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Static baryon properties in a relativistic quark model with centre-of-mass correction

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Abstract. The static properties such as magnetic moments, charge radii and axial vector coupling constant ratios of the quark core of baryons in the nucleon octet have been calculated in an independent-quark model based on the Dirac equation with equally mixed scalar-vector potential in linear form in the current quark mass limit. The results obtained with appropriate corrections due to centre-of-mass motion are in reasonable agreement with experimental data. The magnetic moments of the quark core of baryons in the charmed and b-flavoured sectors have also been calculated with this model and the overall predictions so obtained compare very well with other model predictions.

Keywords. Independent quark model; Dirac equation; linear potential; nucleon octet; magnetic moments; charge radii; axial vector coupling constant; charmed and b-flavoured baryons; centre-of-mass correction.

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1. Introduction

Several quark models inspired by the dynamical theory of quarks and gluons (QCD) have been successfully used to understand the static properties of baryons. Still no unique interquark potential for all ranges has emerged so far either from the first principle theoretical derivations or from simple phenomenological studies. In the study of static baryon properties, the existing data do not seem to prefer uniquely any one of the various potential models suggested so far in the literature. Only a wider range of more accurate data involving certain aspects, more sensitive to the specific features of the models, may enable one to select out some in preference over the others. The Bag model (Chodos *et al* 1974 a, b; De Grand *et al* 1975; Donoghue and Johnson 1980; Thomas *et al* 1981; Theberge and Thomas 1982; Thomas 1983 and Miller 1984) in its different forms, is essentially the only phenomenological model which has been found very extensive in its applications and reasonably successful in the predictions. Yet this model is believed to be neither unique nor entirely free from difficulties and objections. Therefore, there have been in the recent past several attempts at formulating alternative phenomenological schemes with confining potentials of appropriate Lorentz structures. Scalar potentials in linear (Eich *et al* 1985), harmonic and cubic (Tegen *et al* 1982; Tegen and Weise 1983; Osel *et al* 1984) form and equally mixed scalar-vector potentials in linear (Ferreira 1977; Ferreira and Zagury 1977; Kobuskin 1976), harmonic (Ferreira and Zagury 1977; Ferreira *et al* 1980; Barik *et al* 1985; Barik and Dash 1986), logarithmic (Magyari 1980; Jena and Rath 1986a, b) as well as in non-coulombic power-law (Barik and Jena 1980a, b, 1982; Barik and Das

1983a, b) form have been investigated by various authors. All these models used in some limited aspects of baryonic phenomena only meet with success more or less identical to the bag model. However this does not establish the non-uniqueness of bag-like models, unless each of these models is pursued extensively over a much wider range of baryonic phenomena. The present work is only a step in that direction where the potential model, with independent quark potential of the form

$$V_q(r) = \tfrac{1}{2}(1+\gamma^0)(a^2 r + V_0), \quad a>0 \tag{1}$$

has been utilized to study extensively the static properties of baryons in the nucleon octet as well as those in the charmed and b-flavoured sectors. In this work we have calculated the various static properties like the magnetic moments, root mean square charge radii and the axial-vector coupling constant ratio (g_A/g_V) for the ordinary baryons in the nucleon octet making necessary corrections due to centre of mass motion. In addition to this the magnetic moments of charmed and b-flavoured baryons have also been predicted with this model. In fact the potential model (1) has been used earlier by Ferreira (1977), Ferreira and Zagury (1977) and Kobuskin (1976) for calculating the magnetic moments, the ratio $(g_A/g_V)_n$ and the rms charge radii of octet baryons but without taking into account the cm correction. Although the same form of potential model has been used in the present work, it is intended to calculate the static properties of baryons by (a) including cm effects for octet baryons (b) using recent data as inputs, and (c) carrying out the calculations for c- and b-flavoured baryons.

The paper is organized as follows: We present in §2 a brief outline of the potential model and its solutions, leading to a complete description of the relativistic bound states of individually confined quarks of the baryon core, and also to the core contribution to the static properties of baryons in terms of the magnetic moments, rms charge radii and axial vector coupling constant ratios (g_A/g_V) for β-decay processes. We also discuss the prescription adopted here to consider the effect of cm motion on the above quantities. In §3 we estimate the potential parameters and the quark-masses suitably in order to yield an appropriate ground-state energy for the independent quarks, which gives the average nucleon and $\Delta(1232)$ mass approximately, keeping in mind the corrections due to the cm motion. The static properties of octet baryons calculated after the cm corrections turn out to be in reasonable agreement with the experimental results. For the moment we leave aside the effects of the Pion Cloud supposed to surround the assembly of quarks in the baryon. The quark-core contributions to the magnetic moments of charmed and b-flavoured baryons are also predicted in §3. Lastly the §4 is devoted to brief discussion and conclusion.

