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On πd elastic scattering using aligned deuteron targets

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Abstract. A theoretical study is made of the advantages to be gained in polarizing a deuteron target by applying an external electric quadrupole field with non-zero asymmetry parameter for carrying out $\mathbf{d}(\pi, \pi)\mathbf{d}$ experiments, in order to determine empirically the helicity amplitudes A, B, C, D characterizing the scattering, without any phase ambiguities.

Keywords. Aligned deuteron targets; spin transfer parameters; πd helicity amplitudes; polarized spin one system.

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1. Introduction

Considerable experimental interest (Holt *et al* 1979, 1981; Bolger *et al* 1981, 1982; Ulbricht *et al* 1982; Gruebler *et al* 1982; Koenig *et al* 1983; Ungricht *et al* 1984, 1985; Shin *et al* 1985; Smith *et al* 1986, 1987, 1988; Ottermann *et al* 1985, 1988a, b; Stevenson *et al* 1989; SIN proposal) has been evinced during the last decade in studying the polarization observables in elastic πd scattering, which in turn has led to renewed and more incisive theoretical studies (Giraud *et al* 1979, 1980; Rinat *et al* 1979a, b; Mizutani *et al* 1981; Betz and Lee 1981; Rinat and Starkand 1983; Blankleider and Afnan 1981, 1985; Afnan and McLeod 1985; Andrade *et al* 1986; Ferreira *et al* 1987; Lee and Matsuyama 1987; Lamot *et al* 1987; Popping *et al* 1987; Stevenson and Shin 1987; Garcilazo 1984, 1987, 1988; Jennings 1988; Jennings and Rinat 1988; Garcilazo *et al* 1989). However it is found that none of the theoretical models could reproduce all the experimental data simultaneously. Thus in recent attempts emphasis has been laid on model independent analysis of the πd scattering data. Since the process is characterized by four complex helicity amplitudes A, B, C, D (in the notation of Grein and Locher 1981) and since an overall phase is not, in anyway, observable, one has to determine seven real unknowns at each energy and angle from the data. However the important thing to be noted here is that the equations connecting the observables with the amplitudes are not linear, but sesquilinear. Therefore, the expectation that one should be able to solve the seven real unknowns using seven equations need not necessarily be valid although such an expectation seems to underlie some of the investigations. For example, Garcilazo *et al* (1989) claim: "Taking two relations from a theoretical model in addition to the existing five observables, the seven real components of the πd helicity amplitudes have been determined". The two relations from the theoretical model referred to by these authors concern the choice of parameters β and α in their eqs (30) and (44) respectively. However, a careful reading of their paper indicates that more theoretical inputs must have been used, though inadvertently, in several other contexts, as well. For example, (i) given a solution for $\cos\phi$ (where ϕ denotes the phase of an amplitude- say, A or C or D- whose modulus value is known), it determines only the real part of the amplitude unambiguously but leaves a plus/minus ambiguity with respect to the imaginary part. (ii) given ($|A|^2+|C|^2$) and $2|A||C|$, one cannot determine (completely empirically) the sign of $|A|-|C|$ and hence $|A|$ and $|C|$ uniquely. Therefore, a complete empirical determination of the amplitudes A, B, C, D may need (Ramachandran *et al* 1988) measurement of more than 7 observables.

Experiments for which results have already been published are either concerned with the measurement of the differential cross section (equivalently the analyzing powers) using polarized deuteron targets or measurements of the recoil deuteron polarization with initially unpolarized targets. The target polarization is, in all these experiments, achieved through the application of an external magnetic field, which creates essentially an 'oriented' (Ramachandran and Mallesh 1984) type of target characterized by merely three out of the eight available (real) spin polarization degrees of freedom, whereas the application of an external quadrupole field generates an 'aligned' type of target which is (i) 'non oriented' and characterized by five independent spin polarization degrees of freedom, provided the asymmetry parameter η for the field is non-zero and (ii) 'oriented' if $\eta=0$. The spin polarization degrees of freedom for a spin one system can, in general, be enumerated in terms of three axes and two real scalars (Ramachandran and Ravishankar 1986; Ramachandran *et al* 1987).

The purpose of this paper is to study theoretically the advantages to be gained in polarizing a deuteron target by applying an external electric quadrupole field with non-zero asymmetry parameter η, though we are not too sure if the technology (Hamada *et al* 1981; Bruss *et al* 1986; Meyer 1984) for carrying out such an experiment is presently sufficiently advanced; we hope the present theoretical study will provide the needed impetus to such experiments. In §2, we briefly summarize the distinction between the two types of polarized targets and examine in §3 the appropriate geometries at which the πd scattering experiment could be carried out with aligned deuterons. In §4, we discuss the problem of determining (completely empirically) the amplitudes A, B, C, D using data generated by such experiments. Finally in §5, we present numerical estimates for the thermal equilibrium tensor polarization (TETP) that can be achieved using quadrupole fields in various molecules.

