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Model of confinement

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Abstract. Confinement model for quarks and gluons is formulated. An attempt has been made to derive the dielectric function starting from a Lagrangian that also determines the quark and gluon field equations. Deconfinement mechanism is also discussed.

Keywords. Confinement model gluon; anti-deSitter vacuum; deconfinement.

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1. Introduction

The confinement of gluons and quarks has been a central problem in high energy physics and it is clear by now that the confinement is a non-perturbative phenomenon. What is however not clear is whether confinement is true at the classical level itself or it is a quantum phenomenon. Motion of light beam in a medium having spherically symmetric dielectric constant shows that the photon may be confined for a suitable choice of dielectric function. Examples are helical path of light rays in selfoc fibres, formation of perfect image in Maxwell fish eye (Jena and Pradhan 1981) and closed photon orbits in a strong gravitational field. In the realm of high energy physics, strong gravity solutions e.g., Schwarzchild solution, Kerr–Newman solutions (Mielke 1978, 1979, 1980a, b) and geons are examples of classical confining systems consisting of quarks and gluons.

These observations of confined systems lead us to consider the motion of quark and gluon in a colour dielectric medium having space dependent colour dielectric function. Lee first introduced the concept of colour dielectric constant and explained the quark confinement on the assumption that the massless quarks satisfy Maxwell-like equations and move in a colour dielectric medium (Lee 1979). Subsequently several workers developed the idea and treated the quark and gluon confinement with the concept of colour dielectric medium having space dependent dielectric constant. In their models (Jena and Pradhan 1981, 1984; Khadkikar and Vinod Kumar 1987) the form of the dielectric constant, crucial to the understanding of confinement mechanism, is introduced with an assumed form. Moreover modified equations of motion in presence of colour dielectric medium, for quarks and gluons, are also assumed in the above models. However, the ad hoc choice, though phenomenological, is an indication to the way the confinement is achieved. Moreover the same form of dielectric constant used for the confinement of photon (in charge space), quark and gluons (in colour space) suggests that there is a mechanism through which the dielectric constant couples in the same way to all the fields. Recently we developed a model (Biswas and Kumar

1989a, b) to show that tensor gluons and quarks remain bound in a strong gravitational field leading to the confinement in the model. The gravitational field plays the role of medium having, as if, space dependent dielectric permeabilities from a fixed centre. In the approach the metric describing the strong gravitational field plays the role of dielectric constant and is a tensor field and the form of the dielectric constant is obtained by solving Einstein-like equations for the metric. A partial answer to the form of equations in the dielectric medium (gravitational field) is also obtained in the above work.

In this paper we derive the form of dielectric constant as well as the equations of motion starting from a single Lagrangian describing the quark-gluon system. The QCD Lagrangian describing a quark-gluon system is well known. While our understanding of non-abelian quantum chromodynamics (QCD) theory is still incomplete even in the lattice gauge theory formulation and one needs to show in QCD, to explain quark confinement, that the particle spectrum be realized in terms of bound states of quarks and gluons, we proceed with an 'equivalent description'. In the 'equivalent description' the non-abelian gluon field equations are written down in Maxwell-like form with the assumption that the non-linear terms in the field equations can be considered as a supercurrent (in colour space) in analogy with Ginzburg–Landau's theory of superconductivity (Khadkikar and Vinod Kumar 1987). This supercurrent is now treated as polarization current where the polarizability is now described by space dependent dielectric function $\varepsilon(r)$. In this approximation the non-abelian gluon moves in a colour dielectric medium having space dependent dielectric function. This is dealt in §2.

From the point of view of Coulomb's law, the occurrence of a space dependent $\varepsilon(r)$ may be looked upon as a transformation where $r \to \varepsilon^{1/2} r$. Thus from metrical point of view, $\varepsilon(r)$ can be considered as a gravitation-like field, as if

$$(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2) \to \varepsilon(r)(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2).$$

This aspect is dealt in §3. It is worthwhile to point out that as the origin of $\varepsilon(r)$ is due to non-linear terms of the field equations, the term 'dielectric function' is to be used in a restricted sense that it appears in Maxwell-like equations as if it behaves as dielectric constant. It would be judicious to call $\varepsilon(r)$ a 'renormalisation parameter' treated as gravitation-like scalar field. Moreover, the structure of the field equations reveal that the magnetic permeabilities of the medium is also a space dependent function with $\mu(r) = \varepsilon(r)$. In §4 we discuss the motion of massless quarks in the colour dielectric medium. In §5 we construct a Lagrangian, that incorporate the features mentioned in §§2 and 3, describing the quark gluon system and also obtain solution corresponding to the field equation for $\varepsilon(r)$. In §6 we solve the gluon field equations and find that colour waves of selected frequencies are permanently confined. The characteristics of the solutions are that i) frequencies are equally spaced analogous to the equispaced energy spectrum of simple harmonic oscillator ii) solutions with higher frequencies are closer to the centre than those with lower ones. Our investigations lead to the same conclusions arrived at our earlier work (Biswas and Kumar 1989a) and lend support to the claim for the ad hoc form of dielectric constant used by other authors (Jena and Pradhan 1981, 1984; Khadkikar *et al* 1983; Khadkikar and Vinod Kumar 1987). We end up with a concluding section discussing deconfinement mechanism.