2. Theoretical framework

In this section we briefly outline the framework of the model adopted here to study the core contribution to the static properties of the baryons and the prescriptions used to account for the corrections due to the cm motion.

2.1 *The potential model*

We start with the assumption that the quarks in a baryon core move independently in an average flavour-independent potential taken in the form

$$V_q(r)=\tfrac{1}{2}(1+\gamma^0)V(r) \tag{2}$$

where

$$V(r)=a^2r+V_0, \quad \text{with } a>0.$$

The independent quark of rest mass m_q is further assumed to obey the Dirac equation (with $\hbar=c=1$)

$$[\gamma^0 E_q-\boldsymbol{\gamma}\cdot\boldsymbol{p}-m_q-V_q(r)]\Psi_q(\boldsymbol{r})=0. \tag{3}$$

As in bag models if we now assume that all the three quarks in the baryon core are in their ground state with $J^p=\frac{1}{2}^+$ and $J_z=\frac{1}{2}$, then a solution to the independent quark wave function $\psi_q(\boldsymbol{r})$ can be obtained in the two-component form as

$$\Psi_q(\boldsymbol{r})=N_q\begin{bmatrix}\phi_q(\boldsymbol{r})\\ \dfrac{\boldsymbol{\sigma}\cdot\boldsymbol{p}}{\lambda_q}\phi_q(\boldsymbol{r})\end{bmatrix}\chi\uparrow \tag{4}$$

where

$$\lambda_q=E_q+m_q$$

and

$$\phi_q(\boldsymbol{r})=A_q\frac{U_q(r)}{r}Y_0^0(\theta,\phi)=A_q f_q(r)Y_0^0(\theta,\phi) \tag{5}$$

is the normalized radial angular part of $\Psi_q(\boldsymbol{r})$ with normalization constant A_q. The overall normalization constant N_q of $\Psi_q(\boldsymbol{r})$ can be easily obtained as

$$N_q^2=[1+(E_q-m_q-V_0-a^2\ll r\gg_q)/\lambda_q]^{-1} \tag{6}$$

where $\ll r\gg_q$ stands for the expectation value of r with respect to $\phi_q(\boldsymbol{r})$. $\phi_q(\boldsymbol{r})$ satisfies a Schrödinger type equation

$$\phi_q''+\frac{2\phi'}{r}+\lambda_q[E_q-m_q-V(r)]\phi_q=0 \tag{7}$$

which with (5) reduces to

$$U_q''(r)+\lambda_q[E_q-m_q-V(r)]U_q(r)=0. \tag{8}$$

This can be transferred into a convenient dimensionless form

$$U_q''(\rho)+(\varepsilon_{ns}-\rho)U_q(\rho)=0 \tag{9}$$

where $\rho=(r/r_{oq})$ is a dimensionless variable with

$$r_{oq}=(\lambda_q a^2)^{-1/3} \tag{10}$$

and

$$\varepsilon_{ns}=(\lambda_q/a^4)^{1/3}(E_q-m_q-V_0). \tag{11}$$

Now with $z=\rho-\varepsilon_{ns}$, (9) reduces to the Airy equation

$$U_q''(z)-ZU_q(z)=0. \tag{12}$$

The solution $U_q(z)$ of (12) is the Airy function Ai(Z). Since at $r=0$ we require $U_q(r)=0$, we have $\mathrm{Ai}(z)=0$ at $z=-\varepsilon_{ns}$. If z_n are the roots of the Airy function such that $\mathrm{Ai}(z_n)=0$, then we have $z=-\varepsilon_{ns}=Z_n$. For the ground state of the quarks, the ε_{ns} value is given by the first root z_1 of the Airy function so that

$$\varepsilon_{ns}=\varepsilon_{1s}=\varepsilon_q=-z_1 \tag{13}$$

the value of this root being $z_1=-2\cdot 33811$. With the knowledge of ε_q and $U_q(z)$ the expression (5) for normalized $\phi_q(r)$ becomes

$$\phi_q(r)=\frac{(a\sqrt{\lambda_q})^{1/3}}{\mathrm{A'i}(z_1)}\frac{\mathrm{Ai}(z)}{r}Y_0^0(\theta,\phi) \tag{14}$$

where $\mathrm{A'i}(z_1)$ is the derivative of Ai(z) at $z=z_1$. Now the individual quark binding energy E_q can be obtained from (11) through the relation

$$E_q=m_q+V_0+ax_q \tag{15}$$

where x_q is the solution of the root equation obtained through substitution from (11) in the form

$$x_q^4+bx_q^3-\varepsilon_q^3=0 \tag{16}$$

with $b=(2m_q+V_0)/a$. Using the expression (14) the expectation value $\langle\langle r\rangle\rangle_q$ can be easily obtained as

$$\langle\langle r\rangle\rangle_q=\tfrac{2}{3}r_{oq}\varepsilon_q \tag{17}$$

with which the expression (6) for N_q^2 simplifies to

$$N_q^2=\left[1+\left(\frac{a}{\lambda_q}\right)^{4/3}\left(\frac{\varepsilon_q}{3}\right)\right]^{-1}. \tag{18}$$

2.2 *Static properties of the baryon core*

We can now present some consequences of the model in terms of derived expressions for the quark-core contributions to certain measurable quantities of the baryons which are obtained simply by appropriately adding the contributions of each individual quark.