2. Target polarization

The deuteron being a spin one particle, its state of polarization is characterized, apart from vector polarization $\boldsymbol{P}$, by $P_{\alpha\beta}$, $\alpha, \beta=x, y, z$, which constitute a traceless symmetric second rank cartesian tensor or equivalently by spherical tensors t_{kq}, $q=-k, -k+1, \ldots k-1, k$ of rank $k=1,2$. Our notations and the normalizations for t_{kq} follow exactly the Madison convention (Barschall and Haeberli 1970) and the earlier usage of Ramachandran and Umerjee (1964).

The typical experimental arrangement presently being used to generate t_{kq} is described in detail by Smith *et al* (1988) and Ottermann *et al* (1988). Here a magnetic field $\boldsymbol{B}$ is applied making an angle α with $\hat{k}_{\text{in}}$ and an azimuthal angle β with respect to $\hat{k}_{\text{in}}\times\hat{k}_{\text{out}}$, where $\hat{k}_{\text{in}}$ and $\hat{k}_{\text{out}}$ denote the incident and outgoing pion directions in the π-d centre of mass frame. The tensor parameters t_{kq} are then functions of α and β, and by choosing suitable values of the angles, some of the t_{kq}'s could be reduced to zero, so that one can measure the analyzing powers T_{kq} associated with the non-zero t_{kq} using the expression for the differential cross section

$$\sigma_{\text{pol}} = \sigma_{\text{unpol}} \left[1 + \left(\sum_{k=1}^{2} \sum_{q=-k}^{k} (-1)^q t_{k-q} T_{kq} \right) \right] \tag{1}$$

for polarized targets. Here σ_{unpol} denotes the differential cross section (at the same energy and angle) using an unpolarized target.

In this context, it is worth observing that if the spin system is polarized by the application of a uniform magnetic field $\boldsymbol{B}$, the spin states $|1m\rangle^0$ (defined with respect to the axis of quantization chosen along $\boldsymbol{B}$) are the ones which get populated proportional to $\exp(-\mu Bm/KT)$ and consequently the only non-zero spherical tensor parameters t^0_{kq} (with respect to the above z axis) are

$$p_z = \sqrt{\frac{2}{3}}\, t^0_{10} = \frac{1}{n}(n_{+1} - n_{-1}) = \frac{1}{N}[\exp(-\mu B/KT) - \exp(\mu B/KT)] \tag{2}$$

$$= -4 \tanh(\mu B/2KT)/[3 + \tanh^2(\mu B/2KT)] \tag{3}$$

$$p_{zz} = \sqrt{2}\, t^0_{20} = \frac{1}{n}(n_{+1} + n_{-1} - 2n_0)$$

$$= \frac{1}{N}[\exp - (\mu B/KT) + \exp(\mu B/KT) - 2] \tag{4}$$

$$= -2 + 3\sqrt{p_z^2 + \frac{4}{N^2}} \tag{5}$$

with

$$N = 1 + \exp - (\mu B/KT) + \exp(\mu B/KT) \tag{6}$$

where μ denotes the magnetic dipole moment of the deuteron, K the Boltzmann constant, T the absolute temperature and $n = n_{+1} + n_0 + n_{-1}$ denotes the total number, $n_{+1,0,-1}$ being the populations in the $|1m\rangle^0$ states with $m = +1, 0, -1$ respectively. The orientation parameters, t^0_{k0} are thus known in terms of the single parameter $(\mu B/KT)$ and the spin assembly is oriented, with the axis of orientation along $\boldsymbol{B}$. Therefore, it is completely characterized by three independent real parameters viz., $\mu B/KT$ and the two angles α, β defining the axis of orientation with respect to the laboratory frame.

In contrast, if one uses an external electric quadrupole field to polarize the target, the system is aligned and the states $|x\rangle^A$, $|y\rangle^A$ and $|z\rangle^A$ (Ramachandran *et al* 1984) are the ones which get populated proportional to $\exp(-E_x/KT)$, $\exp(-E_y/KT)$ and $\exp(-E_z/KT)$ respectively, where

$$E_x = E_0 + A(1+\eta);\ |x\rangle^A = \frac{1}{\sqrt{2}}(|11\rangle^A + |1-1\rangle^A) \tag{7}$$

$$E_y = E_0 + A(1-\eta);\ |y\rangle^A = \frac{1}{\sqrt{2}}(|11\rangle^A - |1-1\rangle^A) \tag{8}$$

$$E_z = E_0 - 2A;\quad |z\rangle^A = |10\rangle^A \tag{9}$$

are the eigen values and the corresponding eigen states (Ramachandran *et al* 1987) of the quadrupole hamiltonian (Abragam 1961; Lucken 1969; Poole and Farach 1972). Here, E_0 denotes the energy in the absence of the external field and A, the quadrupole coupling constant, while the states $|1m\rangle^A$ are defined by choosing the axis of quantization to coincide with the z axis of the principal axes of alignment frame (PAAF) characterizing the electric quadrupole field. Consequently, the only non-zero spherical tensor parameters in PAAF are

$$t_{20}^A = \frac{1}{\sqrt{2}}\left(1 - \frac{3\exp(2A/KT)}{D}\right) \tag{10}$$

$$\simeq -\sqrt{2}A/KT \tag{11}$$

$$t_{22}^A = t_{2-2}^A = \frac{\sqrt{3}}{2D}(\exp[-A(1+\eta)/KT] - \exp[-A(1-\eta)/KT]) \tag{12}$$

$$\simeq -A\eta/\sqrt{3}KT \tag{13}$$

where

$$D = \exp[-A(1-\eta)/KT] + \exp[-A(1+\eta)/KT] + \exp[2A/KT]. \tag{14}$$

If the asymmetry parameter $\eta = 0$, $E_x = E_y$, then $t_{22}^A = 0$, thus rendering the system oriented. In general, the aligned spin assembly is completely characterized by five independent parameters (A/KT), $(A\eta/KT)$ and the three Euler angles ϕ, θ, ψ prescribing the relative orientation of the PAAF with respect to the laboratory frame.

