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Regge behaviour at high energy and more on boson-fermion interaction

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Abstract. Our approach to the problem of boson-fermion interaction in the conventional RFT stems from a model of super-symmetric version in 2-space and 1 time world. Basically it is stressed here that at super high energy there may not be any distinction between the bosonic and fermionic modes and may be treated on a common footing. Usual renormalization group approach for the vertex function has been adopted and the characteristic functions β_1 and β_2 are calculated and the possibility of having stable points in the theory has been studied.

Keywords. Field theory; reggeon; boson; fermion; non-relativistic; renormalization group equation.

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1. Introduction

There has been a revival of interest in Reggeon field theory (RFT) with the introduction of gluon RFT for spontaneously broken Young Mill's Theory (White 1985, 1987). The present work is an addendum to our studies with the conventional (Gribov's) RFT (Sidhanta and Roy Chowdhury 1987; Roychoudhury and Sidhanta 1986). At present the interest in RFT increases to a large extent with the observation of Regge behaviour in QCD, since diffraction of pomeron is a major theoretical challenge to QCD, which cannot be tackled directly using either of the most familiar formation of the theory i.e. perturbation theory or lattice gauge theory. Recently (White 1987) remarkable success in this direction has been achieved which is based on the maximal exploitation of S-matrix of the multiparticle Regge theory. This approach may ultimately come forward as a powerful tool and be useful for studying the spectrum of other gauge theories besides QCD. It is also interesting enough to note that pomeron describes interaction in a mixed infrared and ultraviolet region of phase space. It is also subject to very precise unitary constraints—the cross-channel constraints of Regge unitarity (Gribov *et al* 1965, 1968) in particular. Within QCD we can expect therefore that the pomeron will make contact with the infrared properties of confinement and chiral symmetry breaking, with the ultraviolet parton model and asymptotic freedom properties and will confront the combination of these properties directly with unitarity. Another point to note is that the critical pomeron was discovered well over a decade ago (Migdal *et al* 1974) (Abarbanel and Bronzan 1974) as a strong coupling solution of the RFT. Still it remains the only known description of the asymptotically rising cross-section which completely satisfies all cross-channel (and direct channel) unitarity constraints. In deriving the conditions under which the critical pomeron occurs in QCD we may argue that the vector gluon is indeed the explanation of the rising cross-sections but only under very special circumstances.

Bearing in mind the background and for that the importance of pomeranchuk Regge behaviour, the present work here is devoted to boson fermion interaction as a pertinent problem. The approach is more or less pragmatic and unlike Bartel's and others a kind of supersymmetric version of a Regge model has been applied to the problem of boson fermion interaction where supersymmetric relation between the coupling constants are not imposed.

2. Boson-fermion interaction

Reggeon field theory with fermion and pomeron has been studied some time ago by Gribov *et al* (Abarbanel *et al* 1975). Savit and Bartels (Savit and Bartels 1974) considered fermion-pomeron interaction in a different way. Recently much interest has been focussed in the supersymmetrization of the non-relativistic quantum fields as well as quantum mechanics (Fayet and Ferrara 1971; Kulish 1988). There one represents the Hamiltonian of some quantum system as the anti-commutator of two (or more) odd operators. To achieve this, one uses a one-dimensional super space consisting of time (t) and Grossman variable (0). Alternatively one may think supersymmetrization of quantum model by using a supersymmetric extension of the 3-dimensional Euclidian symmetry of these theories. The starting point for such generalization may be the supersymmetrization of Schrödinger equation with known Euclidian invariants.

In the 2-space and one time world we define three operators viz, two translations along the axes and also one rotation in that plane as represented by x_1, x_2, x_3, with the algebra

$$[x_i, x_j] = g_{ijk} x_k \tag{1}$$

where g_{ijk} are the structure constants of O(2). The anticommuting operators are V_α where ($\alpha = \frac{1}{2}, \frac{1}{2}$), Now,

$$\{X_m, V\} = \tfrac{1}{2} \tau^m_{\beta\alpha} V_\beta \tag{2}$$

and

$$\{V_\alpha, V_\beta\} = \tfrac{1}{2} (c\tau^\mu)_{\alpha\beta} X_\mu \tag{3}$$

with

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad c = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

with the usual way, the representation of the supersymmetric algebra can be constructed in a superspace (x_k, θ_α). In this superspace the translation (P_k) and supertranslation (Q_α) generators can be realized as differential operators.