(i) *Magnetic moments of the baryon core:* The magnetic moment of a $\frac{1}{2}^+$ baryon primarily consists of contributions from its bare quark-core in terms of the corresponding constituent quark moments which are defined as

$$\mu_q=\tfrac{1}{2}\int \mathrm{d}^3r[\boldsymbol{r}\times \boldsymbol{J}(\boldsymbol{r})]_z \tag{19}$$

where

$$\boldsymbol{J}(\boldsymbol{r})=e_q\Psi_q^+(\boldsymbol{r})\boldsymbol{\alpha}\Psi_q(\boldsymbol{r}).$$

Here e_q is the charge of the quark in units of proton charge. With the help of the expression (4) for $\Psi_q(r)$ eq. (19) simplifies to

$$\mu_q = (2M_p N_q^2/\lambda_q)e_q \text{ nm} \tag{20}$$

where M_p is the mass of the proton and nm stands for nuclear magneton. The bare core contribution to the magnetic moment of a baryon B is given by

$$\mu_B = \left\langle B\uparrow \left| \sum_q \mu_q \sigma_z^q \right| B\uparrow \right\rangle \tag{21}$$

where $|B\uparrow\rangle$ represents the state vectors of the baryons. In the case of octet baryons $|B\uparrow\rangle$ represents the regular SU(6) state vectors. For the charmed and b-flavoured baryons the corresponding state vectors are obtained by the straightforward extensions as given by Singh (1977). The relations for magnetic moments of baryons in terms of constituent-quark moments are well known (Singh 1977, 1979; Pandita *et al* 1981; Choudhury and Joshi 1976; Lichtenberg 1977; Johnson and Shahjahan 1977; Franklin 1968) and can be used to compute the quark-core contribution to the magnetic moments of baryons with the present model.

(ii) *The mean squared electric charge radii of the baryon core*: The distribution of charge within a baryon core is determined by a convenient parameter called the mean square charge radius $\langle r^2\rangle_B$ which is usually defined as

$$\langle r^2\rangle_B = \sum_q e_q \int \Psi_q^+(r) r^2 \Psi_q(r)\, \mathrm{d}^3 r \tag{22}$$

where e_q is the quark electric charge and Ψ_q is the wave function of the quark in $1S_{1/2}$ state.

Using the expression for $\Psi_q(r)$ from (4), (22) can be easily simplified to

$$\langle r^2\rangle_B = \sum_q e_q \frac{N_q^2}{\lambda_q}\left[(2E_q - V_0)\langle\langle r^2\rangle\rangle_q - a^2\langle\langle r^3\rangle\rangle_q + \frac{3}{\lambda_q}\right]. \tag{23}$$

The double angular brackets appearing in (23) are the expectation values with respect to $\phi_q(r)$ which can be easily evaluated as

$$\langle\langle r^2\rangle\rangle_q = \tfrac{8}{15} r_{oq}^2 \varepsilon_q^2 \tag{24}$$

and

$$\langle\langle r^3\rangle\rangle_q = \tfrac{3}{7} r_{oq}^3 (1 + \tfrac{16}{15}\varepsilon_q^3). \tag{25}$$

(iii) *Weak beta decays and the ratio* (g_A/g_V): Now interpreting the weak β-decays of the baryons $B - B' + e^- + \bar{\nu}_e$ as quark β-decays like $q_j - u + e^- + \bar{\nu}_e$ occurring inside the baryon core (where q_j could be d or s quark), one can obtain in a similar manner as bag model calculations (Chodos *et al* 1974b; Donoghue and Johnson 1980), the axial vector coupling constant ratio (g_A/g_V) as

$$(g_A/g_V) = (g_A/g_V)^{\mathrm{NR}}(1 - 2\Delta_{ju}). \tag{26}$$

Here g_V and g_A are the vector and axial vector coupling constants for the baryons. $(g_A/g_V)^{\mathrm{NR}}$ is the non-relativistic SU(6) value (Kokkedee 1969) which for various beta-decays can be computed by using SU(6) wave function for $|B\uparrow\rangle$ states (Quigg 1981). Δ_{ju} is the relativistic correction in the axial coupling for the β-decay of quark 'j' into the quark 'u' and is given by