The spherical tensor parameters t_{kq} in the laboratory frame are thus given by

$$t_{1q} = 0; \quad q = 0, \pm 1 \tag{15}$$

$$t_{20} = \tfrac{1}{2}(3\cos^2\theta - 1)t_{20}^A + \sqrt{\tfrac{3}{2}}\sin^2\theta\cos 2\psi\, t_{22}^A \tag{16}$$

$$t_{21} = \sin\theta\exp(i\phi)[\sqrt{\tfrac{3}{2}}\cos\theta\, t_{20}^A - \cos 2\psi\cos\theta\, t_{22}^A - i\sin 2\psi\, t_{22}^A] \tag{17}$$

$$t_{22} = \exp(2i\phi)[\sqrt{\tfrac{3}{8}}\sin^2\theta\, t_{20}^A + \cos 2\psi(\cos^4\theta/2 + \sin^4\theta/2)t_{22}^A + i\sin 2\psi(\cos^4\theta/2 - \sin^4\theta/2)t_{22}^A]. \tag{18}$$

Looking at the above discussion from the multiaxial point of view (Ramachandran and Ravishankar 1986; Ravishankar 1986), the target system which is polarized using an external magnetic field is characterized by a single axis, viz., the axis of orientation and a single real scalar, while the target polarized by the application of an electric quadrupole field, is characterized by two axes and one real scalar. This can be identified by solving

$$t_{22}^A + \sqrt{6}t_{20}^A Z^2 + t_{22}^A Z^4 = 0 \tag{19}$$

where

$$Z = \cot(\Theta/2)\exp(-i\Phi) \tag{20}$$

and the angles (Θ, Φ), denoting the polar angles, characterize the axes. The solutions are readily seen to be

$$\cot^2(\Theta_{1,2}/2) = -3/\eta \pm [(3/\eta)^2 - 1]^{1/2} \tag{21}$$

and

$$\Phi = 0, \pi.$$

Equation (21) clearly shows that $\Theta_2 = \pi - \Theta_1$. Thus the four solutions of the quadratic eq. (19) viz., $(\Theta_1, 0)$, $(\pi - \Theta_1, 0)$, (Θ_1, π), $(\pi - \Theta_1, \pi)$ define two sets of axes. It is interesting to compare the two types of polarized assemblies considered above. Rewriting (1) in the well known form (Ottermann *et al* 1988a) as

$$\sigma_{\text{pol}} = \sigma_{\text{unpol}} \left[1 + a_{11} iT_{11} + a_{20} T_{20} + a_{21} T_{21} + a_{22} T_{22}\right] \tag{22}$$

we note that

$$a_{11} = \sqrt{3} p_z \sin\alpha \cos\beta \tag{22.1M}$$

$$a_{20} = \frac{p_{zz}}{\sqrt{2}} \frac{(3\cos^2\alpha - 1)}{2} \tag{22.2M}$$

$$a_{21} = \sqrt{3} p_{zz} \sin\alpha \cos\alpha \sin\beta \tag{22.3M}$$

$$a_{22} = -\frac{\sqrt{3}}{2} p_{zz} \sin^2\alpha \cos 2\beta \tag{22.4M}$$

for the magnetic fields.

In the electric quadrupole field also σ_{pol} is given by an expression of the same form (22) but with

$$a_{11} = 0 \tag{22.1Q}$$

$$a_{20} = \frac{p_{zz}^A}{2\sqrt{2}} [3\cos^2\theta + \eta \sin^2\theta \cos 2\psi - 1] \tag{22.2Q}$$

$$a_{21} = \frac{p_{zz}^A}{2\sqrt{3}} [3\sin 2\theta \cos\phi + 2\eta \sin\theta \sin\phi \sin 2\psi - \eta \sin 2\theta \cos\phi \cos 2\psi] \tag{22.3Q}$$

$$a_{22} = \frac{p_{zz}^A}{2\sqrt{3}} [3\sin^2\theta \cos 2\phi + 2\eta \cos^4\theta/2 \cos(2(\phi + \psi)) + 2\eta \sin^4\theta/2 \cos(2(\phi - \psi))] \tag{22.4Q}$$

for the quadrupole field.

The interesting thing to be noted here is that the vector analyzing power T_{11} does not occur at all, when the polarization of the spin-1 system is achieved in this way. This is, in fact, a desirable thing since experiments wherein magnetic fields have been applied lead actually to a stronger vector polarization than the tensor polarization and ingeneous methods have been devised (Bruss *et al* 1986) to reduce vector polarization and increase the tensor polarization. Moreover extensive data on iT_{11} exists already.

