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Differential temperature sensors method for simultaneous determination of thermal conductivity and diffusivity

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Abstract. An attempt has been made to minimize the sources of error involved in the transient probe method for thermal conductivity determination. Two sensors (thermocouples) are mounted parallel to the needle probe at known distances. This modification makes it a device for simultaneous conductivity and diffusivity determination. Thermal conductivity and diffusivity for glycerine, dune sand and mustard seed are determined by this method. Results obtained are compared with those obtained by a calibrated transient probe for conductivity and by a parallel wire method for diffusivity. Analysis of the results prove it to be a better instrument over the traditional ones. The technique can also be used as a direct reading device for conductivity and diffusivity measurements.

Keywords. Thermal conductivity; thermal diffusivity; differential temperature sensors.

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1. Introduction

Transient methods are under constant use for determination of thermal conductivity and diffusivity of liquids, granular substances and insulators in powder form. Various authors (Chaudhary 1968; Lozano *et al* 1979; Singh *et al* 1988a, b; Inaba 1983; Singh *et al* 1985a; Penner *et al* 1975) have used probes of different designs and dimensions according to their need. Wechsler (1966) has described two main sources of error associated with transient probes and the following correction has been suggested.

1.1 *Initial lag error* $(2K/bH)$

It arises due to thermal mass of the probe and contact resistance between the probe and the sample. For small experimental time t, corrected expression giving the temperature rise of the point under consideration become

$$\theta = \frac{q}{4\pi K}\left[\ln\frac{4\alpha t}{r^2} - \gamma + \frac{2K}{bH}\right],$$

where K is the thermal conductivity of the medium, α is its thermal diffusivity, b is the probe radius and H is the contact conductance at the probe medium interface, γ being Euler's constant $= 0{\cdot}5772$.

1.2 *Time correction term* (t_c)

This is due to the finite time required for heating of the probe mass. This error becomes significant for heavier probes of large diameter. For small experimenal time, a correction term should be considered for precise measurements.

If at any point in the medium the rise in temperature is θ_1 at time t_1 and θ_2 at time t_2, then

$$(\theta_2 - \theta_1) = \frac{q}{4\pi K}\left[\ln\frac{t_2 - t_c}{t_1 - t_c}\right].$$

No such correction term has so far been applied for diffusivity measurements. Authors (Sharma *et al* 1984; Kerrisk 1971) have determined diffusivity of substances by independent experiments, but the need for simultaneous determination of these two coefficients has always been felt for exactness of the sample. Many (Agarwal and Bhandari 1970; Singh *et al* 1985b) have described methods for simultaneous determination of these coefficients. But no method could gain acceptance because of either complexity of the apparatus involved or larger time duration needed for experiment.

The method presented here determine both the constants simultaneously by a single experiment of short duration (5–6 min). Two temperature sensors (thermocouples) are mounted parallel, diametrically opposite to the needle. This arrangement improves its performance by eliminating the sources of error discussed above.

To test the performance of the probe, proper substances from various material groups were taken and their thermal coefficients were determined. Glycerine was selected to represent liquids. Dune sand (particle size, mesh No. 80–100) and mustard pusa bold (grain size, mesh No. 5–10) were taken from granular substances. Their thermal conductivity and diffusivity were also determined by transient probe and parallel wire method. All the measurements were done under identical conditions.

2. Theory

When a line source of heat is placed in an inifnite medium, the rise in temperature θ at a near point is given by the expression (Chaudhary 1968, Carslaw and Jaeger 1959)

$$\theta = \frac{q}{2\pi K}\left[-\frac{\gamma}{2} - \ln\frac{r}{\sqrt{4\alpha t}} + \frac{1}{2}\frac{r^2}{4\alpha t} - \frac{1}{8}\frac{r^4}{(4\alpha t)^2}\right]. \tag{1}$$

where r is the distance of the point from the line source, q, the power generated per unit length of probe, K being thermal conductivity of sample, α being its diffusivity and γ the Euler's constant ($= 0{\cdot}5772$). The terms containing third and higher powers of $(1/t)$ have been neglected.

