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Q matter in the field of an abelian dyon admitting parity violating anomalous couplings

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Abstract. The properties of a configuration of two scalar doublets admitting a global SO(2) symmetry is studied in the presence of an abelian dyon core when the scalar doublets are coupled to the electromagnetic field through parity violating and parity preserving terms. The mass and the conserved global charge of the Q configuration are computed in terms of the parameters of the lagrangian

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1. Introduction

The existence of global symmetries in nature prods us to think if they are really the result of a precise gauge theory at a high energy scale with the global symmetry of the theory remaining after the gauge symmetry has been broken (Peccei 1986). In fact, many theorists believe that changing the field by the same amount in causally disconnected regions is very unnatural and cannot have any dynamical properties associated with it (Safian, Personal Communication). In spite of this fact, global symmetries have been useful in studying certain problems in particle theory, notably, the $U(1)_{PQ}$ (Peccei Quinn) symmetry to resolve the strong CP problem (Sikvie 1982) and $SU(3)_L \times SU(3)_R$ chiral flavor symmetry to aid in the understanding of the mechanism of quark mass generation (Wilczek 1982). With conserved global charges like baryon and lepton number being unavoidable consequences of the structure of particle theory, it seems relevant that we look for situations where these conserved charges play a role when a large number of particles are present. In response to this sentiment, Coleman suggested that conserved global charges might stabilize a configuration of fields against collapse or decay (Coleman 1984). This is the essential idea of the Q ball, namely a configuration of fields admitting a global symmetry wherein the mass of the configuration is less than that of the corresponding pions of the theory specified by the global charge Q. The original discussion of Coleman drew analogy with a conserved angular momentum preventing a particle's orbit from destabilizing. In an interesting paper, Cohen *et al* discuss the stability of L balls which are configurations of fermi fields and scalar fields inspired by the Gelmini Roncadelli model of neutrino mass generation (Cohen *et al* 1986). The result of their analysis was that L balls were stable except for slow decay into fermions at the surface. Also, in curved space, we have discussed the stability of Q balls in general relativity with the result that the Q ball is stable providing its radius remains small (Wolf 1986). If gauge forces come into play, Q balls or "Q matter" has the additive feature of admitting internal repulsive forces in much the same way that a high Z nucleus becomes unstable to electrostatic repulsion. In this spirit, Lee *et al* have discussed gauged Q balls with a charged scalar field carrying the Q charge, the configuration is indeed stable provided the gauge charge does not grow large enough to destabilize the configuration (Lee *et al* 1989). In a previous note, we studied a shell of Q matter in the field of an abelian dyon with anomalous couplings between the scalar fields and electromagnetic field (Wolf 1989), the couplings we studied violated parity and in the present paper we study the more general case with both parity violating terms and parity conserving terms present. The interesting feature of this study is that the conserved Q charge is coupled to the magnetic charge of the central dyon core. Such a result suggests the indirect effect of magnetic charge on astrophysical situations. Of interest, also, is the fact that all non-abelian dyons appear as abelian for distances far removed from the dyon core, this justifies us describing the dyon as abelian when it interacts with the Q matter (Prassard and Sommerfield 1975). The interest in this investigation lies in the fact that it may simulate a configuration of Higgs fields when GUT dyons are present in the early universe. Such dyons or monopoles are inevitable consequences of GUT theories when the GUT group is broken to a sub-group with surviving U(1) factor, the simple problem we study here could serve as a spring-board from which we may investigate more complex Q configurations in the presence of a dyon core.