$$\Delta_{ju}=\int \Psi_u^+ L_z \Psi_j \, d^3 r \tag{27}$$

which on putting the expression for Ψ_q from (4) for quarks in the $1S_{1/2}$ state becomes

$$\Delta_{ju}=\left(\frac{2N_u N_j}{3\lambda_u \lambda_j}\right) I_{uj} \tag{28}$$

with

$$I_{uj}=\int_0^\infty r^2 \, dr f'_u(r) f'_j(r) \tag{29}$$

integrating by parts and using (8) with

$$\frac{U_q(r)}{r}=f_q(r) \text{ one gets}$$

$$I_{uj}=\frac{1}{r_{oj}^2}\int_0^\infty dr[\varepsilon_q-(r/r_{oj})]U_j(r)U_u(r). \tag{30}$$

Since the Airy function solution for $U_q(r)$ makes the evaluation of the integral I_{uj} complicated, we use for simplicity the WKB solution for $U_q(r)$ (Quigg and Rosner 1979) which is given by

$$U_q(\rho)=\frac{B_q}{(\varepsilon_q-\rho)^{1/4}}\cos\eta_q \tag{31}$$

where

$$\eta_q=\int_0^\rho d\rho' \, (\varepsilon_q-\rho')^{-1/2}-\frac{\pi}{4}$$

and B_q = normalization constant $=(\sqrt{\varepsilon_q}/r_{oq})^{1/2}$. With a reasonable approximation $\cos\eta_u \cdot \cos\eta_j \simeq \cos^2\eta \simeq \frac{1}{2}$, I_{uj} can be obtained as

$$I_{uj}=\frac{2}{7}\frac{\varepsilon_q K}{r_{oj}^2}F(\tfrac{1}{4},1,\tfrac{11}{4},K^2) \tag{32}$$

where $K^2=(r_{oj}/r_{ou})$ and $F(\frac{1}{4}, 1, \frac{11}{4}, K^2)$ in the hypergeometric function. Thus (g_A/g_V) values for various β-decays can be computed with the help of the relations (26), (28) and (32). For example, for neutron beta-decay

$n-pe\bar{\nu}$ (or $d-u+e+\bar{\nu}_e$) eq. (26) reduces to

$$(g_A/g_V)_n-pe\bar{\nu}_e=\frac{5}{3}\left[1-\frac{4N_u^2\varepsilon_q}{3\lambda_u^2 r_{ou}^2}\left(\frac{1}{3}\right)\right] \tag{33}$$

which ultimately simplifies to

$$(g_A/g_V)_n-pe\bar{\nu}_e=\tfrac{5}{9}(4N_u^2-1). \tag{34}$$

2.3 Centre of mass correction

In this model there would be a sizeable spurious contribution to the energy from the motion of the centre of mass of the three quark system. The expressions for static quantities which we have evaluated above can now be modified taking into account the three quarks centre of mass motion.

Clearly our shell type relativistic quark model is not translationally invariant. This means that the independent motion of the quarks inside the potential does not lead to a state of definite total momentum as it should in order to represent a physical baryon state. The problem appears in the same way in nuclear physics in the case of He and also in the MIT bag model (Donoghue and Johnson 1980) and has to be resolved accordingly (Hill and Wheeler 1953; Wong 1975 and Bartelski *et al* 1984). Though there is still some controversy on this subject we adopt here the prescription followed by Wong (Wong 1981; Duck 1978) and other workers (Bartelski *et al* 1984 and Eich *et al* 1983), which is just one way of accounting for the cm motion. We take the static three-quark baryon-core state with the core centre at x as a superposition of plane wave momentum eigenstates $\chi(\boldsymbol{p})$

$$|3q, \mathbf{x}\rangle = \int d^3\boldsymbol{p}\left(\frac{M_B}{E_B}\right)\chi(\boldsymbol{p})\exp(i\boldsymbol{p}\cdot\mathbf{x})\,|B(\boldsymbol{p})\rangle \tag{35}$$

where the momentum eigen states $|B(\boldsymbol{p})\rangle$ of the baryon core B are normalized usually as

$$B(\boldsymbol{p}')|B(\boldsymbol{p})\rangle = (2\pi)^3\delta(\boldsymbol{p}-\boldsymbol{p}')\left(\frac{E_B}{M_B}\right) \tag{36}$$

with M_B and E_B denoting respectively the mass and energy of the baryon core state under consideration. The momentum-profile function $\chi(\boldsymbol{p})$ can be obtained from (35) and (36) as

$$|\chi(\boldsymbol{p})|^2 = \left(\frac{E_B}{M_B}\right)\frac{\tilde{I}(\boldsymbol{p})}{(2\pi)^3} \tag{37}$$

where

$$\tilde{I}(\boldsymbol{p}) = \frac{1}{(2\pi)^3}\int d^3\boldsymbol{r}\exp(-i\boldsymbol{p}\cdot\boldsymbol{r})\langle 3q, 0|3q, \boldsymbol{r}\rangle \tag{38}$$

is the Fourier transform of the Hill-Wheeler overlap function (Wong 1981 and Duck 1978). This result along with (35) can be used to evaluate the expectation values of any $F(\boldsymbol{p})$ as

$$\langle 3q, 0|F(\boldsymbol{p})|3q, 0\rangle = \int d^3\mathbf{p}\tilde{I}(\mathbf{p})F(\mathbf{p}). \tag{39}$$