3. Choice of geometries

With the existing arrangement of polarizing deuteron targets using magnetic fields, T_{20} has been measured directly by choosing $\alpha = 0$ while the combination

$$\tau_{21} \equiv T_{21} + \tfrac{1}{2}(T_{20}/\sqrt{6} + T_{22}) \tag{23}$$

has been determined (Smith *et al* 1988; Ottermann *et al* 1988a) by choosing $\alpha=\pi/4$ and $\beta=\pi/2$, and the combination

$$\tau_{22} \equiv T_{22} + T_{20}/\sqrt{6} \tag{24}$$

has been determined (Ottermann *et al* 1988b) by choosing $\alpha=\pi/2$, $\beta=0$ and $\alpha=\pi/2$, $\beta=\pi$ in (22·1M–22·4M). Moreover, the recoil deuteron tensor polarization t_{20} (Holt *et al* 1979, 1981; Ungricht *et al* 1984, 1985; Shin *et al* 1985) with unpolarized targets has also been measured and since (centre of mass observable) $t_{20}=T_{20}$ (because of the time reversal invariance, Grein and Locher 1981) reliable data on T_{20} is available. The measurements involved in τ_{21} and τ_{22} are those of $\sigma_{\text{pol}}(\alpha,\beta)$ and σ_{unpol} (at the same angle and energy) and p_{zz}. Since T_{20} term is weighted by $1/\sqrt{6}$ in (24), it is usually neglected or one may use measured values of T_{20} in estimating T_{22} from τ_{22}. Likewise the measured τ_{21} is approximated to T_{21}, by neglecting, T_{20}, as it is weighted by $1/2\sqrt{6}$ and T_{22} which is small in the backward hemisphere where the experiments are performed.

As the programme to determine the amplitudes A, B, C, D purely from experiment gains momentum, the need for more accurate estimates for T_{21} and T_{22} are likely to be felt and it is worthwhile to look around for alternative methods of determining T_{21} and T_{22} with possibly greater accuracy.

In this context, it is worth recalling that in the case of d-^{4}He scattering (Schwandt and Haeberli 1968; Koenig and Gruebler 1970) estimates of the analyzing powers T_{kq} have been obtained using

$$iT_{11} = \frac{1}{2\sqrt{3}p_z}\left(\frac{\sigma_+ - \sigma_-}{\sigma_{\text{unpol}}}\right) \tag{25}$$

$$T_{20} = \frac{\sqrt{2}}{p_{zz}}\left(\frac{\sigma_0}{\sigma_{\text{unpol}}} - 1\right) \tag{26}$$

$$T_{21} = \frac{1}{\sqrt{3}\,p_{zz}}\left(\frac{\sigma_R - \sigma_L}{\sigma_{\text{unpol}}}\right) \tag{27}$$

$$T_{22} = \frac{1}{\sqrt{3}p_{zz}}\left(3 - \frac{\sigma_+ + \sigma_- + \sigma_0}{\sigma_{\text{unpol}}}\right) \tag{28}$$

where $\sigma_{+,-,0}$ and $\sigma_{R,L}$ denote σ_{pol} with the angles (α,β) taking values respectively $(\pi/2,0)$, $(\pi/2,\pi)$, $(0,0)$, $(\pi/4,\pi/2)$ and $(\pi/4,-\pi/2)$. It is clear that one has to make measurements at more than one set of (α,β) to isolate iT_{11}, T_{21}, T_{22}, while T_{20} alone can be directly measured by choosing $\alpha=0$. Although σ_L and σ_R are found to be sensitive to small uncertainties in α, these get cancelled out in the combination $(\sigma_R-\sigma_L)$ which is needed to measure T_{21}.

We may also point out that T_{21} can be isolated by measuring σ_{pol} at (α,β) and $(\pi-\alpha,\beta)$ and subtracting one from the other.

Considering the case of the external electric quadrupole field—eqs (22.1Q) to (22.4Q)—we notice, once again, that T_{20} can be measured directly by choosing $\theta=0$ and $\phi+\psi=45°$. More interestingly, it is possible here to measure directly several families, which contain linear combinations of T_{22} and T_{20}. Choosing $\theta=0$ or π

$$\tau_{22}^{a,\pm}(\phi,\psi) \equiv \frac{\sqrt{2}}{p_{zz}^A}\left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}} - 1\right) = T_{20} + \sqrt{\tfrac{2}{3}}\,\eta\cos(2\phi \pm 2\psi)\,T_{22}. \tag{29}$$

Likewise choosing $\phi=\pi/2$, $\psi=0$ or $\pi/2$

$$\tau_{22}^{b,\pm}(\theta)\equiv\frac{-\sqrt{8}}{p_{zz}^A}\left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}}-1\right)=(1-3\cos^2\theta\mp\eta\sin^2\theta)T_{20}$$
$$+\sqrt{\tfrac{2}{3}}(3\sin^2\theta\pm2\eta(\cos^4\theta/2+\sin^4\theta/2))T_{22}. \tag{30}$$

Similarly when $\theta=\pi/2$, $\psi=0$ or $\pi/2$

$$\tau_{22}^{c,\pm}(\phi)\equiv\frac{-\sqrt{8}}{p_{zz}^A}\left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}}-1\right)=(1\mp\eta)T_{20}-\sqrt{\tfrac{2}{3}}\cos2\phi(3\pm\eta)T_{22}. \tag{31}$$