2. Q matter in the presence of a dyon core admitting anomalous scalar EM couplings

We begin our discussion by considering two scalar doublets transforming under the same SO(2) symmetry.

$$\phi = \begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}, \quad \psi = \begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

$$\phi \to \left[1 + \varepsilon\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\right]\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix}$$

$$\psi \to \left[1 + \varepsilon\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}\right]\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}. \tag{1}$$

For the lagrangian of scalar fields interacting with electromagnetism and gravity, we have

$$\mathscr{L} = \frac{C^4}{16\pi G} R\sqrt{-g} + \left[\frac{\partial^\mu \phi^T \partial_\mu \phi}{2} + \frac{\partial^\mu \psi^T \partial_\mu \psi}{2} - \frac{A_2}{4}\left(\phi^T\phi - \frac{A_1}{A_2}\right)^2 - \frac{\bar{A}_2}{4}\left(\psi^T\psi - \frac{\bar{A}_1}{\bar{A}_2}\right)^2 + \alpha\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\partial_\mu\phi^T\partial_\nu\psi}{\sqrt{-g}}\right) + \beta\left(\frac{\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}\partial_\mu\phi^T\partial_\nu\psi}{\sqrt{-g}}\right)^2\right]\sqrt{-g} - \frac{1}{16\pi}F_{\mu\nu}F^{\mu\nu}\sqrt{-g}. \tag{2}$$

Here, $F_{\mu\nu} = (\partial A_\mu/\partial x^\nu) - (\partial A_\nu/\partial x^\mu)$ where A_μ = vector potential and A_1, A_2, $\bar{A}_1$, $\bar{A}_2$ are the parameters in the scalar potentials. The constants α, β measure the anomalous scalar —EM couplings with the α term being parity violating. The anomalous terms are phenomenological but can be thought of as generated by a Feynmann diagram representing a photon coupled to a quark look with two external scalars. The same type of term arises in an axion-photon coupling only in our case we may think of replacing the axion with a photon and the two photons in the axion diagram with the scalars in the present theory (Sikivie 1984). In this paper, we understand these couplings to be purely phenomenological and study what effect they have on the configuration of scalar fields.

For the boundary conditions, we put an abelian dyon at $r=0$ with electric charge e and magnetic charge q. The mass of the dyon we represent by M_x. For the region $R_1 > r > 0$ we have the true vacuum of each scalar and have the static solution

$$\phi = \begin{pmatrix} \sqrt{A_1/A_2} \\ 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} \sqrt{\bar{A}_1/\bar{A}_2} \\ 0 \end{pmatrix}. \tag{3}$$

For the region of $R_2 \geqslant r \geqslant R_1$ we have the stationary Q matter solution

$$\phi = \begin{pmatrix} \phi(r)\cos wt \\ \phi(r)\sin wt \end{pmatrix}, \quad \psi = \begin{pmatrix} -\psi(r)\sin wt \\ \psi(r)\cos wt \end{pmatrix}. \tag{4}$$

We choose the true vacuum at $r = R_1$, ϕ, $(R_1)=\sqrt{A_1/A_2}$, $\psi(R_1)=\sqrt{\bar{A}_1/\bar{A}_2}$, at $r=R_2$ we have the false vacuum $\phi(R_2)=0$, $\psi(R_2)=0$. For $r > R_2$ we have the static false vacuum

$$\phi = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \psi = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{5}$$

The metric takes the spherically symmetric form

$$(\mathrm{d}s)^2 = \mathrm{e}^{\nu}(\mathrm{d}x^4)^2 - \mathrm{e}^{\lambda}(\mathrm{d}r)^2 - r^2\,\mathrm{d}\theta^2 - r^2\sin^2\theta(\mathrm{d}\phi)^2 \tag{6}$$

with $x^4 = ct$, $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$. The radial electric and magnetic fields are related to the field tensor as $F_{14} = E(r)$, $F_{23} = r^2 \sin\theta B_r$. The antisymmetry of $F_{\mu\nu}$ implies

$$\frac{\partial}{\partial x^{\nu}}(\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}) = 0 \tag{7}$$

$$\frac{\partial}{\partial r}(r^2 \sin\theta B_r) = 0,$$

$$B_r = \frac{q}{r^2} \tag{8}$$

for all $r > 0$. Here q is the magnetic charge of dyon. Varying (2) with respect to the electromagnetic field gives