2. Abelian approximation to non-abelian gluon

In this section we show that the non-abelian gluon fields **E** and **B** can be considered as quasi-Maxwellian fields such that the field equations resemble Maxwell's equations of electrodynamics in dielectric medium. The Lagrangian density for a pure colour field is given by

$$L = -\tfrac{1}{4} F^l_{\mu\nu} F^{\mu\nu}_l \tag{1}$$

in which

$$F^l_{\mu\nu} = f^l_{\mu\nu} + G^l_{\mu\nu} \tag{2}$$

$$f^l_{\mu\nu} = \partial_\mu A^l_\nu - \partial_\nu A^l_\mu \tag{3}$$

$$G^l_{\mu\nu} = g f^{lmn} A^m_\mu A^n_\nu. \tag{4}$$

Here l, m, $n = 1, 2, \ldots 8$ refer to colour indices and A^l_μ is the gluon field potential. The variational approach yields the field equation

$$\partial_\mu F^l_{\mu\nu} + g f^{lmn} A^m_\mu F^n_{\mu\nu} = 0. \tag{5}$$

Substituting for $F^n_{\mu\nu}$ from (2) to (4) and transferring all the nonlinear terms like A. A. A., A∂A to r.h.s. and identifying it as a supercurrent J^l_ν we get

$$\partial_\mu f^l_{\mu\nu} = -J^l_\nu. \tag{6}$$

To correlate (6) with superconductivity type approach we note that in London theory of superconductivity the current is linearly related to vector potential **A** so that curl **J** becomes proportional to **B**. In Ginzburg–Landau theory of superconductivity, the supercurrent is again written as

$$\mathbf{J} = \mathbf{F}(\psi) - b|\psi|^2 \mathbf{A}$$

where ψ is an order parameter. With this idea in mind we put the r.h.s. of (6) as a supercurrent in colour space. The dielectric constant $\varepsilon(r)$ is then viewed as an order parameter, as mentioned earlier, the microscopic basis of which is clearly known from QCD-like theory. In our approach it is related to vacuum polarization effects. To the extent that the theory is QCD-like non-abelian and satisfies the properties of asymptotic freedom, the microscopic basis of the order parameter at short distances, lies in the processes gluon→quark + anti-quark and gluon→gluon + gluon, etc. This perturbative explanation need not be taken as a final answer to the microscopic basis as it is believed that the colour dynamics is a non-perturbative one as well as the perturbative method becomes meaningless at large distances (large coupling constant). In QCD, one generally calculates the running coupling constant $\alpha_s(q^2)$ in momentum space; in our approach we like to find out an effective charge in co-ordinate space. In our approach we write the supercurrent

$$J^l_\nu = -g f^{lmn}[A^m_\mu \partial_\mu A^n_\nu + \partial_\mu A^m_\mu A^n_\nu + A^m_\mu \partial_\mu A^n_\nu - A^m_\mu \partial_\nu A^n_\mu + g f^{nl'm'} A^m_\mu A^{l'}_\mu A^{m'}_\nu],$$

in low momentum approximation. In that case the supercurrent can be approximated as

$$J^l_\nu \simeq \theta_{\mu\nu} A^l_\mu \tag{7}$$

and restrict ourselves to co-ordinate space configurations so that $\theta_{\mu\nu}$ are space dependent functions. It has been shown that for a particular choice (Khadkikar and Vinod Kumar 1987)

$$\theta_{\mu\nu} = -\delta_{\mu\nu}(2\alpha\delta_{\mu 0} - \alpha^2 r^2)$$

the dielectric constant comes out to be

$$\varepsilon(r, \nabla) = 1 - \frac{\alpha^2 r^2}{\omega^2}[1 - \nabla\{1/(\omega^2 + \nabla^2)\}\nabla]$$

with $\mathbf{D} = \varepsilon(r, \nabla)\mathbf{E}$. Neglecting the non-local part, the equations of motion of the quasi-gluons are now written as

$$\nabla \cdot \mathbf{D} = 0 \tag{8}$$

$$\nabla \times \mathbf{B} = \partial \mathbf{D}/\partial t \tag{9}$$

with $\mathbf{D} = \varepsilon(r)\mathbf{E}$ where $\varepsilon(r) \cong [(1 - \alpha^2 r^2/\omega^2)]$. To generate the space dependent dielectric function from QCD, one fourier transforms the running coupling constant $\alpha_s(q^2)$ (Schmitz *et al* 1987) and the co-efficient of $\alpha_s(q_0^2)$ is identified as $1/\varepsilon(r)$. This type of result may be well suited for small distances but is hopeless for large distances. Our approach is quited different. The present work is directed to find out a form of $\varepsilon(r)$ satisfying the requirements of both the asymptotic freedom and confinement. Though the ultraviolet limit of QCD appears to be well understood, the infrared properties remain very unclear. Attempts are always being done (Zachariasen 1980; Muller 1978) to study the infrared behaviour in different ways. On the lattice QCD side, we have a signal of confinement but still within abelian approximation. The numerical results through Monte Carlo simulation do not convincingly indicate the co-existence of the asymptotic freedom and colour confinement in a single phase of QCD. In our approach we develop a model in such a way that the confined phase as well as the asymptotic phase do co-exist in a single phase. In the model we simulate the form of $\varepsilon(r)$ from a Lagrangian that, under variational approach, will take into account the quark gluon interaction as well as generates the field equations (8) and (9). The asymptotic freedom and confinement is arrived at through the boundary conditions imposed on $\varepsilon(r)$.