Finally, when $\theta=\pi/2$, $\phi=0$ or π

$$\tau_{22}^{d}(\psi)\equiv\frac{-\sqrt{8}}{p_{zz}^A}\left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}}-1\right)$$
$$=(1-\eta\cos2\psi)T_{20}-\sqrt{\tfrac{2}{3}}(3+\eta\cos2\psi)T_{22}. \tag{32}$$

Likewise, one can get several families which are linear combinations of T_{21} and T_{20}. For example, when $\phi=\pi/4$, $\psi=0$ or $\pi/2$

$$\tau_{21}^{a,\pm}(\theta)\equiv-\frac{\sqrt{8}}{p_{zz}^A}\left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}}-1\right)=(1-3\cos^2\theta\mp\eta\sin^2\theta)T_{20}$$
$$-\frac{1}{\sqrt{3}}\sin2\theta(3\mp\eta)T_{21} \tag{33}$$

and when $\phi=\pi/4$ and $\theta=\pi/2$

$$\tau_{21}^{b}(\psi)\equiv\frac{-\sqrt{8}}{p_{zz}^A}\left(\frac{\sigma_{\text{pol}}}{\sigma_{\text{unpol}}}-1\right)=(1-\eta\cos2\psi)T_{20}-\frac{2\eta}{\sqrt{3}}\sin2\psi T_{21}. \tag{34}$$

It is interesting to note that by choosing $\theta=54\cdot7^\circ$ and $\psi=\pm\pi/4$ or $\pm3\pi/4$, one can altogether eliminate the T_{20} term, so that several linear combinations of T_{21} and T_{22} can also be measured. Similar situations can also be envisaged by choosing $\alpha=54\cdot7^\circ$ in the magnetic fields. But such experiments have not been carried out so far, as such a choice of α, is not allowed by the target geometry. Hopefully, such a difficulty is not likely to arise in the quadrupole fields. Moreover, there is more freedom here in the choice of geometries as it is not necessary to uniquely specify all the angles ϕ, θ, ψ to generate the required form of the linear combinations. T_{21} and T_{22} can individually be estimated either by eliminating T_{20} from a pair of such measurements of τ_{21} and τ_{22} respectively or by using the known estimates of T_{20}, which can be isolated and therefore capable of being measured highly accurately.

4. Determination of the scattering amplitudes

The scattering matrix M describing πd elastic scattering is well known (Grein and Locher 1981) and the differential cross section σ_{unpol} (using an unpolarized target) is given by

$$3\sigma_{\text{unpol}} = \text{tr}(MM^\dagger) = 2|A|^2 + 4|B|^2 + 2|C|^2 + |D|^2 \equiv \alpha_1 \tag{35}$$

while the analyzing powers T_{2q} are given by

$$\left(\frac{3}{\sqrt{2}}\sigma_{\text{unpol}}\right) T_{20} = |A|^2 - |B|^2 + |C|^2 - |D|^2 \equiv \alpha_2 \tag{36}$$

$$\left(-\sqrt{\frac{3}{2}}\sigma_{\text{unpol}}\right) T_{21} = \text{Re}\,[B^*(A - C - D)] \equiv \alpha_3 \tag{37}$$

$$(\sqrt{3}\,\sigma_{\text{unpol}}) T_{22} = 2\,\text{Re}\,A^*C - |B|^2 \equiv \alpha_4 \tag{38}$$

Besides the differential cross section and analyzing powers T_{2q}, a set of polarization transfer observables is required for the complete empirical determination of the elements A, B, C, D of the scattering matrix. Since the overall phase of M is not an observable one has effectively to determine seven real unknowns. However, because of the bilinear nature of the observables, one needs, in practice, a set containing more than seven observables to determine the scattering amplitudes completely. Further, the experimental accuracy and the statistical nature of the data cause limitations and additional measurements may be needed to ensure the consistency of the determined set of amplitudes.

If target polarization is achieved using a quadrupole field, one can in principle, measure a set of nine real observables, namely, $t^f_{kq}(\phi, \theta, \psi)$ with $k = 0, 1, 2$ and $q = -k \cdots k$, for a given set of Euler angles (ϕ, θ, ψ) and A, η. Hence by varying (ϕ, θ, ψ) one can in fact make as many measurements as needed. However, measurement of some of the observables yield identical results (at a given scattering angle and incident energy), because of the time reversal invariance and parity invariance of the scattering matrix.