$$\frac{\partial}{\partial x^{\nu}}\left(\frac{1}{4\pi}\sqrt{-g}\,F^{\mu\nu}\right) - \frac{\partial}{\partial x^{\nu}}(2\varepsilon^{ab\mu\nu}\partial_a\phi^T\partial_b\psi)$$
$$-\frac{\partial}{\partial x^{\nu}}\left(4\frac{(\varepsilon^{ab\alpha\beta}F_{\alpha\beta}\partial_a\phi^T\partial_b\psi)}{\sqrt{-g}}\varepsilon^{st\mu\nu}\partial_s\phi^T\partial_t\psi\right) = 0 \tag{9}$$

since ϕ, ψ are independent of θ, ϕ the anomalous terms do not contribute in (9) giving

$$\frac{\partial}{\partial x^{v}}\left(\frac{1}{4\pi}\sqrt{-g}\,F^{\mu v}\right)=0. \tag{10}$$

We next calculate the energy momentum tensor from the matter part of the lagrangian in (2),

$$\begin{aligned}
T_{\mu v} &= \frac{2}{\sqrt{-g}}\frac{\partial \mathscr{L}}{\partial g^{\mu v}} \\
&= \partial_{\mu}\phi^{T}\partial_{v}\phi + \partial_{\mu}\psi^{T}\partial_{v}\psi - \frac{g_{\mu v}}{2}(\partial^{\alpha}\phi^{T}\partial_{\alpha}\phi + \partial^{\alpha}\psi^{T}\partial_{\alpha}\psi) \\
&\quad + g_{\mu v}\frac{A_2}{4}\left(\phi^{T}\phi - \frac{A_1}{A_2}\right)^2 + \frac{g_{\mu v}}{4}\bar{A}_2\left(\psi^{T}\psi - \frac{\bar{A}_1}{\bar{A}_2}\right)^2 \\
&\quad + \frac{1}{16\pi}g_{\mu v}F_{\alpha\beta}F^{\alpha\beta} - \frac{1}{4\pi}F_{\mu\alpha}F_{v}^{\ \alpha} + \beta\frac{(\varepsilon^{ab\alpha\beta}F_{\alpha\beta}\partial_{a}\phi^{T}\partial_{b}\psi)^2}{(-g)}g_{\mu v}.
\end{aligned}$$

For $R_1 > r$ and $r > R_2$ we have from (11), $T_1^1 = T_4^4$, this implies $\lambda + v = 0$ from the Einstein equation for the metric. From (10) we have

$$\frac{\partial}{\partial r}(r^2 \exp[(-v+\lambda)/2]E(r)\sin\theta) = 0, \quad \text{or } E(r) = e/r^2 \tag{12}$$

for $R_1 > r, r > R_2$. Here e = electric charge of dyon. To calculate the metric for $R_1 > r$, we have, from $\binom{1}{1}$ and the $\binom{4}{4}$ component of the Einstein equation

$$\frac{\mathrm{d}}{\mathrm{d}r}(r\exp(-\lambda)) = 1 - \frac{8\pi G}{C^4}T_4^4 r^2$$

$$T_4^4 = \frac{E^2}{8\pi} + \frac{B^2}{8\pi}. \tag{13}$$

Integrating gives

$$\exp(-\lambda) = 1 - \frac{2GM_x}{rC^2} + \frac{Gg^2}{r^2C^4} + \frac{Ge^2}{r^2C^4}. \tag{14}$$

Here we identify M_x as the mass of the dyon. For $r > R_2$ we have

$$T_4^4 = \frac{E^2}{8\pi} + \frac{B^2}{8\pi} + \frac{A_1^2}{4A_2} + \frac{\bar{A}_1^2}{4\bar{A}_2}$$

this gives upon integrating (13)

$$\begin{aligned}
\exp(-\lambda) = 1 &- \frac{2G(M_x + M_Q)}{rC^2} + \frac{Ge^2}{r^2C^4} + \frac{Gq^2}{r^2C^4} \\
&- \frac{8\pi G}{3C^4}r^2\left(\frac{A_1^2}{4A_2} + \frac{\bar{A}_1^2}{4\bar{A}_2}\right).
\end{aligned} \tag{15}$$