3. Dielectric constant as gravitational field

Many years back Dicke (Dicke 1957) considered a flat theory of gravitation with the idea that the gravitation can be considered as some sort of electromagnetic effect. The bending of a light beam due to gravitational field of the sun is a well known phenomenon. Dicke argued that the amount of bending could also be calculated considering the space (considered as electromagnetic vacuum) surrounding the sun as polarised medium having space dependent dielectric permeabilities. Thus the gravitation is seen as electromagnetic effect.

We extend Dicke's idea in an analogous way to deal with the colour force arising out of colour supercurrent as mentioned in (6). The quasi chromoelectromagnetic vacuum is now polarized by the colour supercurrent. This polarized vacuum simulated by the supercurrent is now described by space dependent colour dielectric function.

For space dependent dielectric constant the gradient of dielectric constant will be nonvanishing. As a result there will be more induced charge on one side of a particle than on the other and the electrostatic interaction with the induced charges leads to a force acting on the particle in the direction of increasing gradient. This would result in a change in energy or frequency of a particle depending upon the location of the particle in the field. Not only that, the Bohr radius and other atomic lengths would be function of position. The origin of such a variation can be traced back to the modification of coulomb force in dielectric medium where $r^2 \to \varepsilon(r) r^2$. This would lead to a bending of meter sticks depending upon their location in space time. If such meter sticks are defined as unchanged in length, the variation is then interpreted due to curved space-time. We have then Einstein-like theory for the field $\varepsilon(r)$. Another approach is to scale down the length and time at every space time points so that we are still in flat space-time. In view of the assumption that $\varepsilon(r)$ affects atomic lengths and times, what is needed is a local measure of $\varepsilon(r)$. It will appear as a derived result of the theory that $\varepsilon(r)$ is in principle locally measurable by determining the ratio of electrical to gravitational force. In Dicke's approach the source of gravitational field is the vacuum polarization and is described by classical field quantities $\varepsilon(r)$ and $\mu(r)$. We extend the approach for the colour dielectric vacuum. Here "charge" will refer to colour charge and electromagnetic fields will refer to chromo-electric and chromomagnetic fields.

We work with flat space-time units with

$$\eta_{00} = 1, \quad \eta_{11} = \eta_{22} = \eta_{33} = -1. \tag{10}$$

We call this system of unit as Newtonian unit. But locally infinitesimal interval will read

$$ds^2 = f_{\mu\nu}\, dx^\mu\, dx^\nu, \tag{11}$$

where $f_{00} = \varepsilon^{-1}$ and $f_{11} = f_{22} = f_{33} = -\varepsilon$, $f_{\mu\nu}$ occurring in (11) should not be confused with $f_{\mu\nu}$ in (3). The equation (11) specifies the local units. As we want to work with flat units, a scaling of length and time is made at every space-time points so that local units of length and frequency when expressed in Newtonian units vary as

$$L = L_0/\varepsilon^{1/2}, \tag{12}$$

$$\omega = \omega_0/\varepsilon^{1/2}.$$

where L_0 and ω_0 are constants and $\varepsilon(r)$ is the colour dielectric constants. As the choice of co-ordinate system is not unique, we invoke Mach's principle such that the co-ordinate system (10) is determined by the dielectric matter distribution in the system. Further assuming cosmological principle to be valid (in the realm of hadronic mini-cosmos) we fix a unique time direction which is taken to be parallel to our Newtonian co-ordinate system (10). In this co-ordinate system the dielectric medium (polarized vacuum) will appear uniform from any fixed position point. This high degree of symmetry, special to the co-ordinate system, allows us to treat the colour dielectric constant as a scalar field. The arguments leading to (11) and (12) allow us to treat $\varepsilon(r)$ as an auxillary gravitational field. The distinction between the abelian and non-abelian field is maintained through the behaviour of $\varepsilon(r)$ at the origin and at infinity. In abelian case

$$\varepsilon(r) \xrightarrow[r \to \infty]{} 1$$

whereas in non-abelian case

$$\varepsilon(r) \xrightarrow[r\to\infty]{} \text{negative}$$

leading to confinement. At small r, $\varepsilon \to 1$ so that the asymptotic freedom is recovered in the model and quarks are almost free. We now summarize the basic assumption in our model.