$$P_k = i \frac{\partial}{\partial x_k} = -i\partial_k \tag{4}$$

$$Q_\alpha = i \frac{\partial}{\partial \theta^\alpha} + \frac{iN}{2} (\sigma^k \theta_\alpha) P_k. \tag{5}$$

Another important differential operator is the spinor covariant derivative

$$\mathscr{D}_\alpha = i\frac{\partial}{\partial\theta^\alpha} - \frac{iN}{2}(\sigma^k\theta)_\alpha P_k. \tag{6}$$

It satisfies an algebra similar and that of Q

$$\{\mathscr{D}_\alpha \mathscr{D}_\beta\} = -N(\sigma^k\varepsilon)_{\alpha\beta}P_k \tag{7}$$

and anticommutes with Q_α

$$\{\mathscr{D}_\alpha Q_\beta\} = 0. \tag{8}$$

For describing quantum mechanical systems we have to add a time variable (t) to our superspace

$$(t_j, x_k, \theta_\alpha).$$

We note that (t) does not transform under supersymmetry and 0(2), therefore, we have a non-relativistic framework.

The superfields (wave functions) $\phi(t, x, 0)$ are given by their θ-expansion,

$$\phi(t,x,\theta) = \mathrm{A}(t_j x) + \theta^\alpha\phi_\alpha(t_j,x) + \theta^\alpha\theta_\alpha \mathrm{B}(t_j,x). \tag{9}$$

Here A, B are complex scalar fields and ϕ_α is a spinor one. Our next task is to find a nonlinear equation which generalizes the usual Schrödinger equation

$$i\frac{\partial\phi}{\partial t} = K\phi \tag{10}$$

where $K = \alpha\mathscr{D}^\alpha\mathscr{D}_\alpha$. And $\mathscr{D}_\alpha$ = super-covariant derivative.

To introduce nonlinearity we consider a term of the form $H = \lambda\int|\phi|^4$ in the Hamiltonian which will give rise to a cubic term in the Hamiltonian. After the expansion of the superfield and elimination of the extra variables, the whole theory reduces to

$$H = \int d^4x\,\{\alpha_0^B\phi_\mu^+\phi_\mu + 2\alpha_0^F\psi_\mu^+\psi_\mu + \alpha_1\phi^+\phi^+\phi\phi + i\alpha_2^1\phi^+\phi\psi^+\psi$$
$$-i\alpha_2^{1/2}(\phi\psi^+\beta_\mu\partial_\mu\psi + \phi^+\psi\beta_\mu\partial_\mu\psi) + \Delta_0(\phi\phi^+ + \psi^+\psi)\}. \tag{11}$$

The set up equation of motion is given by

$$i\partial_t\phi = -\alpha_0^B\partial_\mu^2\phi + 2\alpha_1\phi^+\phi\phi + i\alpha'_2\psi^+\psi\phi + i\alpha_2^{1/2}\beta_\mu\psi^+\partial_\mu\psi + \Delta_0\phi \tag{12}$$
$$i\partial_t\psi = -2\alpha_0^F\partial_\mu^2\psi + i\alpha'_2\phi^+\phi\psi + i\alpha_2^{1/2}(2\phi\beta_\mu\partial_\mu\psi^+ + \psi^+\beta_\mu\partial_\mu\phi) + \Delta_0\psi. \tag{13}$$

Here we extend the special dimension to D for adopting the technique of dimensional regularisation. Supersymmetric relation between the coupling constants has been relaxed. The coupling constants α_2^1 and $\alpha_2^{1/2}$ are considered to be independent. The free Regge boson and Regge fermion propagators are considered in the usual form and the mass like quantities Δ_0 are considered the same for both. Unrenormalized slopes of the trajectory of bosonic and fermionic types are α_0^B and α_0^F respectively.

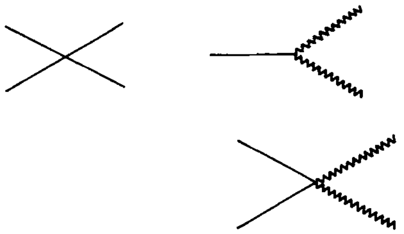

Figure 1. Different type of interactions.

