical nuclei. It is shown that quark density is related in a simple way with the probability of finding a nucleon in a nucleus. The quark density is used to determine the ratio of average distance between two quarks to the average distance between two nucleons.

Keywords. Quark density; shell model; cluster model.

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One of the most important aspects of a many-body system is to study its various properties starting from its basic constituent particles. As our knowledge of the particles which are used to build a nucleon advances, we get more and more interested in building a theory of nucleus starting from this basic picture. As a first step in this direction we would like to show how we can derive an analytic expression for the density of these constituent particles in a heavy spherical nucleus. We shall do this using non relativistic quark shell model (NRQSM).

Let us consider a nucleus of mass number A, then its wave function using NRQSM can be written as

$$\Phi = \int \mathscr{A}\left[\prod_{i=1}^{A} \chi_{i,c=0,s=\frac{1}{2},m_{s_i},I=\frac{1}{2},m_{I_i}}(\bar{\rho}_i, \bar{\lambda}_i)\,\phi_{n_i l_i m_{l_i}}\left(\frac{1}{3}\sum_{k=0}^{2} \bar{r}_{i+k}\right)\right]\delta(\bar{R})\,\mathrm{d}\bar{R}, \tag{1}$$

where

$$\bar{R} = \sum_{i=1}^{3A} \bar{r}_i, \tag{2a}$$

and χ_i denotes the internal wave function of a nucleon consisting of three constituent quarks. It is an antisymmetric wave function having colour quantum number $c = 0$, spin $s = \frac{1}{2}$, projection of spin m_{s_i}, isospin $I = \frac{1}{2}$ and projection of isospin m_{I_i}. The radial part consists of two Gaussian wave functions

$$\left(\frac{2}{3\pi b^2}\right)^{3/4}\left(\frac{2}{4\pi b^2}\right)^{3/4}\exp\left(-\frac{\rho_i^2}{4b^2}-\frac{\lambda_i^2}{3b^2}\right), \tag{2b}$$

b, being the harmonic oscillator parameter, and $\bar{\rho}_i = \bar{r}_i - \bar{r}_{i+1}, \bar{\lambda}_i = \bar{r}_{i+2} - \frac{1}{2}(\bar{r}_i + \bar{r}_{i+1})$. The wave function $\phi_{n_i l_i m_{l_i}}$ is the centre-of-mass wave function of ith nucleon having quantum numbers n_i, l_i; m_{l_i}. The integration over δ function takes care of the centre-of-mass motion of the nucleus while $\mathscr{A}$ antisymmetrizes quarks between diff?rent nucleons.

Since we shall be considering closed shell nuclei ($J = T = 0$) further coupling of angular momenta is not shown in expression (1).

We first note that the wave function Φ gives the usual wave function for deutron which has been studied in detail in the past (Warke and Shanker 1980; Yamamouchi *et al* 1985; Ito and Faessler 1987).

The density of quarks $\rho(\bar{r})$ is given by

$$\rho(\bar{r}) = \left\langle \Phi \left| \sum_{i=1}^{3A} \delta(\bar{r} - \bar{r}_i) \right| \Phi \right\rangle. \tag{3}$$

It is normalized such that

$$\int \rho(\bar{r})\,\mathrm{d}\bar{r} = 3A. \tag{4}$$

In order to obtain a simple expression for $\rho(\bar{r})$ we make two further approximations. The first approximation is to neglect the centre-of-mass correction. This approximation is justified since we shall be considering heavy nuclei. The second approximation is to use a partial antisymmetrization operator $\mathscr{A}$ which produces antisymmetry between pairs of nucleons i and j. This is the same kind of approximation which one sometime makes in R-matrix theory of nuclear reactions. What this means here is that in the wave function Φ we are not including higher configurations arising from Δ (Warke 1985).