(i) We work with flat space-time. (ii) Mach's principle and cosmological principles are assumed to be valid. (iii) Colour quarks are massless. (iv) Chromoelectric and chromomagnetic fields satisfy Maxwell-like equations in dielectric medium. (v) The dielectric properties of the medium are described by classical field quantities $\varepsilon(r)$ and $\mu(r)$, treated as scalar field. (vi) We assume local Lorentz invariance.

There is a basic difference between our approach and Dicke's approach. In Dicke's approach gravitation is seen as an electromagnetic effect, we look at the nonlinear aspects of chromo-dynamical equation through a space dependent dielectric constant $\varepsilon(r)$ and magnetic permeabilities $\mu(r)$ treated as gravitation-like scalar fields.

4. Equation of motion

We first limit ourselves to a simple problem rather than considering immediately the formation of general field equations. Let us consider the motion of mass-less quarks in dielectric medium, the Lagrangian density of which is now written as Local Lorentz invariant

$$L_q = -\tfrac{1}{2}(-i\bar{\psi}\tilde{\gamma}^\mu\partial_\mu\psi) - \tfrac{1}{2}(i\partial_\mu\bar{\psi}\tilde{\gamma}^\mu\psi). \tag{13}$$

Here $\tilde{\gamma}^\mu$ are space-dependent gamma matrices fixed such that

$$\tilde{\gamma}^\mu\tilde{\gamma}^\nu + \tilde{\gamma}^\nu\tilde{\gamma}^\mu = 2f^{\mu\nu}, \tag{14}$$

so that

$$\tilde{\gamma}^0 = \gamma^0\varepsilon^{1/2}, \quad \gamma^i = \gamma^i\varepsilon^{-1/2}. \tag{15}$$

The γ^i are usual gamma matrices with

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2\eta^{\mu\nu}. \tag{16}$$

In our Newtonian co-ordinate system, the variational principle reads

$$\delta\int L_q\sqrt{-\eta}\,\mathrm{d}^4x = 0. \tag{17}$$

Euler-Lagrange equation corresponding to (17) is

$$\tilde{\gamma}^\mu\partial_\mu\psi + \tfrac{1}{2}\gamma^\mu\partial_\mu(\varepsilon^{-1/2})\psi = 0, \tag{18}$$

where $\xi = \varepsilon^{-1/2}$. Using

$$\psi \sim \exp(-i\omega t)\begin{pmatrix}\varphi\\ \chi\end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} \quad \gamma^i = \begin{pmatrix}0 & \sigma^i\\ \sigma^i & 0\end{pmatrix}$$

and (15), we get

$$(\boldsymbol{\sigma}\cdot\mathbf{P})\varphi - i(\boldsymbol{\sigma}\cdot\mathbf{r})\frac{\partial\xi/\partial r}{\xi r}\varphi = \omega\xi^{-1}\chi, \tag{19}$$

$$(\boldsymbol{\sigma}\cdot\mathbf{P})\chi - i(\boldsymbol{\sigma}\cdot\mathbf{r})\frac{\partial\xi/\partial r}{\xi r}\chi = \omega\xi^{-1}\varphi. \tag{20}$$

It has been shown (Khare and Pradhan 1983) that the form $\varepsilon(r)=\exp(-\lambda r^2)$ confine gluons provided $\varepsilon\mu=1$ condition is satisfied whereas Khadkikar and Vinod Kumar (1987) tried to obtain a form of $\varepsilon(r)\simeq 1-\Lambda r^2$ as in §2. In our ealier work (Biswas and Kumar 1989a) we treat $\varepsilon(r)$ as tensor field and solved Einstein equation to obtain $\varepsilon(r)=1-\Lambda r^2$, where Λ is a cosmological constant and as usual is related to colour energy density. To get the equations as mentioned in our earlier work and make it suitable for future application like charmed meson decays we put $\varepsilon=\exp(-\lambda r^2)$ in (19) and (20). Using the standard technique of reducing the above pair to two first order radial equations, we get

$$f\lambda r + f' = \frac{k-1}{r}f - \omega\varepsilon g, \tag{21}$$

$$g\lambda r + g' = \omega\varepsilon f - \frac{k+1}{r}g. \tag{22}$$

Furthermore, putting $(f,g)\sim\exp(-\lambda r^2/2)\,(\xi,\eta)$, we get

$$\xi' = \frac{k-1}{r}\xi - \omega\varepsilon\eta, \tag{23}$$

$$\eta' = \omega\varepsilon\xi - \frac{k+1}{r}\eta. \tag{24}$$

These types of equations were already used by us in dealing with quark confinement. Now we proceed with formulation of the Lagrangian that will determine ε as well as the field equations.