Where M_Q = mass of Q matter, and the false vacuum is present for $r > R_2$. For the region $R_2 \geqslant r \geqslant R_1$ we have $\lambda + \nu \neq 0$ since $T_1^1 \neq T_4^4$ from (11), however we will approximate $E_r = e/r^2$ in this region and also have $B = q/r^2$ from (8) which is an exact result following from the anti-symmetric condition in (7). We next proceed to solve for the field $\phi(r)$, $\psi(r)$ for $R_1 \leqslant r \leqslant R_2$, from the lagrangian (2) and the stationary solution in (4) we have upon substituting in (2) for the matter electromagnetic part of the lagrangian

$$\mathscr{L} = \left[\begin{array}{c}\dfrac{\exp(-\nu)(\phi(r))^2 w^2}{2c^2} + \dfrac{\exp(-\nu)(\psi(r))^2 w^2}{2c^2} - \dfrac{\exp(-\lambda)(\phi_{,r})^2}{2} \\ \dfrac{-\exp(-\lambda)(\psi_{,r})^2}{2} - \dfrac{A_2}{4}\left(\phi^2 - \dfrac{A_1}{A_2}\right)^2 - \dfrac{\bar{A}_2}{4}\left(\psi^2 - \dfrac{\bar{A}_1}{\bar{A}_2}\right)^2\end{array}\right] r^2 \exp\left(\frac{\lambda+\nu}{2}\right)\sin\theta$$
$$+ \frac{\alpha}{c}(2r^2 B(r))\left[\frac{w\phi\psi_{,r}}{c} + \frac{w\phi_{,r}\psi}{c}\right]\sin\theta$$
$$+ \beta 4(B(r))^2\left[\frac{w\phi\psi_{,r}}{c} + \frac{w\phi_{,r}\psi}{c}\right]^2 r^2 \exp\left(\frac{\lambda+\nu}{2}\right)\sin\theta \tag{16}$$

where we have approximated $(\lambda + \nu) = 0$ in the denominator of the second anomalous term, varying (16) with respect to ϕ, ψ gives upon setting $e^\nu \simeq e^\lambda \simeq 1$.

$$r^2\phi_{,rr} + 2r\phi_{,r} + \frac{r^2 w^2}{c^2}\phi - r^2 A_2 \phi\left(\phi^2 - \frac{A_1}{A_2}\right)$$
$$+ 8\beta\left(\frac{q^2}{r^2}\right)\left(\frac{w^2}{c^2}\right)(\phi(\psi_{,r})^2 + \psi\psi_{,r}\phi_{,r})$$
$$+ 16\beta\left(\frac{q^2}{r^3}\right)\left(\frac{w^2}{c^2}\right)(\phi\psi\psi_{,r} + \phi_{,r}(\psi)^2)$$
$$- \frac{8\beta q^2}{r^2}\left(\frac{w^2}{c^2}\right)(3\psi\psi_{,r}\phi_{,r} + \phi(\psi_{,r})^2 + \phi\psi\psi_{,rr} + \phi_{,rr}(\psi)^2) = 0 \tag{17}$$

$$r^2\psi_{,rr} + 2r\psi_{,r} + \frac{r^2 w^2 \psi}{c^2} - r^2 \bar{A}_2 \psi(\psi^2 - (\bar{A}_1/\bar{A}_2)$$
$$+ 8\beta\left(\frac{q^2}{r^2}\right)\left(\frac{w^2}{c^2}\right)(\psi(\phi_{,r})^2 + \phi\phi_{,r}\psi_{,r})$$
$$+ 16\beta\left(\frac{q^2}{r^3}\right)\left(\frac{w^2}{c^2}\right)(\psi\phi\phi_{,r} + \psi_{,r}\phi^2)$$
$$- \frac{8\beta q^2}{r^2}\left(\frac{w^2}{c^2}\right)\left(\begin{array}{c}3\phi\phi_{,r}\psi_{,r} + \psi(\phi_{,r})^2 \\ + \psi\phi\phi_{,rr} + \psi_{,rr}\phi^2\end{array}\right) = 0. \tag{18}$$