Then it is a simple matter to evaluate various cm corrections, once the Hill-Wheeler function $\tilde{I}(\boldsymbol{p})$ is calculated. This overlap function permits ready estimates of cm effects in terms of the expectation values $\langle M_B^2/E_B^2\rangle$, $\langle M_B/E_B\rangle$, $\langle \boldsymbol{p}^2\rangle$, and $\langle \boldsymbol{R}^2\rangle$, where

$$\boldsymbol{R} = \sum_q E_q r_q \Big/ \sum_q E_q \tag{40}$$

evaluated according to (39). In fact the physical mass M_B of the baryon core is related to the relativistic energy E_B of the three quark state as

$$M_B=(E_B^2-\langle p_B^2\rangle)^{1/2} \tag{41}$$

where $E_B=\sum_q E_q$, p_B is the centre of mass momentum and $\langle p_B^2\rangle$ is evaluated with the usual approximation as

$$\langle p_B^2\rangle=\sum_q \langle p^2\rangle_q. \tag{42}$$

Here $\langle p^2\rangle_q$ is the average value of the square of the individual quark momentum taken over the 1S single-quark states $\Psi_q(r)$ and is obtained in this model as

$$\langle p^2\rangle_q=N_q^2[2E_q(E_q-m_q)-(3E_q-m_q-V_0)V_0 \\ -(3E_q-m_q-2V_0)a^2\langle\langle r\rangle\rangle_q+a^4\langle\langle r^2\rangle\rangle_q] \tag{43}$$

where the double angular brackets represent the expectation values with respect to $\phi_q(r)$ and are given by (17) and (24) respectively. Then with (41), (42) and (43) one easily gets

$$\langle M_B^2/E_B^2\rangle=\left[1-\sum_q p_q^2/E_B^2\right]=\delta_B^2. \tag{44}$$

Now taking $\langle M_B/E_B\rangle$ as roughly equal to $\delta_B=\langle M_B^2/E_B^2\rangle^{1/2}$, one can compute the corrected (Bartelski *et al* 1984 and Eich *et al* 1983) static properties as

$$\mu_B'=[3\mu_B+Q_B(M_p/M_B)(1-\delta_B)]/(1+\delta_B+\delta_B^2) \tag{45}$$

$$\langle r^2\rangle_B'=3(\langle r^2\rangle_B-Q_B\langle R^2\rangle)/(2+\delta_B^2) \tag{46}$$

$$(g_A/g_V)'=3(g_A/g_V)/(1+2\delta_B) \tag{47}$$

where Q_B is the total charge of the baryon, the unprimed quantities are the uncorrected static properties, and the primed quantities are the corresponding corrected ones. Now with the evaluation of $\langle R^2\rangle$, the expression (46) can be expressed as

$$\langle r^2\rangle_B'=\left[3\langle r^2\rangle_B-3\sum_q\left\{2e_q\left(\frac{E_q}{E_B}\right)-Q_B\left(\frac{E_q}{E_B}\right)^2\right\}\langle r^2\rangle_q\right]\Big/(2+\delta_B^2). \tag{48}$$

For the nucleon it is quite consistent to put

$$\left(\frac{E_q}{E_B}\right)\approx\frac{1}{3},$$

thus (48) may be cast into the form

$$\langle r^2\rangle_N'=\left[3\langle r^2\rangle_N-\sum_q\{2e_q+Q_B/3\}\langle r^2\rangle_q\right]\Big/(2+\delta_N^2). \tag{49}$$

3. Calculation and results

We make the usual assumption that the average potential taken in this model for the confined independent quarks inside the baryons is flavour independent. Therefore, we use the same set of potential parameters and quark masses for the calculation of static baryon properties. The expressions for these properties such as magnetic moments, mean square charge radii and axial vector coupling constant ratios as given in §2 are found to depend on potential parameters a and V_0, the quark mass parameters m_q and the single-quark energy eigenvalue E_q. Although the parameters a, V_0 and m_q are a priori unconstrained, we have to make a suitable choice by reasonable assumptions.