2.1 *Lag error correction*

Considering this correction term the rise in temperature will be

$$\theta = \frac{q}{2\pi K}\left[-\frac{\gamma}{2} - \ln\frac{r}{\sqrt{4\alpha t}} + \frac{1}{2}\frac{r^2}{4\alpha t} - \frac{1}{8}\frac{r^4}{(4\alpha t)^2} + \frac{K}{bH}\right]. \tag{2}$$

Determining the respective rise in temperatures θ_1 and θ_2 at same time t, at two near points r_1 and r_2 from the above equation and subtracting the two we get,

$$(\theta_1-\theta_2)=\frac{q}{2\pi K}\left[\ln\left(\frac{r_2}{r_1}\right)-\frac{r_2^2-r_1^2}{8\alpha}\cdot\frac{1}{t}+\frac{r_2^4-r_1^4}{128\alpha^2}\cdot\frac{1}{t^2}\right]. \tag{3}$$

So the lag error term is eliminated from the final expression.

2.2 *Time correction term* (t_c)

This term is to be subracted from the observed time t, Hence the (3) becomes

$$(\theta_1-\theta_2)=\frac{q}{2\pi K}\left[\ln\left(\frac{r_2}{r_1}\right)-\frac{r_2^2-r_1^2}{8\alpha}\cdot\frac{1}{(t-t_c)}+\frac{r_2^4-r_1^4}{128\alpha^2}\cdot\frac{1}{(t-t_c)^2}\right].$$

As $t \gg t_c$, hence, simplifying and neglecting higher power terms of (t_c/t) in the above equation we get

$$(\theta_1-\theta_2)=\frac{q}{2\pi K}\left[\ln\left(\frac{r_2}{r_1}\right)-\frac{r_2^2-r_1^2}{8\alpha}\frac{1}{t}+\frac{r_2^2-r_1^2}{8\alpha}\left(\frac{r_2^2+r_1^2}{16\alpha}-t_c\right)\frac{1}{t^2}\right]. \tag{4}$$

Here we have retained terms up to second power of $(1/t)$. Equation (4) shows that correction term appears in the coefficient of $(1/t^2)$. The values of the coefficient can be determined by data fitting technique by a computer.

2.3 *Computation of thermal conductivity and diffusivity*

Equation (4) between $(\theta_1-\theta_2)$ and reciprocal of time $(1/t)$, represents parabola of the form

$$y=B_0+B_1x+B_2x^2. \tag{5}$$

The coefficients B_0, B_1, and B_2 lead to the following expressions for obtaining thermal conductivity and thermal diffusivity

$$K=\frac{q}{2\pi B_0}\ln\left(\frac{r_2}{r_1}\right) \tag{6}$$

$$\alpha=\frac{1}{8}\frac{B_0}{B_1}\frac{(r_2^2-r_1^2)}{\ln(r_2/r_1)} \tag{7}$$

$$\alpha=\frac{1}{16}\frac{B_1}{B_2}(r_2^2+r_1^2) \tag{8}$$

$$\alpha=\frac{1}{8}\left[\frac{B_0}{B_2}\frac{(r_2^4-r_1^4)}{2\ln(r_2/r_1)}\right]^{1/2}. \tag{9}$$

The correction term t_c in (4) has been omitted for sake of simplicity while evaluating (8) and (9). Its order can be estimated by the knowledge of constant B_1 and B_2. It comes out to be,

$$t_c = \frac{(r_2^2 + r_1^2)}{16\alpha} + \left(\frac{B_2}{B_1}\right).$$

For the probe used for glycerine, correction term comes out to be 5·8 s, whereas the term $(r_2^2+r_1^2)/(16\alpha)$ works out to be 25·3 s.

3. Fabrication of differential sensor probe

A hypodermic needle of 13·5 cm long and 2 mm diameter was selected for the purpose. A silk insulated constantan wire (SWG 42) of about 6 times the needle length was taken as the heater of probe. It is made into six folds and wrapped on 13·3 cm long insulated copper wire (SWG 32). The ends of the heater wire are connected to heavy copper leads. The junctions are soldered and insulated by a sticking tape. The whole assembly was then pushed inside the needle carefully. Air pockets left if any, were filled by zircona powder and ends of the needle were sealed.