5. Field equations

The Lagrangian density for the scalar field ε is taken to be an invariant quadric in gradient of ε,

$$L_1 = \tfrac{1}{2} f^{\mu\nu}\varepsilon_{,\mu}\varepsilon_{,\nu}. \tag{25}$$

Here $f^{\mu\nu}$ is the reciprocal of the tensor $f_{\mu\nu}$ i.e.,

$$f_{\mu\nu}f^{\nu\beta} = \delta^{\beta}_{\mu},$$

and

$$\varepsilon_{,\mu} = \partial\varepsilon/\partial x^{\mu}.$$

The Lagrangian density of abelian gluon is taken, in the spirit of §2, as

$$L_3 = -\frac{\varepsilon}{16\pi}F_{\mu\nu}F_{\alpha\beta}f^{\alpha\mu}f^{\beta\nu}, \tag{26}$$

and the Lagrangian density for the quark field is taken as before as

$$L_2 = \tfrac{1}{2}(-i\bar{\psi}\tilde{\gamma}^\mu \partial_\mu \psi) + \tfrac{1}{2}(i\partial_\mu \bar{\psi}\tilde{\gamma}^\mu \psi), \tag{27}$$

so that the total Lagrangian density now reads

$$L = L_1 + L_2 + L_3. \tag{28}$$

The variational principle

$$\delta \int L\sqrt{-\eta}\, \mathrm{d}^4x = 0$$

gives back the equation of motion for the quark field as mentioned in §2. The variational principle also gives the two Maxwell equations that are not identities

$$\nabla \times (\mathbf{B}/\varepsilon) - \frac{\partial}{\partial t}(\varepsilon \mathbf{E}) = 0, \tag{29}$$

$$\nabla \cdot (\varepsilon \mathbf{E}) = 0. \tag{30}$$

The strucure of equations (29) and (30) now reveals that these resemble the equations of motion in dielectric medium with $\varepsilon(r) = \mu(r)$. The variational principle also gives the field equation for the scalar field ε as (assuming ε time-independent)

$$\nabla^2 \varepsilon = -\left\{ \bar{\psi}\omega\varepsilon^{1/2}\gamma^0\psi + \frac{1}{8\pi}\left(\varepsilon E^2 + \frac{1}{\varepsilon}B^2\right) - \frac{(\nabla\varepsilon)^2}{2\varepsilon} \right\}. \tag{31}$$

In deriving (31) we have used the field equation (18) to reduce the $\partial L_2/\partial\varepsilon$ term as the first term in the right hand side. To interpret, (31) we calculate the energy momentum tensor from the relation

$$T^\mu_\nu = \varepsilon_{,\nu} \frac{\partial L}{\partial \phi_{,\mu}} - \delta^\mu_\nu L. \tag{32}$$

In (32), ϕ stands for ε, ψ and A_μ fields. We calculate only the T^0_0 components

$$T_{00}(\varepsilon) = \frac{1}{2}\left[\varepsilon\left(\frac{\partial\varepsilon}{\partial t}\right)^2 + \frac{1}{\varepsilon}(\nabla\varepsilon)^2 \right] \tag{33}$$

$$T_{00}(A_\mu) = \frac{1}{8\pi}(\varepsilon E^2 + B^2/\varepsilon) \tag{34}$$

$$T_{00}(\psi) = \bar{\psi}\omega\varepsilon^{1/2}\gamma^0\psi. \tag{35}$$

It is interesting to note that (29) and (30) are exactly of the same form that one obtains writing down Maxwell equation in 3-dimensional form in a gravitational background (Landau and Lifshitz 1975). Now going back to (31) we find that the electromagnetic energy density and the gravitational energy density (basically energy density of multi-gluon exchange) serve as source term for the field ε. As the second and third term are opposite in sign, the self-electric and gravitational energies could be large compared with particle energy and still generate the proper ε. Thus in our framework there is possibility of getting a confined system of charged (colour) particles held together by own gravitational forces within dimension of the order of gravitational radius.

To obtain a solution for (31) we take ε as time independent and spherically symmetric. Equation (31) then simplifies to

$$\nabla^2\varepsilon - \frac{1}{2\varepsilon}(\nabla\varepsilon)^2 = -\left[\omega\bar{\psi}\gamma_0\varepsilon^{1/2}\psi + \frac{1}{8\pi}\left(\varepsilon E^2 + \frac{1}{\varepsilon}B^2\right)\right]. \tag{36}$$

It may be argued that one can multiply L_1, L_2 and L_3 by arbitrary function of ε still keeping the local Lorentz invariance intact. However on physical ground (in the sense of Einstein equations), to keep the r.h.s. of (31) as energy density expressions, the multiplication by arbitrary function of ε must be excluded. For the confined phase we assume that the second term of (36) in r.h.s. is much greater than the first term and in the static approximation we write (36) as

$$\nabla^2\varepsilon - \frac{1}{2\varepsilon}(\nabla\varepsilon)^2 + \frac{E^2}{8\pi}\varepsilon = 0. \tag{37}$$

Let us put $\varepsilon^{1/2} = \exp(2\lambda(r))$ in (37). We get

$$2\exp(2\lambda)\left[\lambda'' + \frac{2}{r}\lambda' + 2\lambda'^2 + \frac{E^2}{16\pi}\right] = 0. \tag{38}$$