The quark mass parameters are assumed to be of the order of current quark masses and are chosen according to the prediction of current algebra, which would restrict m_q for the u-or-d-quark not to exceed some 10 or 20 meV. However, since no physical quantity considered here depends appreciably on these parameters as long as they are small, we keep them fixed as $m_u \simeq m_d = 10$ meV. Then the potential parameters a and V_0 have to be adjusted properly to yield the appropriate single-quark ground state energy E_q. Normally $E_u = E_d$ is expected to be of the order of $\frac{1}{3}\bar{M}_N$, where

$$\bar{M}_N = \left[\frac{4M_N + 16M_\Delta}{20}\right] = 1173\,\text{meV}.$$

is the spin-isospin average mass of the nucleon and Δ (1232). However, if one admits that spurious cm motion corrections and other possible corrections such as pion-Cloud effects not considered in this model, are to be accounted for at appropriate stages, then E_u must be somewhat larger than $\frac{1}{3}\bar{M}_N$. Since we are not particularly interested here in the detailed baryonic mass spectrum, the parameters a, V_0 and hence E_q are so chosen that the static nucleon properties such as μ_p, $\langle r^2 \rangle_p^{1/2}$ and $(g_A/g_V)_n$ obtained after the cm motion corrections agree closely with the corresponding experimental values. In other words we choose a, V_0 and hence E_q appropriately to obtain an estimate for the quark-core contributions to the nucleon properties like μ_p, $\langle r^2 \rangle_p^{1/2}$ and $(g_A/g_V)_n$ which with the inclusion of cm corrections would lead to the corresponding quantities in close agreement with the experiment.

We find that with

$$(a, V_0) = (254, 94)\ \text{MeV}, \tag{50}$$

and

$$m_u = m_d = 10\ \text{MeV}. \tag{51}$$

the energy eigenvalue condition (15) yields

$$E_u = E_d = 558{\cdot}114\ \text{MeV} \tag{52}$$

which results in the nucleon core properties

$$\mu_p = 2{\cdot}6082\ \text{n.m.}\ \langle r^2 \rangle_p^{1/2} = 1{\cdot}0725\ \text{fm}.$$
$$(g_A/g_V)_{n \to Pe\bar{\nu}} = 1{\cdot}1991 \tag{53}$$

before the cm correction is applied. Now with the assumed flavour independence of the potential, the parameters a, V_0 obtained in (50) along with $m_s = 225$ MeV chosen well within the limits of current algebra predictions, give $E_s = 704{\cdot}398$ MeV from (15). This yields a value of $\mu_\Lambda = -0{\cdot}5913$ nm before the cm correction is included.

With the parameters a, V_0 and m_q along with the corresponding binding energy

Table 1. Magnetic moments of the nucleon octet calculated by the present model with and without cm correction as compared with the results of Cloudy-bag-model (CBM) and the experimental data (all members in nuclear magnetons).

Baryon	Present calculation: Uncorrected	Present calculation: After cm correction	CBM Calculation (Thomas *et al* 1981; Theberge *et al* 1982)	Experiment (Particle Data Group 1986)
p	2·6082	2·7694	2·60	2·7928444 ± 0·0000011
n	−1·7388	−1·8386	−2·01	−1·91304308 ± 0·00000054
Λ^0	−0·5913	−0·6246	−0·58	−0·613 ± 0·004
Σ^+	2·5155	2·6675	2·34	2·379 ± 0·020
Σ^0	0·7767	0·8204	—	0·61 ± 0·08*
Σ^-	−0·9621	−1·0267	−1·08	−1·14 ± 0·05
Ξ^0	−1·3680	−1·4430	−1·27	−1·250 ± 0·014
Ξ^-	−0·4986	−0·5353	−0·51	−0·69 ± 0·04
(Λ, Σ^0)	−1·5058	−1·5923	—	$-1{\cdot}82^{+0\cdot18}_{-0\cdot25}$**

*Values computed according to the definition $\mu_{\Sigma^0} = \frac{1}{2}(\mu_{\Sigma^+} + \mu_{\Sigma^+})_{exp}$ which is quite often referred to as the measured value.
**Dydak *et al* 1977

E_q being known, all the relevant quantities leading to the predictions of the static baryon core properties before cm correction can be calculated. First of all we obtain from (20) the constituent quark magnetic moments as

$$\mu_u = -2\mu_d = 1{\cdot}7388\,\text{nm} \tag{54}$$

and

$$\mu_s = -0{\cdot}5913\,\text{nm}$$

which can be used to compute the uncorrected magnetic moments of the quark core for the octet baryons. The results calculated in this way are presented in table 1.