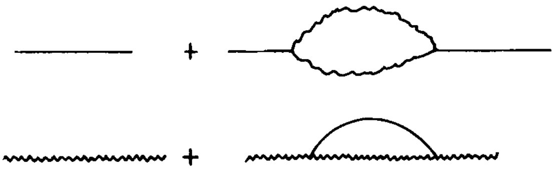

Figure 2. Radiative correction for Regge boson and Regge fermion.

Regge bosons are represented by solid lines and Regge fermions are represented by wavy lines. Different types of interactions are shown in figure 1 and radiative correction for Regge bosons and Regge fermions are shown in figure 2.

3. The renormalization group equations

The renormalization procedure consists of replacing the unrenormalized boson and fermion field operator by renormalized operators (Itzykson and Zuber 1985; Abarbanel *et al* 1975). The dimensionless coupling constants are defined to be

$$x=\alpha_2^1\frac{E^{(D/2)-1}}{(\alpha_0^{\rm B})^{D/2}}\ \text{and}\ y^{1/2}=\alpha_2^{1/2}\frac{E^{(1/2)((D/2)-1)}}{(\alpha_0^{\rm B})^{1/2(1+D/2)}}.$$

Again following the standard procedure one can deduce the renormalization group equation for the (m) bosons $\rightarrow$ (n) fermions $+$ (n) anti-fermions vertex function $\Gamma^{(m,n\bar{n})}$ as given by

$$\left[E_N\frac{\partial}{\partial E_N}+\beta_1\frac{\partial}{\partial\alpha_2'}+\beta_2\frac{\partial}{\partial\alpha_2^{1/2}}+\rho_1\frac{\partial}{\partial\alpha_{\rm F}}+\rho_2\frac{\partial}{\partial\alpha_{\rm B}}-\frac{m}{2}\gamma_{\rm B}-n\gamma_{\rm F}\right]\Gamma^{(m,n\bar{n})}=0 \tag{14}$$

where

$$\beta_1=E_N\frac{\partial}{\partial E_N}(i\alpha_2^1)\quad \beta_2=E_N\frac{\partial}{\partial E_N}(\log\alpha_2^{1/2})$$

$$\gamma_{\rm F}=E_N\frac{\partial}{\partial E_N}(\log z_1)\quad \gamma_{\rm B}=E_N\frac{\partial}{\partial E_N}(\log z_2)$$

$$\rho_{\rm F}=E_N\frac{\partial}{\partial E_N}\alpha_{\rm F}\quad \rho_{\rm B}=E_N\frac{\partial}{\partial E_N}\alpha_{\rm B}.$$

The radiatively corrected bosonic graph is evaluated from the diagram (figure 2). The inverse propagator yields

$$iT^{(B\to B)}(E,k^2)=E-\alpha_0^{B}k^2-\Delta_0+\tfrac{1}{2}(-i\alpha_2^{1/2})^2$$

$$\int\frac{i^2\,\mathrm{d}^{D}k_1\,\mathrm{d}E_1k_{1\mu}\beta_\mu(k-k_1)_\nu\beta_\nu}{(E-2\alpha_0^{F}k^2-\Delta_0+i\varepsilon)(E-E_1-2\alpha_0^{F}(k-k_1)^2-\Delta_0+i\varepsilon)}$$

$$=E-\alpha_0^{B}k^2-\Delta_0-\frac{2^{(D/2)+1}D\Gamma(-D/2)(2\Delta_0-E+\alpha_0^{F}k^2)\,T_1\alpha_2}{(4\alpha_0^{F})^{(D/2)+1}} \quad (15)$$

where, $T_i=\text{Trace }(\beta_\lambda\beta_\mu)$.

Radiatively corrected Regge-fermion propagator due to exchanges to the lowest nontrivial order is calculated from the Feynman graph (figure 2). The inverse propagator is given by

$$i\Gamma^{(F\to F)}(E,k^2)=E-2\alpha_0^{F}k^2-\Delta_0+\tfrac{1}{2}(-i\alpha_2^{1/2})^2$$

$$\int\frac{\mathrm{d}^{D}k_1\,\mathrm{d}E_1k_{1\mu}\beta_\mu(k-k_1)_\nu\beta_\nu}{(E-\alpha_0^{B}k^2-\Delta_0+i)(E-E_1-2\alpha_0^{F}(k-k_1)^2-\Delta_0+i\varepsilon)}$$

$$=E-2\alpha_0^{F}k^2-\Delta_0+\frac{\pi^{(D/2)+1}D\Gamma(-D/2)\alpha_2^{1}}{(\alpha_0^{B}+2\alpha_0^{F})^{D+1}}$$

$$\times[(2\alpha_0^{F}k^2-E+2\Delta_0)(\alpha_0^{B}+2\alpha_0^{F})-4\alpha_0^{F}k^2]^{D/2}. \quad (16)$$