The needle was mounted in the centre of a triangular bakelite plate of side 5 cm each. The ends of the plate were rounded off to remove sharp corners. Two such identical plates were also taken for mounting the sensor assembly. These were supported on 14.5 cm long iron bars, such that distance between them was 13.5 cm. Holes were drilled at the centre for insertion of the needle. The thermocouples are made by soldering constantan wire and steel wire (each SWG 28). Steel wire meant for string musical instruments were selected for the purpose. Such thermocouples could be easily stretched without producing a sag up to 100°C. The thermo emf generated for these thermocouples is 39.45 μV/°C, not very much different to that produced by copper constantan thermocouples (40.8 μV/°C). Two similar sensors were mounted at distances 0.2 cm and 0.6 cm from the central hole. Their junctions were placed midway between the two plates. The sensors were kept tight by inserting springs in the supporting rods. The needle is then inserted within the assembly and screwed properly. Distance of the thermocouples from the needle is determined accurately by a travelling microscope. In figure 1 the design of the probe is given.

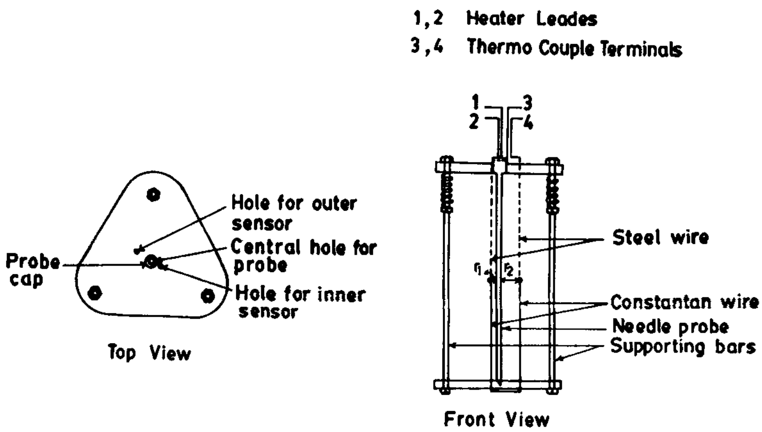

Figure 1. Design of differential temperature sensors probe.

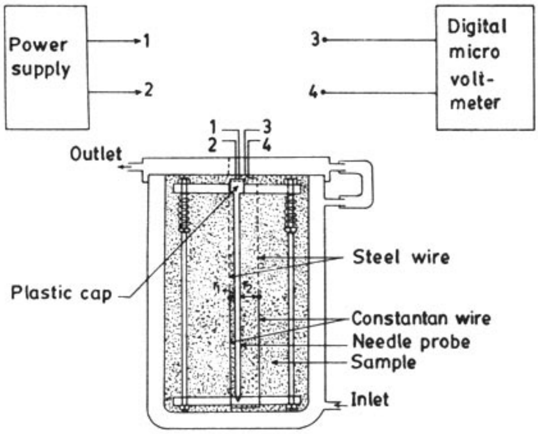

Figure 2. Experimental arrangement for conductivity and diffusivity determination.

4. Experimental set-up

The experimental arrangement consisted of mainly three parts.

4.1 *Constant current power supply*

It provides a constant current to the probe heater wire.

4.2 *Sample container*

It consists of a cylindrical copper vessel 15.5 cm long and diameter of 7 cm. It is placed in a chamber maintained at constant temperature. The temperature of the container can be kept constant at a desired value within $\pm 1°C$ by circulating liquid from a constant temperature bath.

4.3 *Temperature measuring unit*

The thermo emf generated at the two ends of thermocouple assembly can be registered by a digital microvoltmeter with an accuracy of 0.1 μV. In figure 2 the experimental arrangement is shown schematically.

5. Procedure

After switching on the current in the heater wire, the temperature difference in the sample at the midpoints of the sensors is noted with time. A plot between reciprocal of time $(1/t)$ and temperature difference $(\theta_1 - \theta_2)$ will be a parabola. Its coefficients are determined by data fitting technique by a personal computer.

Conductivity and diffusivity can be determined by formulae discussed above. The computer programme is designed to display the values of various coefficients, i.e., thermal conductivity and diffusivity directly.

The experimental arrangement for conductivity determination by transient probe is similar to that used by Singh *et al* (1988b). For diffusivity measurements parallel wire method as described by Sharma *et al* (1984) was used.