In the framework of this model we calculate the expressions (24) and (25) for u and s quarks as

$$\begin{aligned} &\langle\langle r^2 \rangle\rangle_u = 1{\cdot}0289\,\text{fm}^2, \quad \langle\langle r^3 \rangle\rangle_u = 1{\cdot}3148\,\text{fm}^3 \\ &\langle\langle r^2 \rangle\rangle_s = 0{\cdot}7411\,\text{fm}^2, \quad \langle\langle r^3 \rangle\rangle_s = 0{\cdot}8037\,\text{fm}^3 \end{aligned} \tag{55}$$

which lead to the evaluation of rms charge radii $\langle r^2 \rangle_B^{1/2}$ of the octet baryons from (23). The results obtained here without cm correction are given in table 2.

With this model we also compute the expression (32) to obtain the values

$$(I_{uu}, I_{us}) = (0{\cdot}0859, 0{\cdot}1054)\,\text{GeV}^2 \tag{56}$$

which through (28) and (26) provide the axial vector coupling constant ratio (g_A/g_V) for the β-decay processes corresponding to the members of the baryon octet, with appropriate values (Kokkedee 1969) of $(g_A/g_V)^{NR}$. These results without cm corrections are presented in table 3.

For introducing cm corrections we estimate the factor δ_B from expressions (43) and (44) which certainly depend on the flavour combinations of the quark core in baryons.

Table 2. $\langle r^2 \rangle_B^{1/2}$ rms charge radii of octet baryons calculated in the present model with and without cm correction as compared to the experiment and Ferreira-Helayel-Zagury (FHZ) model values (all numbers in fm).

Baryon	$\langle r^2 \rangle_B^{1/2}$ uncorrected	$\langle r^2 \rangle_B^{1/2}$ with cm correction	Experiment	FHZ model (Ferreira *et al* 1980)
p	1·0725	0·8918	0.87 ± 0.02	0·932
n	0	0	-0.341	0
Λ^0	0·3458	0·3031	—	0·346
Σ^+	1·1268	0·9487	—	0·994
Σ^0	0·3458	0·3031	—	0·346
Σ^-	1·0152	0·8463	—	0·86
Ξ^0	0·4890	0·4340	—	0·48
Ξ^-	0·9545	0·7956	—	0·79

Table 3. Axial vector coupling constant ratio (g_A/g_V) calculated in the present model with and without cm correction as compared to the experiment.

Decay mode	Axial vector coupling constant ratio: Uncorrected (g_A/g_V)	Axial vector coupling constant ratio: with cm corrected (g_A/g_V)	(g_A/g_V) Experiment Ref. 21 (Particle Data group 1986)
$n \rightarrow p e \bar{\nu}$	1·1991	1·2450	1.254 ± 0.006
$\Lambda \rightarrow p e \bar{\nu}$	0·7783	0·8076	0.694 ± 0.025
$\Sigma^- \rightarrow n e \bar{\nu}$	-0.2594	-0.2692	-0.362 ± 0.043
$\Xi^- \rightarrow \Lambda e \bar{\nu}$	0·1730	0·1793	0.25 ± 0.05
$\Xi^- \rightarrow \Sigma^0 e \bar{\nu}$	1·2972	1·3447	—
$\Xi^0 \rightarrow \Sigma^+ e \bar{\nu}$	1·2972	1·3447	—

Accordingly we evaluate $\delta_B = 0.9447$, 0·9457, 0·9471 for baryon core with three non-strange quarks, two non-strange quarks with one strange quark and one non-strange quark with two strange quarks respectively. Then it is straight forward to obtain the corrected values of the magnetic moments, rms charge radii and axial vector coupling constant ratio of octet baryons from expressions (45), (46) and (47) respectively. These quantities, so obtained, after cm correction are presented in appropriate tables (1, 2 and 3) in comparison with the corresponding uncorrected values as well as the experimental values.

Finally we use the same set of potential parameters a and V_0 as obtained in (50) along with the quark masses

$$(m_c, m_b) = (1.5, 4.5)\,\mathrm{GeV} \tag{57}$$

chosen well within the limits of current algebra predictions to obtain from (15) the energy eigenvalues

$$(E_c, E_b) = (1.8455, 4.7730)\,\mathrm{GeV} \tag{58}$$

Table 4. Magnetic moment of charmed baryons computed in the present model as compared to the values obtained in the bag model and DeRujula-Georgi-Glashow (DGG) model (all numbers in nuclear magneton).