In (17) and (18) we have approximated $e^\nu \simeq e^\lambda \simeq 1$. To solve these equations we obtain an approximate solution as follows, we set

$$\begin{aligned}\phi &= a_0 + a_1 r + a_2 r^2 + a_3 r^3 \\ \psi &= b_0 + b_1 r + b_2 r^2 + b_3 r^3\end{aligned} \tag{19}$$

we next insist that (19) obeys (17), (18) up to the third order in r, we also insist that (19) obeys the boundary condition

$$\phi(R_1)=\sqrt{\frac{A_1}{A_2}},\quad \phi(R_2)=0$$
$$\psi(R_1)=\sqrt{\frac{\bar{A}_1}{\bar{A}_2}},\quad \psi(R_2)=0. \tag{20}$$

To solve for the coefficients we must impose a relation between the coefficient a_0, b_0 given by $a_0=\bar{\alpha}b_0$ because of the finite number of terms we use. The results up to the cubic term are for ϕ, ψ

$$\phi=a_0+a_1r+a_2r^2+a_3r^3$$
$$a_0=-\frac{R_2^3}{R_1^3}\sqrt{\frac{A_1}{A_2}}+\frac{R_2^3}{R_1^3}\left(\frac{\bar{\alpha}}{2}\right)\left(\frac{1}{\bar{A}}\sqrt{\frac{A_1}{A_2}}-\sqrt{\frac{\bar{A}_1}{\bar{\alpha}_2}}\right)$$
$$a_1=0$$
$$a_2=\frac{-\frac{R_2^3}{R_1^3}\left[\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}-\frac{1}{\bar{\alpha}}\sqrt{\frac{A_1}{A_2}}\right]}{\frac{2}{\alpha}\left(\frac{R_2^3}{R_1}-R_2^2\right)} \tag{21}$$
$$a_3=\frac{1}{R_1^3}\left[\sqrt{\frac{A_1}{A_2}}-a_0-a_2R_1^2\right]$$
$$\psi=b_0+b_1r+b_2r^2+b_3r^3$$
$$b_0=\frac{1}{\bar{\alpha}}a_0$$
$$b_1=0$$
$$b_2=-\frac{1}{\bar{\alpha}}a_2$$
$$b_3=\frac{1}{R_1^3}\left[\sqrt{\frac{\bar{A}_1}{\bar{A}_2}}-\frac{1}{\bar{\alpha}}a_0+\frac{1}{\bar{\alpha}}a_2R_1^2\right]. \tag{22}$$

Our next task is to evaluate the conserved Q charge, we have from the invariance under SO(2) of the lagrangian in (2)

$$\frac{\partial\mathscr{L}}{\partial\phi_1}\delta\phi_1+\frac{\partial\mathscr{L}}{\partial\phi_2}\delta\phi_2+\frac{\partial\mathscr{L}}{\partial\phi_{1,\mu}}\delta\phi_{1,\mu}+\frac{\partial\mathscr{L}}{\partial\phi_{2,\mu}}\delta\phi_{2,\mu}$$
$$+\frac{\partial\mathscr{L}}{\partial\psi_1}\delta\psi_1+\frac{\partial\mathscr{L}}{\partial\psi_2}\delta\psi_2+\frac{\partial\mathscr{L}}{\partial\psi_{1,\mu}}\delta\psi_{1,\mu}+\frac{\partial\mathscr{L}}{\partial\psi_{2,\mu}}\delta\psi_{2,\mu}=0 \tag{23}$$

Inserting the field equations for ϕ_1, ϕ_2, ψ_1, ψ_2 into (23) and integrating over r, θ, ϕ gives the conserved Q charged

$$Q = \frac{4\pi w}{c}\int_{R_1}^{R_2} |\phi(r)|^2 r^2\,\mathrm{d}r + \frac{4\pi w}{c}\int_{R_1}^{R_2} |\psi(r)|^2 r^2\,\mathrm{d}r + 8\pi q\alpha\int_{R_1}^{R_2} (\psi_{,r}\phi + \phi_{,r}\psi)\,\mathrm{d}r + \frac{32\pi q^2\beta}{C}\int_{R_1}^{R_2} \frac{w}{r^2}(\psi_{,r}\phi + \phi_{,r}\psi)^2\,\mathrm{d}r. \tag{24}$$