Observing that the recoil deuteron spin observables, with initially polarized deuteron targets, are given by

$$t^f_{kq}(\phi, \theta, \psi) = \text{tr}\,[(M\rho(\phi, \theta, \psi)M^\dagger \tau_{kq}(\mathbf{J}))/\text{tr}\,(M\rho(\phi, \theta, \psi)M^\dagger)] \tag{39}$$

where $\rho(\phi, \theta, \psi)$ denotes the initial spin density matrix and $\tau_{kq}(\mathbf{J})$, the spherical tensor operators normalized following Madison convention (Barschall and Haeberli 1970). The explicit expressions for the spin transfer observables, when the deuteron polarization is achieved using quadrupole fields, are

$$3\sigma_{\text{pol}}(\phi, \theta, \psi) t^f_{10}(\phi, \theta, \psi) = u_1\,\text{Im}\,B^*A + u_2\,\text{Im}\,B^*C + u_3\,\text{Im}\,AC^* \tag{40}$$

where $\sigma_{\text{pol}}(\phi, \theta, \psi)$ is given by (22) and

$$u_1 = u_2 = \frac{\sqrt{3}}{2} p^A_{zz}(-3\sin 2\theta \sin\phi + \eta \sin 2\theta \sin\phi \cos 2\psi$$
$$+ 2\eta \sin\theta \cos\phi \sin 2\psi)$$

$$u_3 = \sqrt{\tfrac{3}{2}}\, p^A_{zz}(3\sin^2\theta \sin 2\phi + 2\eta \cos^4\theta/2 \sin[2(\phi + \psi)]$$
$$+ 2\eta \sin^4\theta/2 \sin[2(\phi - \psi)])$$

$$3\sigma_{\text{pol}}(\phi, \theta, \psi) t^f_{11}(\phi, \theta, \psi) = w_1\,\text{Im}\,B^*A + w_2\,\text{Im}\,B^*C + w_3\,\text{Im}\,B^*D$$
$$+ w_4\,\text{Im}\,AD^* + w_5\,\text{Im}\,CD^* \tag{41}$$

where

$$w_1 = w_2^* = -i\sqrt{6} + i\sqrt{\tfrac{3}{8}}\, p_{zz}^A[(1 - 3\cos^2\theta + 3\sin^2\theta\exp(2i\phi))$$
$$- \eta\sin^2\theta\cos 2\psi + 2\eta\cos^4\theta/2\exp 2i(\phi+\psi)$$
$$+ 2\eta\sin^4\theta/2\exp 2i(\phi-\psi)]$$

$$w_3 = -i\sqrt{6} - i\sqrt{\tfrac{3}{2}}\, p_{zz}^A(1 - 3\cos^2\theta - \eta\sin^2\theta\cos 2\psi)$$

and

$$w_4 = w_5^* = -i\frac{\sqrt{3}}{2} p_{zz}^A \sin\theta\exp(i\phi)[3\cos\theta - \eta\cos\theta\cos 2\psi - i\eta\sin 2\psi]$$

$$3\sigma_{\text{pol}}(\phi,\theta,\psi)t_{20}^f(\phi,\theta,\psi) = x_1 \operatorname{Re} A^*C + x_2 \operatorname{Re} B^*A + x_3 \operatorname{Re} B^*C$$
$$+ x_4 \operatorname{Re} B^*D + x_5|A|^2 + x_6|B|^2 + x_7|C|^2 + x_8|D|^2 \qquad (42)$$

where

$$x_1 = \sqrt{2}\, p_{zz}^A\, [\tfrac{3}{2}\sin^2\theta\cos 2\phi + \eta\cos^4\theta/2\cos(2\phi+2\psi)$$
$$+ \eta\sin^4\theta/2\cos(2\phi-2\psi)]$$

$$x_2 = -x_3 = \frac{x_4}{2} = -p_{zz}^A\sin\theta[3\cos\theta\cos\phi - \eta\cos\theta\cos\phi\cos 2\psi$$
$$+ \eta\sin\phi\sin 2\psi]$$

$$x_5 = x_7 = \sqrt{2} + \frac{p_{zz}^A}{2\sqrt{2}}[3\cos^2\theta + \eta\sin^2\theta\cos 2\psi - 1]$$

$$x_6 = -\sqrt{2} - \sqrt{2}\, p_{zz}^A[(3\cos^2\theta - \tfrac{3}{2}\sin^2\theta\cos 2\phi - 1)$$
$$+ \eta(\sin^2\theta\cos 2\psi - \cos^4\theta/2\cos(2\phi+2\psi) - \sin^4\theta/2\cos(2\phi-2\psi)]$$

and

$$x_8 = -\sqrt{2} + \frac{p_{zz}^A}{\sqrt{2}}[3\cos^2\theta + \eta\sin^2\theta\cos 2\psi - 1]$$

$$3\sigma_{\text{pol}}(\phi,\theta,\psi)t_{21}^f(\phi,\theta,\psi) = y_1 \operatorname{Re} B^*A + y_2 \operatorname{Re} B^*C + y_3 \operatorname{Re} B^*D$$
$$+ y_4 \operatorname{Re} AD^* + y_5 \operatorname{Re} CD^* + y_6|B|^2 \qquad (43)$$

where

$$y_1 = iw_1, \quad y_2 = iw_1^*, \quad y_3 = -iw_3, \quad y_4 = iw_4, \quad y_5 = iw_4^*, \quad y_6 = \sqrt{3}x_2$$

$$3\sigma_{\text{pol}}(\phi,\theta,\psi)t_{22}^f(\phi,\theta,\psi) = z_1 \operatorname{Re} A^*C + z_2 \operatorname{Re} B^*A + z_3 \operatorname{Re} B^*C$$
$$+ z_4|A|^2 + z_5|B|^2 + z_6|C|^2 \qquad (44)$$

where

$$z_1 = \sqrt{6}x_5, \quad z_2 = -z_3^* = \sqrt{2}iw_4, \quad z_5 = \sqrt{6}x_5 - 3\sqrt{3}$$

$$z_4 = z_6^* = \frac{\sqrt{3}}{2} p_{zz}^A \exp(2i\phi)[\tfrac{3}{2}\sin^2\theta + \eta\cos^4\theta/2\exp(2i\psi)$$
$$+ \eta\sin^4\theta/2\exp(-2i\psi)].$$