Using these two simplifications we can write $\rho(\bar{r})$ using (1), (2), (3) and some algebraic manipulations as

$$\rho(\bar{r}) = K \int \mathrm{d}\bar{R} \sum_{nlm_l} \left| \phi_{nlm_l}(\bar{R}) \right|^2 \left[\exp - \frac{1}{2b^2}(\bar{R} - \bar{r})^2 + \sqrt{2} \exp - \frac{1}{4b^2}(\bar{R} - \bar{r})^2 \right], \tag{5}$$

where K is the normalization constant. Since $\Sigma|\phi_{nlm_l}(\bar{R})|^2$ is the probability $P(\bar{R})$ of finding a nucleon at $\bar{R}$, (5) can be written as

$$\rho(\bar{r}) = K \int \mathrm{d}\bar{R} P(\bar{R}) \left[\exp - \frac{1}{2b^2}(\bar{R} - \bar{r})^2 + \sqrt{2} \exp - \frac{1}{4b^2}(\bar{R} - \bar{r})^2 \right]. \tag{6}$$

This is one of the main results of the present investigation.

We now use expression (6) for closed shell nuclei, for which $P(\bar{R})$ is spherically symmetric. This gives

$$\rho(\bar{r}) = K \left[\exp\left(-\frac{r^2}{2b^2} \right) \int \mathrm{d}R R^2 P(R) j_0\left(\frac{Rr}{b^2} \right) \exp\left(-\frac{R^2}{2b^2} \right) + \sqrt{2} \exp\left(-\frac{r^2}{4b^2} \right) \int \mathrm{d}R R^2 P(R) j_0\left(\frac{Rr}{2b^2} \right) \exp\left(-\frac{R^2}{4b^2} \right) \right]. \tag{7}$$

where j_0 is spherical Bessel function.

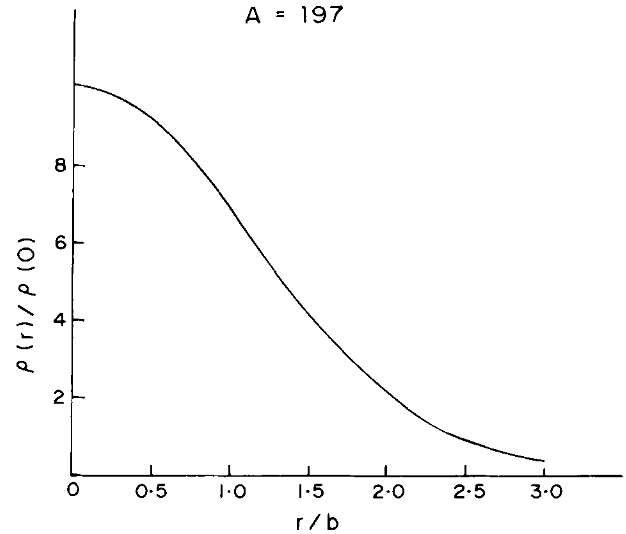

Figure 1. A plot of $\rho(r)/\rho(0)$ versus r/b, where b is the harmonic oscillator parameter.

For heavy spherical nuclei $P(R)$ is given by (Ullah 1987).

$$P(R)=\left[\frac{(\frac{3}{2}A)^{1/3}}{3/2b^2}-R^2\right]^{3/2} \tag{8}$$

Since A is large, this finally gives us

$$\rho(r)=K\left[1+\frac{r^2}{\lambda^2 b^2}\right][\exp(-r^2/b^2)+4\exp(-r^2/2b^2)], \tag{9}$$

where

$$\lambda=\left[\frac{2}{3}\left(\frac{3}{2}A\right)^{1/3}\right]^{1/2}. \tag{10a}$$

A plot of this density for $A=197$ is shown in figure 1.

As an application of the present formulation we now calculate the average distance between two quarks d_q in a heavy spherical nucleus. Using (10a) and an earlier result (Ullah 1988) which gives average distance between two nucleons d_n, we find that

$$\frac{d_q}{d_n}=0{\cdot}56, \tag{10b}$$

which shows that most of the time quarks are confined within the nucleons.

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