Symbol	Quark content	Present calculation	Bag model (Bose and Singh 1980; Eich *et al* 1985)	DGG model (Pandit *et al* 1981)
Σ_c^{++}	*cuu*	2·1968	1·955	2·36
Σ_c^{+}	*cud*	0·4580	0·363	0·43
Σ_{c}^{0}	*cdd*	−1·2808	−1·23	−1·43
Ω_c^0	*css*	−0·9100	−0·98	−0·89
Ξ_c^+	*csu*	0·6431	0·475	0·73
Ξ_c^0	*csd*	−1·0954	−1·09	−1·16
Ξ_{cc}^{++}	*ccu*	−0·0932	−0·167	−0·12
Ξ_{cc}^{+}	*ccd*	0·7762	0·865	0·82
Ω_{cc}^{+}	*ccs*	0·6835	0·838	0·69
Λ_c^+	*c(ud)a*	0·3648	0·503	0·37
$\Xi_c'^{+}$	*c(us)a*	0·3648	0·503	0·37
$\Xi_c'^{0}$	*c(ds)a*	0·3648	0·503	0·37

Table 5. Magnetic moments of *b*-flavoured baryons computed in the present model as compared to the values obtained in the bag model and DGG model (all members in nuclear magneton).

Symbol	Quark content	Present calculation	Bag model (Bose and Singh 1980; Eich *et al* 1985)	DGG model (Pandita *et al* 1981)
Σ_b^+	*buu*	2·3407	2·318	2·5
Σ_b^0	*bud*	0·6019	0·587	0·61
Σ_b^-	*bdd*	−1·1369	−1·117	−1·28
Ω_b^-	*bss*	−0·7661	−0·838	−0·55
Ξ_{cb}^0	*bcd*	−0·3141	−0·39	−0·38
Ξ_{cb}^+	*bcu*	1·4247	2·04	1·5
Ω_{ccb}^+	*bcc*	0·5088	0·894	0·51
Ξ_{bb}^0	*bbu*	−0·6690	−0·614	−0·7
Ξ_{bb}^-	*bbd*	0·2004	0·14	0·23
Ω_{bb}^-	*bbs*	0·1077	0·084	0·105
Ω_{cbb}^0	*bbc*	−0·2110	−0·31	−0·21
Ξ_b^0	*bsu*	0·7873	0·73	0·87
Ξ_b^-	*bsd*	−0·9515	−0·977	−1·05
Ω_{cb}^0	*bcs*	−0·1287	−0·223	−0·11

which yield the constituent quark magnetic moments from (20) as

$$(\mu_c, \mu_b) = (0{\cdot}3648,\ -0{\cdot}0670)\,\text{nm}. \tag{59}$$

The magnetic moments of the baryons in charmed and *b*-flavoured sectors are computed in a straightforward way using the constituent-quark moments obtained in (54) and (59). Since no experimental data for the magnetic moments are available in the case of charmed and *b*-flavoured baryons we do not incorporate the cm corrections in these cases and the uncorrected results are displayed in tables 4 and 5 in comparison with the predictions of some other models (Pandita *et al* 1981; Bose and Singh 1980; Eich *et al* 1985). The symbols used in these tables for charmed and *b*-flavoured baryons are according to Pandita *et al* (1981).

4. Discussion and conclusion

We observe that our cm corrected results for the magnetic moments, rms charge radii and the axial vector coupling constant ratios for octet baryons, compare reasonably well with the existing experimental values. The proton charge radius $\langle r^2 \rangle_p^{1/2}$, which was 1·0725 fm before the cm correction becomes 0·8918 fm as against the experimental value of 0·87 fm. Furthermore we find $\langle r^2 \rangle_n^{1/2} = 0$ contrary to the experimental value of $-0{\cdot}341$ fm. This is of course the case with most of the models of this kind including the bag model. In the absence of experimental data of $\langle r^2 \rangle_B^{1/2}$ for other members of the octet baryons we compare our results with those obtained by Ferreira *et al* (1980). Any departure of our calculated values in the (u, d) sector from the experimental data can hopefully be accounted for by the pionic contribution, which will be taken up in a subsequent work. But in the strange-quark sector we expect the pionic contribution to be less significant in view of its being heavier than the pion. In any case our overall prediction for the static properties of the baryons in the nucleon octet show a significant improvement over the results obtained earlier in a similar model (Ferreira 1977; Ferreira and Zagury 1977; Kobuskin 1976) without cm correction, which differs from ours in the choice of potential parameters. Our predictions for charmed and *b*-flavoured baryons are not drastically different from those of other models.

In the present work we thus find that an independent quark model based on the Diraç equation with an equally mixed scalar-vector linear confining potential of the form given by (1) is found to provide a simple and straight forward approach for the study of static properties of light, charmed and *b*-flavoured baryons in a favour-independent manner. In view of the simplicity of the model the results obtained are quite encouraging.

The purpose of this work is not so much to claim that nature prefers such a linear confinement scheme over other schemes advanced earlier in this line, but as much as to present an analysis that complements studies based on various potential models. Such studies from various quark-models when extended to describe hadronic properties in a much wider sector, may provide an opportunity to resolve the question regarding the true nature of quark confinement which is not straight forward to conclude on purely theoretical grounds based on quantum chromodynamics (QCD).

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