To achieve confinement and the asymptotic freedom we take the boundary conditions on ε as

$$\varepsilon(r) \xrightarrow[r\to 0]{} 1$$

$$\varepsilon(r) \xrightarrow[r\to R]{} 0.$$

Here R is a characteristic length of confinement. In bag model's language R is the effective radius of hadronic bag. To solve (38) we need a suitable ansatz for E^2. As the gluon field energy must vanish after some distance (taken to be R) and be finite at $r=0$ we put in (38)

$$\frac{E^2}{16\pi} = \Lambda_f \exp(2\lambda), \tag{39}$$

where Λ_f is a constant. The gluon field energy vanishes outside a certain distance and is equal to a constant Λ_f at the centre of confinement. Putting the value of $E^2/16\pi$ and with the substitution,

$$F = re^{2\lambda}, \tag{40}$$

and the above requirements, we get

$$F'' + \Lambda_f \frac{F^2}{r} = 0. \tag{41}$$

Putting the nonlinear term $F^2 \simeq 0$ in (37) we have the solution

$$F = ar + b$$

As $\exp(2\lambda)\to 1$ as $r\to 0$, we take $F\to r$ as $r\to 0$. Putting this behaviour of in (41) we get

$$F'' + \Lambda_f r = 0$$

with the solution

$$F = a + br - \tfrac{1}{6}\Lambda_f r^3. \tag{42}$$

To satisfy the boundary conditions we put $b=1$ and $a=0$ so that

$$\varepsilon = 1 - \tfrac{1}{3}\Lambda_f r^2$$

neglecting terms $\sim r^4$ on the assumption that $\sqrt{\Lambda_f}\, r \ll 1$.

Some characteristics of the solutions are the following:

(i) As $r\to 0$, $\varepsilon\to 1$ the quarks are almost free at the centre of confinement signalling asymptotic freedom. However, one must show that the running coupling constant (in our case it is effective charge) must approach zero as $r\to 0$ and infinity as $r\to R$. At first sight it may appear that in our model the fine structure constant defined as

$$\alpha_c = g^2/\varepsilon hc$$

tends to a constant due to the relation $c=(\mu\varepsilon)^{-1/2}$ and the apparent behaviour $\varepsilon=\mu$. It was mentioned by Landau and Lifshitz (1975) that the analogy of (29) and (30) to the Maxwell equations for electromagnetic field in material media is purely formal. Thus μ appearing in the expression of c is not the same as $\mu=\varepsilon$ appearing in the Maxwell equations in (29). Now the quantum field theory vacuum differs from the ordinary polarizable medium on a very important point: it is relativistically invariant i.e., $\mu\varepsilon=1$ is always satisfied giving $c=1$. So instead of calling ε as dielectric constant, it should be called as renormalization parameter. Thus $\alpha_c = g^2/\varepsilon hc \to g^2/hc$ as $r\to 0$ and $\alpha_c\to\infty$ as $r\to R=(3/\Lambda_f)^{1/2}$. This result follows from (42) when $\varepsilon\to 0$ as $r\to R$. Thus we have asymptotic freedom in our model.

(ii) We find later through the explicit solution of **E** from (29) and (30) that $E\sim\exp(-\frac{1}{3}\Lambda_f r^2)$ suggesting that our ansatz (39) is quite a successful one.

(iii) The solution is valid for very small r region and leads to confined phase for $r<(3/\Lambda_f)^{1/2}$ i.e., for $\sqrt{\Lambda_f}\, r \ll 1$.

(iv) The interval (11) will now read ($\sqrt{\Lambda_f}\, r\ll 1$).

$$ds^2 = (1+\tfrac{1}{3}\Lambda_f r^2)\,dt^2 - (1+\tfrac{1}{3}\Lambda_f r^2)^{-1}(dx^2+dy^2+dz^2)$$

$$ds^2 = (1+\tfrac{1}{3}\Lambda_f r^2)\,dt^2 - (1-\tfrac{1}{3}\Lambda_f r^2)(dx^2+dy^2+dz^2).$$

This is basically an anti-deSitter manifold with negative cosmological constant. Now it is widely known that the confining aspects (Salam and Strathdee 1978; Gasperini 1987, 1988) of strong interaction dynamics can be represented formally by embedding the quark in an anti-deSitter manifold. What we have excess in our model that one does not require the negative cosmological constant as an input, the sign of Λ_f is fixed by the positive energy density of gluon field. The inclusion of the first term in (37) will not alter the sign of Λ_f.

(v) Equations (23) and (24) have been applied to charmed meson decays with remarkable success. This is dealt in a separate paper.