Renormalization of the bosonic slope is calculated to be

$$\alpha_B=\frac{(4\alpha_0^{F})^{(D/2)+1}-\alpha_2\alpha_0^{F}\pi^{(D/2)+1}D\Gamma[1-(D/2)](2\Delta_0+E_n)^{(D/2)-1}T_1}{(4\alpha_0^{F})^{(D/2)+1}-\alpha_2\pi^{(D/2)+1}D\Gamma(1-D/2)(2\Delta_0+E_N)^{(D/2)-1}T_1}. \quad (17)$$

To simplify the analysis we consider the special cases $\alpha_0^{B}=\alpha_0^{F}$ and $\Delta_0=0$.

To examine the infrared behaviour of the vertex function we scale the energy by the factor ξ and putting $t=\log\xi$ the RGE equation takes the form,

$$0=\left[\frac{\partial}{\partial t}-\beta_1(x)\frac{\partial}{\partial x}-\beta_2(y)-\frac{\partial}{\partial y}+(\alpha_F-\rho_1)\frac{\partial}{\partial\alpha_F}-\rho_2\frac{\partial}{\partial\alpha_B}\right.$$

$$\left.+\left\{\frac{m}{2}\gamma_B+n\gamma_F-1\right\}\right]\Gamma^{(m,n\bar{n})}(\xi_i,k_ix,y,\alpha_B,\alpha_FE_N). \quad (18)$$

The solution to this equation in the usual standard form is given by

$$\Gamma_R^{(m,n\bar{n})}(\xi_iE_i,k_i,x,y,\alpha_F,\alpha_B,E_N)$$

$$=\Gamma_R^{(m,n\bar{n})}(E_i,k_i,y(-t),\,x(-t),\,F(-t),\,E_N)$$

$$\times\exp\int_{-t}^{0}\mathrm{d}t^1\left[1-\tfrac{1}{2}\left(\frac{m}{2}\gamma_B(y(t'))+n\gamma_F(x(t'))\right)\right]. \quad (19)$$

Therefore

$$\frac{\partial t}{1}=\frac{\partial\tilde{x}}{-\beta_1}=\frac{\partial\tilde{y}}{-\beta_2}=\frac{\partial\alpha_F}{\alpha_F-\rho_F}=\frac{\partial\tilde{\alpha}_B}{\alpha_B}=\frac{\partial\Gamma_R}{\left(\frac{m}{2}\gamma_B+n\gamma_F-1\right)\Gamma_R} \quad (20)$$

And we have,

$$\frac{\partial \tilde{x}(t)}{\partial t} = -\beta_1(\tilde{x}(t))$$

$$\frac{\partial \tilde{y}(t)}{\partial t} = -\beta_2(\tilde{y}(t))$$

$$\frac{\partial \tilde{\alpha}_B}{\partial t} = -\rho_B(\tilde{\alpha}_B(t), \tilde{y}(t)$$

$$\frac{\partial \tilde{\alpha}_F}{\partial t} = \tilde{\alpha}_F(t) - \rho_F(\tilde{\alpha}_F(t), \tilde{x}(t)). \tag{21}$$

One cannot find the behaviour of (19) unless the auxilliary functions given by (20) are known. Again in the infrared limit $\xi \to 0$ ($t \to -\infty$) may be governed by the zeros of the function β_1 and β_2. These zeros are searched by using perturbation theory. A priori, it is not obvious that such a perturbation expansion is justified but if the zeros of β_1 and β_2 occur for small values of the coupling constant, then the perturbation expansion in the neighbourhood of this point will be justified a posteriori.

4. Conclusion

To have some impression about using the modified supersymmetric model in realm of boson-fermion interaction we analyze the stability of the solutions.