Here, we have approximated $e^{\nu} \simeq e^{\lambda} \simeq 1$ and used the fact that the spatial component of the conserved current generated from (23) gives zero contribution to the spatial integral from the invariance condition in (1). The global charge in (24) we see is coupled both to the magnetic charge and the square of the magnetic charge, the linear dependence on the magnetic charge is parity violating while the quadratic coupling is parity respecting. Equation (24) can be used to solve for ω in terms of the global charge Q and the magnetic charge q. Our next task is to evaluate the mass of the Q matter, we first write down the $(^4_4)$ component of the energy momentum tensor from (11) for $R_2 \geqslant r \geqslant R_1$

$$T^4_4 = \frac{(\phi(r))^2 w^2}{2c^2} + \frac{(\psi(r))^2 w^2}{2c^2} + \frac{(\phi_{,r})^2}{2} + \frac{(\psi_{,r})^2}{2} + \frac{A_2}{4}\left((\phi(r))^2 - \frac{A_1}{A_2}\right)^2 + \frac{\bar{A}_2}{4}\left((\psi(r))^2 - \frac{\bar{A}_1}{\bar{A}_2}\right)^2 + \frac{E^2}{8\pi} + \frac{B^2}{8\pi} + 4\beta(B^2)\left(\frac{\phi\psi_{,r}}{c} + \frac{\psi_{,r}\phi}{c}\right)^2 w^2. \tag{25}$$

Here, once again, we have approximated $e^{\nu} \simeq e^{\lambda} \simeq 1$ for $R_2 \leqslant r \leqslant R_1$. We can now insert into (25) E, B from

$$B = \frac{q}{r^2}, \quad E = \frac{e}{r^2}$$

where we are using the approximate solution for E for $R_2 \leqslant r \leqslant R_1$, we also use the expression for $\phi(r)$, $\psi(r)$ from (21) and (22) with w being expressible in terms of the Q charge from (24). This series of substitutions will specify T^4_4 in terms of e, q, α, β and the parameters in the two scalar potentials for $R_2 \geqslant r \geqslant R_1$. Inserting (25) into the $(^4_4)$ component of the Einstein equations gives

$$\frac{\mathrm{d}}{\mathrm{d}r}(re^{-\lambda}) = 1 - \frac{8\pi G}{c^4} T^4_4 r^2, \tag{26}$$

integrating between R_2 and R_1 gives

$$(re^{-\lambda})_{R_2} - (re^{-\lambda})_{R_1} = R_2 - R_1 - \frac{8\pi G}{c^4}\int_{R_1}^{R_2} r^2 T^4_4\,\mathrm{d}r. \tag{27}$$

Substituting (14) for the metric $e^{-\lambda}$ at $r = R_1$ and (15) for the metric $e^{-\lambda}$ at $r = R_2$ allows us to solve (27) for the mass of the Q matter since T^4_4 is known from (25).

3. Conclusion

The above calculation provides an interesting model upon which to build Q matter calculations whenever a central dyon or monopole core interacts with the scalar fields through the above anomalous coupling. Such a coupling might arise in super-symmetric extensions of the standard model where two Higgs doublets are required, to form such a Q matter configuration about a dyon core we need only have a gradient in the potential between the outer and inner boundary as well as the condition that the mass of the configuration is less than that of the free pions of the theory. Even if this last condition is not met, the above solution represents an interesting classical configuration of scalar fields. If the above configuration becomes unstable, either gravitationally, or through surface effects, it could provide a source of high energy gamma rays or massive particles with the energy liberated being a function of the global charge Q and the magnetic charge of the central dyon. It would be interesting to speculate that a series of these objects might appear in a cosmological setting with the energy liberated having the characteristic dependence on Q and q mentioned above.

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