Apart from the existing experimental data on $\alpha_1 \ldots \alpha_4$, one can now choose for measurement, several sets t_{kq}^f. For example, consider the measurement of the real and imaginary parts of the vector polarization $t_{11}^f(\phi,\theta,\psi)$ and tensor polarization

$t^f_{21}(\phi,\theta,\psi)$ of the recoil deuteron, such that $\theta=\phi=0$ and $\psi=\pi/4$. We then have, after equating the real and imaginary parts

$$k_1 \,\mathrm{Re}\, t^f_{11}=\mathrm{Im}\,[B^*(A+C)]\equiv\alpha_5 \tag{45}$$

$$k_2 \,\mathrm{Im}\, t^f_{11}=\mathrm{Im}\,[B^*(A-C)]+k_3\,\mathrm{Im}\,B^*D\equiv\alpha_6 \tag{46}$$

$$-k_2 \,\mathrm{Re}\, t^f_{21}=\mathrm{Re}\,[B^*(A-C)]-k_3\,\mathrm{Re}\,B^*D\equiv\alpha_7 \tag{47}$$

$$k_1 \,\mathrm{Im}\, t^f_{21}=\mathrm{Re}\,[B^*(A+C)]\equiv\alpha_8 \tag{48}$$

where

$$k_1=-\sqrt{6}\sigma_{\mathrm{unpol}}\left(1+\frac{p^A_{zz}}{\sqrt{2}}T_{20}\right)\Big/\eta p^A_{zz} \tag{49}$$

$$k_2=-\sqrt{6}\sigma_{\mathrm{unpol}}\left[1+\frac{p^A_{zz}}{\sqrt{2}}T_{20}\right]\Big/(2+p^A_{zz}) \tag{50}$$

$$k_3=2(1-p^A_{zz})/(2+p^A_{zz}). \tag{51}$$

Since the over all phase of M is unobservable, one can choose for example B to be real and positive and proceed to reconstruct the four helicity amplitudes using a set of eight real observables. From (35), (36) and (38) and also from (45) and (48), we have

$$B=\left[\frac{3(\alpha_5^2+\alpha_8^2)}{\alpha_1+\alpha_2+3\alpha_4}\right]^{1/2} \tag{52}$$

and

$$2\,\mathrm{Re}\;A^*C=\alpha_4+B^2. \tag{53}$$

Hence we have

$$|A-C|^2=(\alpha_1+\alpha_2-3\alpha_4-6B^2)/3 \tag{54}$$

and

$$|D^2|=(\alpha_1-2\alpha_1-6B^2)/3. \tag{55}$$

From (45) and (48), we get

$$A+C=(\alpha_8+i\alpha_5)/B. \tag{56}$$

Thus determination of B, $|D|$ and complex number $(A+C)$ is complete. Next, we choose eqs (46) and (47), from which we have

$$A-C-k_3D^*=(\alpha_7+i\alpha_6)/B. \tag{57}$$

The complex number $A-C-k_3D^*$ can be looked upon as a vector in the Argand (complex) plane, being the resultant of $(A-C)$ and $-k_3D^*$, whose modulii are known. The trigonometrical ambiguity associated with the absolute determination of $(A-C)$ and D is resolved, by noting that

$$\mathrm{Re}\,D=(\alpha_7-\alpha_3)/B(1-k_3) \tag{58}$$

$$\mathrm{Re}\,(A-C)=(\alpha_7-k_3\alpha_3)/B(1-k_3) \tag{59}$$

Thus D and $(A-C)$ are known unambiguously. Then the complex numbers A and C will be separately known, using (56).

5. Numerical estimates and outlook

Compounds containing deuterons are usually employed as the target in πd elastic scattering experiments and tensor polarization is achieved using normal magnetic field of strength 2·5T at very low temperatures below 1°K. The chemical bonds in these compounds provide sufficient electric field gradient so as to cause interaction with the deuteron quadrupole moment and it is worthwhile examining the resulting tensor polarization in the absence of an external magnetic field.