It is worthwhile to look at the vacuum solutions, i.e., with $T_{00}=0$. The solution is with $\varepsilon \xrightarrow[r\to\infty]{} 1$,

$$\varepsilon=\left(1+\frac{b}{r}\right)^2. \tag{43}$$

If we assume that the vacuum energy density does not play any role at large distances, we have the solution

$$\varepsilon=-a^2+\frac{b}{r} \tag{44}$$

with the boundary condition that $\varepsilon\to$negative value at large distances. It is interesting to write down the solution on (43) in spherical polar co-ordinate system from isotropic co-ordinates. The result is the Schwarzschild solution for $b/r \ll 1$. Thus we can set $b=2GM$ where G is the strong gravity constant $\simeq 10^{38} G_N$ and M is the mass of hadron. Thus outside the hadron ε plays the role of strong gravity (Salam and Strathdee 1978; Biswas *et al* 1983; Sivaram and Sinha 1979) whereas inside the hadron it acts like dielectric constant to simulate confinement as well as asymptotic freedom. Thus the colour is 'transcendent' in this picture and we do not require colour singlet postulate as is the case with QCD. This type of result is also obtained by Mielke and Scherzer (1980) in the context of colour geometrodynamics. The solution (44) is used by Jena and Pradhan (Jena and Pradhan 1981, 1984) to deal with confinement of photon and quark considering the motion of particles in dielectric medium. We favour the solution (43) as it reproduces the results of strong gravity as well as the confinement and asymptotic freedom. The flexibility of the solutions outside the hadron is due to the fact that the infrared behaviour of QCD is not tested well as we have not seen any colour state free in nature. The outside solution may be settled well if one understands the quark deconfinement transition well. We will discuss this in §7 after carrying out confinement solutions.

6. Solutions of field equations

Assuming the time variation of the field as $\exp(-iwt)$, we obtain the stationary wave equation for $\mathbf{A}$ as

$$\nabla^2\mathbf{A}+\frac{\varepsilon'}{\varepsilon r}(\mathbf{r}\times\nabla\times\mathbf{A})+\omega^2\varepsilon^2\mathbf{A}=\nabla(\nabla\cdot\mathbf{A})-i\omega\varepsilon^2\nabla\varphi. \tag{45}$$

We have used the fact

$$\mathbf{E}=-\partial\mathbf{A}/\partial t-\nabla\varphi \tag{46}$$

$$\mathbf{B}=\nabla\times\mathbf{A} \tag{47}$$

$$\varepsilon'=\partial\varepsilon/\partial r. \tag{48}$$

To make (45) homogeneous to get bag-like cavity eigenmodes we enforce the gauge condition (Khadkikar and Vinod Kumar 1987) such that

$$\nabla(\nabla\cdot\mathbf{A}) - i\omega\varepsilon^2\nabla\varphi = 0.$$

Obviously, the Lorentz condition cannot provide this requirement for general. As the quasigluon approaches the region $r \to (3/\Lambda_f)^{1/2}$, i.e., the confinement region, the condition $\nabla\cdot\mathbf{A}$ is satisfied well. Equation (45) then reduces to

$$\nabla^2\mathbf{A} + \omega^2\varepsilon^2\mathbf{A} + \frac{\varepsilon'}{\varepsilon r}(\mathbf{r}\times\nabla\times\mathbf{A}) = 0. \tag{49}$$

With $\nabla\cdot\mathbf{A} = 0$, $\mathbf{r}\times\nabla\times\mathbf{A} = -r\partial\mathbf{A}/\partial r$, so we get

$$\nabla^2\mathbf{A} - \frac{\varepsilon'}{\varepsilon}\frac{\partial\mathbf{A}}{\partial r} + \omega^2\varepsilon^2\mathbf{A} = 0. \tag{50}$$

Thus in the region $\nabla\cdot\mathbf{A} = 0$ eigenmodes can be defined as

$$\mathbf{A}^{\mathrm{TE}} = \mathbf{L}\psi_{nlm} \tag{51}$$

$$\mathbf{A}^{\mathrm{TM}} = \nabla\times\mathbf{L}\psi_{nlm} \tag{52}$$

where $\psi_{nlm} = R_{nl}Y_{lm}(\theta,\varphi)$ and R_{nl} is the solution of scalar wave equation

$$R''_{nl} + \left(\frac{2}{r} - \frac{\varepsilon'}{\varepsilon}\right)R'_{nl} + \left(\omega^2\varepsilon^2 - \frac{l(l+1)}{r^2}\right)R_{nl} = 0. \tag{53}$$

Before proceeding to the solution, let us now look at the quark field solution. Let us put

$$\begin{pmatrix} f \\ g \end{pmatrix} = \begin{pmatrix} \xi \\ \eta \end{pmatrix}\exp(-\Lambda_f r^2/2)$$

in (21) and (22) to get

$$\xi' = \frac{k-1}{r}\xi - \omega\epsilon\eta \tag{54}$$

$$\eta' = \omega\varepsilon\xi - \frac{k+1}{r}\eta. \tag{55}$$

Equations (54) and (55) can be converted as

$$\xi'' + \left(\frac{2}{r} - \frac{\varepsilon'}{\varepsilon}\right)\xi' + \left(\omega^2 t^2 - \frac{k(k+1)}{r^2}\right)\xi = 0. \tag{56}$$