From the expressions for β_1 and β_2 we find that four sets solution may occur for simultaneous zeros of

$$\text{(i) } y=0, \quad x=0, \quad \text{(ii) } y=0, \quad x=\sigma/\gamma$$

$$\text{(iii) } y=\lambda/\mu, \quad x=0; \quad \text{(iv) } y=\lambda/\mu; \quad x=\frac{1}{\gamma}\left(\sigma-\frac{\delta\lambda}{\mu}\right)$$

where

$$\sigma=2\lambda=\left(\frac{D}{2}-1\right), \quad \gamma=\frac{\pi^2}{(4)^{D/2}}$$

$$\delta=D\Gamma(2-D/2)\pi^{(D/2)+1}T_1\left(1+\frac{1}{4^{D/2}}\right)$$

$$\mu=\Gamma(2-D/2)\pi^{(D/2)+1}\left\{D\left(\frac{1}{3^{d/2}+1}+\frac{1}{2(4)^{D/2+1}}\right)T_1+\frac{2T_2}{\Gamma(D/2)}\right\}.$$

And the relevant matrix is evaluated to be

$$S=\begin{pmatrix} \frac{\partial\beta_1}{\partial y} & \frac{\partial\beta_2}{\partial y} \\ \frac{\partial\beta_1}{\partial x} & \frac{\partial\beta_2}{\partial x} \end{pmatrix}=\begin{pmatrix} \frac{1}{2}\lambda y^{-1/2}-3\frac{\mu}{2}y^{1/2} & 0 \\ -\delta x & \sigma-2\gamma x-\delta y \end{pmatrix}.$$

For the first two cases the matrix becomes undefined. And we are left with the last two cases to be physically interesting. For the case (iii) we have the matrix (S) given by

$$S=\begin{pmatrix}\sqrt{\mu\lambda} & 0\\ 0 & \lambda(1-\delta/\mu)\end{pmatrix}=\begin{pmatrix}\lambda_{11}^{3} & 0\\ 0 & \lambda_{22}^{3}\end{pmatrix}.$$

The diagonal elements (i.e. the eigenvalues) are given by

$$\lambda_{11}^{3}=\left[\frac{\pi^{(D/2)+1}}{2}\Gamma(2-D/2)\left(\frac{D}{2}-1\right)\left\{\left(\frac{1}{3^{D/2+1}}+\frac{1}{2(4)^{D/2+1}}\right)T_1+\frac{2T_2}{\Gamma(D/2)}\right\}\right]^{1/2}$$

$$\lambda_{22}^{3}=\tfrac{1}{2}\left(\frac{D}{2}-1\right)\left[1-\frac{DT_1\left(1+\frac{1}{4^{D/2}}\right)}{\left(\frac{1}{3^{D/2+1}}+\frac{1}{2(4)^{D/2+1}}\right)T_1+\frac{2T_2}{\Gamma(D/2)}}\right].$$

For the case (iv) the (S) matrix is given by

$$S=\begin{pmatrix}\sqrt{\mu\chi} & 0\\ -\frac{\delta}{\gamma}\left(\sigma-\frac{\delta\lambda}{\mu}\right) & 2\lambda\left(\frac{\delta}{\mu}-1\right)\end{pmatrix}.$$

It is important to note that the diagonal element in the last two cases bear a simple relation between each other. For the last case, $\lambda_{22}^{4}>0$ when $\delta>\mu$ which is approximately equivalent to $T_1>2T_2$ while T_1 and T_2 are dependent upon the particular representation of the matrices. If in a certain representation $T_1>2T_2$ we have a stable situation. On the other hand if $\lambda_{22}^{4}<0$ we have $\lambda_{22}^{3}>0$ and this will yield stable situation for the 3rd solution. So in any case we have at least one stable point unlike instability obtained by Savit and Bartels.

Appendix

The vertex function is calculated from the graph (figure 2).

$$\Gamma^{(1,2)}(E_1,k_1;E_2k_2;E_3,k_3)=(-i\alpha_2^{1/2})^3$$

$$\times\frac{(-i\alpha_2^{1/2})^3 i^3\int\beta_\lambda k_\lambda\beta_\mu(k_1-k)_\mu\beta_\nu(k-k_2)_\nu\,\mathrm{d}^Dk\,\mathrm{d}E}{(E-2\alpha_0^{\mathrm{F}}k^2-\Delta_0)(E_1-E-2\alpha_0^{\mathrm{F}}(k_1-k)^2-\Delta_0)\,(E-E_2-\alpha_0^{\mathrm{B}}(k-k_2)^2-\Delta_0}$$