The estimate of the thermal equilibrium vector polarization p_z, in the deuterated compounds, achieved using a magnetic field of 2·5T, at 1°K is $5{\cdot}3 \times 10^{-4}$ (Wait *et al* 1989) and at 0·65°K is $8{\cdot}05 \times 10^{-4}$ (Ottermann *et al* 1988a). Therefore, the corresponding values of TETP (p_{zz}) calculated using (5), are respectively, $2{\cdot}05 \times 10^{-7}$ and $4{\cdot}86 \times 10^{-7}$. At 50° mK, which is often realized (Ottermann *et al* 1988a; Hamada *et al* 1981; Meyer 1984), the estimate of TETP at 2·5T magnetic field is $0{\cdot}821 \times 10^{-4}$. This can be compared with the estimates (presented in table 1) of the TETP at the same temperature (viz., 50° mK) generated (in the absence of an external magnetic field) by the electric quadrupole fields which exist in the various molecules. Since the deuteron polarimeters are sensitive up to 10^{-5} (Garcilazo *et al* 1989), it should be possible to measure the TETP, generated by the chemical bonds inside the molecule. Since the TETP generated by the quadrupole fields at the same temperature compare favourably with those generated by the external magnetic fields, we may point out that the method we are advocating is free from some of the difficulties encountered

Table 1. Thermal equilibrium tensor polarization (TETP) of the deuteron in various substances without any external magnetic field.

*Molecule	e^2qQ/h in kHz	η	$p_{zz} \times 10^4$	$(p_{xx} - p_{yy}) \times 10^4$
Deuterated propanediol D6 and D8 (for the C–D bond)	166·3	0·03	−0·8	−0·024
Deuterated propanediol D6 and D8 (for the O–D bond)	197·9	0·17	−0·952	−0·162
$C_6D_3F_3$	180	0·05	−0·864	−0·043
Anthracene-d_{10}	181	0·064	−0·869	−0·056
DCOOH	161	0·04	−0·773	−0·031
$C_6D_5NH_3^+$	188	0·05	−0·903	−0·045
$(CDO_2)_2Sr{\cdot}2D_2O$	167·6	0·043	−0·805	−0·035
$(CDO_2)_2Sr{\cdot}2D_2O$	170·3	0·025	−0·818	−0·02
$(DCO_2)_2Cu{\cdot}4D_2O$	161·4	0·034	−0·775	−0·026
CD_3F	133	0·03	−0·639	−0·019
$CD_2(COOD)_2$	168·1	0·038	−0·807	−0·03
$CD_2(COOD)_2$	165	0·01	−0·792	−0·008
$(CD_3CO_2)_2Cu$	168	0·05	−0·807	−0·04

*The data for quadrupole coupling constants are taken from Hamada *et al* (1981) for the deuterated propanediol and Mantsch *et al* (1977) for the other compounds.

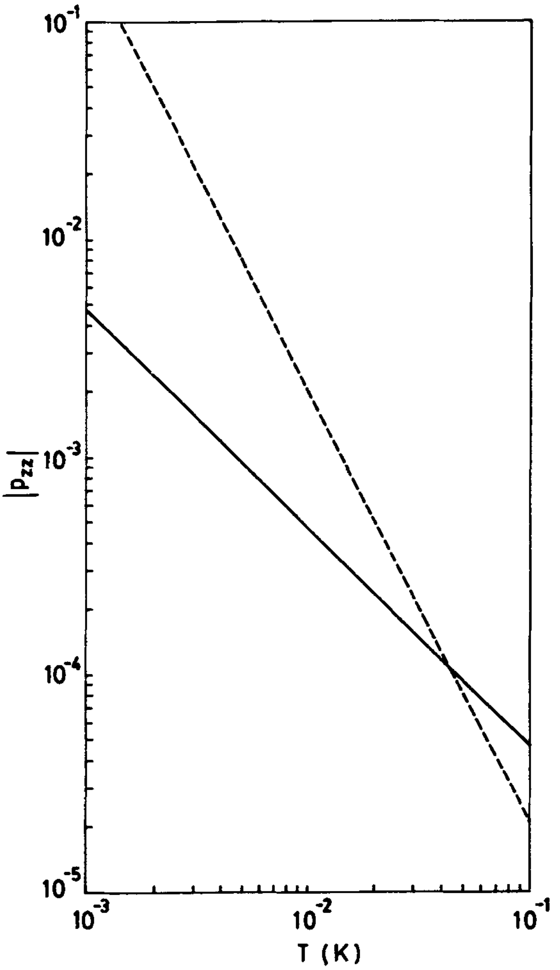

Figure 1. The variation of TETP (p_{zz}) of the deuteron with temperature in deuterated propanediol. For comparison the TETP generated by an external magnetic field of 2·5 T is shown as a broken line.

in employing magnetic fields viz., (i) The large superconducting magnetic coils restrict the angular range made available for experiment, (ii) the external magnetic fields affect the trajectories of the incoming and outgoing charged pions. Moreover, it may be worth pointing out that the effects of the electric quadrupole interaction has generally been neglected so far, as being small in comparison with the magnetic field interaction. How far this is justified from the point of view of the TETP will be taken up for investigation in a later contribution. Since the TETP (p_{zz}) in a pure electric quadrupole interaction is proportional to the ratio A/KT, one can hope to achieve higher values of TETP, by suitably lowering the temperature. Figure 1 shows for example, the variation of TETP with temperature for a given molecule of deuterated propanediol. At temperatures of order 50° mK, which are usually available in present day experiments (Meyer 1984; Ottermann *et al* 1988a), it is seen that the magnitude of TETP (p_{zz}) generated by the electric quadrupole field is in fact somewhat higher than that generated by the application of an external magnetic field of 2·5T. Hopefully, therefore, the suggestions made in this paper, to employ electric quadrupole fields, deserve the attention of the experimental groups working at such low temperatures.

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