The structure of (55) and (56) suggests that the quarks and gluons are confined in the same way. This is due to the resemblance of quark field equations with that of Maxwell equations as mentioned in our earlier work (Biswas and Kumar 1989b, c). To carry out the solution we put $\varepsilon'/\varepsilon \simeq -\frac{2}{3}\Lambda_f r$ for $\sqrt{\Lambda_f}r \ll 1$ and

$$R_{nl} = \exp(-\Lambda_f r^2/6)U_{nl}$$

to get

$$U''_{nl} + \frac{2}{r} U'_{nl} + \left(\omega^2 - (\Lambda_f^4/9) r^2 - \tfrac{2}{3} \Lambda_f \omega^2 r^2 - \frac{l(l+1)}{r^2} \right) U_{nl} = 0 \tag{57}$$

The solutions (57) have already been carried out (Biswas and Kumar 1989a, b). The solutions

$$R_{nl} \sim r^l \exp(-\tfrac{1}{3}\Lambda_f r^2)\,{}_1F_1(n, l+\tfrac{3}{2}; \Lambda_f r^2), \tag{58}$$

$$E_{nl} = (8\Lambda_f/3)^{1/2}(2n + l + \tfrac{3}{2}), \quad \text{for } \omega^2 \gg \Lambda_f^3/6$$

$$E_{nl} = 3\sqrt{2}(\Lambda_f/3)(2n + l + \tfrac{3}{2})^{1/2}, \quad \text{for } \omega^2 \ll \Lambda_f^3/6$$

suggest that the fields are mostly confined in the region $\sqrt{\Lambda_f}r \ll 1$ i.e., for $r \ll 10^{-14}$ cm if we restrict ourselves to $\Lambda_f \sim 10^{28}\,\text{cm}^{-2}$, characteristic cosmological constant of strong gravity. We can now proceed with the costruction of glueball states along the lines carried out by Khadkikar and Vinod Kumar (Khadkikar and Vinod Kumar 1987).

7. Quark deconfinement

According to bag model it is argued that at finite internal temperature the bag radius acquires a thermal dependence. At a critical temperature T_c the radius diverges resulting in quark deconfinement. In our approach the radius is related to the constant Λ_f and the characteristic length of the confinement is now given as $\Lambda_f = 3/R^2$. A thermal dependence of Λ_f implies that it is now a function of temperature T. It has been shown (Gasperni 1987) that the cosmological constant of the anti-deSitter vacuum is modified as

$$\Lambda_f(T) = \frac{3}{R^2} - 12\pi^2 T^2$$

where T denotes the intrinsic temperature of the geometric background. It is evident from (58) that for perfect deconfinement $\Lambda_f(T) = 0$ so that the critical temperature is

$$T_c = 1/2\pi R$$

and

$$R(T) = R(1 - T^2/T_c^2)^{-1/2}.$$

At zero temperature, experimental data on charmed meson decays give $R \sim 1\,\text{GeV}^{-1}$ fixing

$$T_c \simeq 160\,\text{MeV}.$$

This result is in confirmity with the QCD calculations and estimates based on bag model calculations, which suggest $T_c \simeq 150 - 200$ MeV.

The result mentioned above now awaits experimental confirmations.

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References

Biswas S and Kumar S 1989a *Pramāṇa – J. Phys.* **33** 277
Biswas S and Kumar S 1989b *Pramāṇa – J. Phys.* **33** 289
Biswas S and Kumar S 1989c *Indian J. Phys.* **A63** 633
Bediaga I, Gasperini M and Predazzi E 1988 *Phys. Rev.* **D38** 1626
Dicke R H 1957 *Rev. Mod. Phys.* **20** 263
Gasperini M 1987 *Phys. Lett.* **B195** 453
Gasperini M 1988 *Class. Quantum Gravit.* **5** 521
Jena P K and Pradhan T 1981 *Pramāṇa – J. Phys.* **17** 287
Jena P K and Pradhan T 1984 *Pramāṇa – J. Phys.* **22** 237
Khare A and Pradhan T 1983 *Phys. Lett.* **B120** 145
Khadkikar S B and Vinodkumar P C 1983 *Phys. Lett.* **B120** 145
Khadkikar S B and Vinodkumar P C 1987 *Pramāṇa – J. Phys.* **29** 39
Lee T D 1979 *Phys. Rev.* **D19** 1802
Landau L D and Lifshitz E M 1980 *The classical theory of fields* (NY: Pergamon) p. 257
Mielke E W 1979 *Lett. Nuovo Cimento* **25** 424
Mielke E W 1978 *Phys. Rev* **D18** 4525
Mielke E W 1980a *The eightfold way to colour geometrodynamics* ICTP Preprint IC/80/158
Mielke E W 1980b *Int. J. Theor. Phys.* **19** 189
Mielke E W and Scherzer R 1980 *Geon type solution of the nonlinear H K G equation* ICTP Preprint IC/80/158
Muller A 1978 *Phys. Rev.* **D18** 3705
Sivaram C and Sinha K P 1979 *Phys. Rep.* **51** 111
Salam A and Strathdee J 1978 *Phys. Rev.* **D18** 4596
Zachariasen F 1980 *In hadronic matter at extreme density* (Cabbibo and Sertirio, etc.) (Plenum Press) p. 313
Stefan Schmitz, Beavis D and Kaus P 1987 *Phys. Rev.* **D36** 184