$$+(-i\alpha_2^{1/2})^3 i^3\int\frac{\beta_\lambda k_\lambda\beta_\mu(k_1-k)_\mu\beta_\nu(k_2-k)_\nu\,\mathrm{d}^Dk\,\mathrm{d}E}{(E-2\alpha_0^{\mathrm{F}}k^2-\Delta_0)(E_1-E-2\alpha_0^{\mathrm{F}}(k_1-k)^2-\Delta_0)\,(E_2-E-\alpha_0^{\mathrm{B}}(k_2-k)^2-\Delta_0)}$$

$$\Gamma^{(1,2)}(E_1k_1;E_2k_2;E_3k_3)=(-i\alpha_2^{1/2})^3-(-i\alpha_2^{1/2})^3\frac{4\pi^{D/2+1}}{\Gamma(D/2)}T_2$$

$$U\Gamma(1-D/2)f(\alpha_0^{\mathrm{B}},\alpha_0^{\mathrm{F}})$$

(U) is a unit vector of dimension K^{D-2} and is expressed as $(E^{D/2-1}/(\alpha_0^{\mathrm{B}})^{D/2-1})$ and $T_2 = \mathrm{Trace}\ (\beta_\lambda \beta_\mu \beta_\nu)$. Again for the particular case $\alpha_0^{\mathrm{F}} = \alpha_0^{\mathrm{B}}$ we have

$$f(\alpha_0^{\mathrm{F}}, \alpha_0^{\mathrm{B}}) = \frac{1}{3\alpha_0^{\mathrm{B}2}}$$

Hence,

$$\beta_2 = E_N \frac{\partial}{\partial E_N} \frac{E_N^{(1/2)((D/2)-1)}}{(\alpha_0^{\mathrm{B}})^{\frac{1}{2}(D/2+1)}} \left\{ \alpha_2^{1/2} - \alpha_2^{3/2} \frac{E_N^{((D/2)-1)}}{(\alpha_0^{\mathrm{B}})^{D/2+1}} \times \frac{4\pi^{((D/2)+1)}}{3T(D/2)} \Gamma(1-D/2) \right\} Z_1 Z_2^{1/2}$$

$$= \frac{1}{2}\left(\frac{D}{2} - 1\right) y^{1/2} - \pi^{((D/2)+1)} \Gamma(2-D/2) \left\{ D T_1 \frac{1}{3^{D/2+1}} + \frac{1}{2(4)^{D/2+1}} + \frac{2T_2}{\Gamma(D/2)} \right\} y^{3/2}.$$

Again to calculate (β_1) we evaluate the vertex function from figure 2.

$$\Gamma^{(2\mathrm{B}\to 2\mathrm{B})} = i\alpha_2^1 - \tfrac{1}{2}(i\alpha_2')^2 \int \frac{\mathrm{d}E\,\mathrm{d}^D k}{(E_1 + E_2 - E - 2\alpha_0^{\mathrm{F}}(k_1 + k_2 - k) - \Delta_0)(E - 2\alpha_0^{\mathrm{F}} k^2 - \Delta_0)}$$

$$= i\alpha_2' + \frac{i\pi^{D/2+1}\alpha_2'^2}{(4\alpha_0^{\mathrm{F}})^{D/2}} (2\Delta_0 - E_1 - E_2)^{D/2-1} \Gamma(1 - D/2).$$

Putting $E_1 = E_2 = E_N/2$ and $\Delta_0 = 0$, $\alpha_0^{\mathrm{B}} = \alpha_0^{\mathrm{F}}$

$$\Gamma^{(2\mathrm{B}\to 2\mathrm{B})} = i\alpha_2' + \frac{i\pi^{D/2+1}\alpha_2' E_N^{D/2-1}}{(4\alpha_0^{\mathrm{F}})^{D/2}} \Gamma(1 - D/2).$$

Hence,

$$\beta_1 = E_N \frac{\partial}{\partial E_N} \left[\frac{E_N^{D/2-1}}{(\alpha_0^{\mathrm{B}})^{D/2}} \left\{ \alpha_2' + \frac{\pi^{D/2+1}\alpha_2'^2}{(4\alpha_0^{\mathrm{B}})^{D/2}} E_N^{D/2-1} \Gamma(1 - D/2) \right\} Z_1 \right]$$

$$\beta_1 = \left[\left(\frac{D}{2} - 1\right) x - \frac{xyD\Gamma(2 - D/2)}{3^{D/2+1}} \pi^{D/2+1}\, T_1 \left(1 + \frac{1}{4^{D/2}}\right) - \frac{\pi^2 x^2}{4^{D/2}} \right].$$

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