6. Results and discussion

Two sets of observations for the samples investigated were taken in the temperature range −20°C to 90°C. Equation (6) was used for conductivity determination. For diffusivity measurements any expression represented by (7), (8) and (9) can be used, but (7) was preferred for calculations. Equations (8) and (9) were not used because of the following reasons.

6.1. Both expressions contain B_2, which is the coefficient of second order term in $(1/t)$. Thus any error in time measurement will have two fold effect on it. It also includes the time correction term, so diffusivity given by these expressions will not be accurate.

6.2. Equation (8) contains the sum of squares of distances r_1 and r_2. Any error in their measurement will have additive effect on the final result. Similar reasoning rejects (9) which contain fourth power of r_1 and r_2.

Equation (7) is free from such discrepancies. Moreover here occurrence of squares of r_1 and r_2 is in subtractive mode, so the inaccuracy due to errors involved in measurement of r_1 and r_2 is minimum.

The results of conductivity and diffusivity determination by two methods are shown by graphs in figures 3 and 4. To ascertain the closeness of the results with the mean

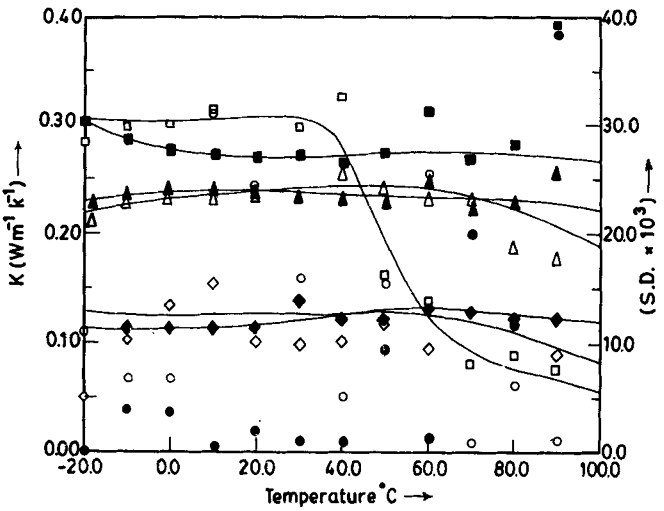

Figure 3. Variation in thermal conductivity of substances with temperature.

	Differential temp. sensors probe	Transient probe
K Glycerine	■	□
K Dune sand	▲	△
K Mustard pusa bold	◆	◇
Standard deviation for glycerine	●	○

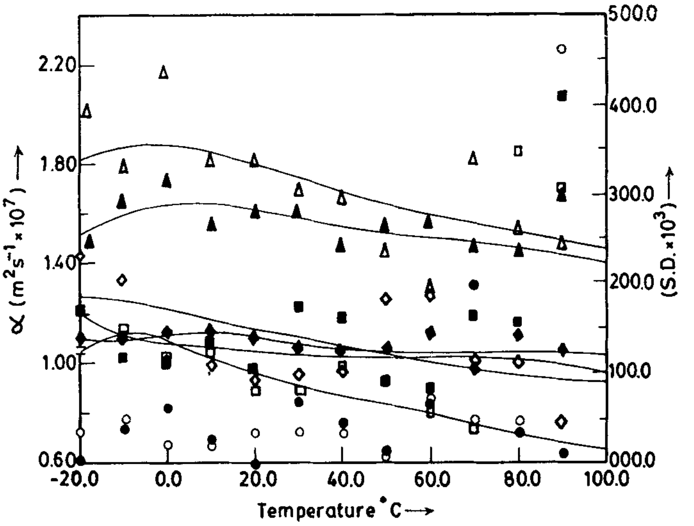

Figure 4. Variation in thermal diffusivity of substances with temperature.

	Differential temp. sensors probe	Parallel wire method
Glycerine	■	□
Dune sand	▲	△
Mustard pusa bold	◆	◇
Standard deviation for glycerine	●	○

values, the standard deviation was also calculated for all the samples. But here on graphs, standard deviation for glycerine alone is depicted, to avoid too much accumulation of points on graphs.

The two graphs show that the values of the respective thermophysical constants determined by differential temperature sensors probe are relatively closer to the expected behaviour of the samples. Moreover the comparison of standard deviation also indicate that the values given by it are nearer to the mean value. The same is found true for other substances also. The following considerations also favour its superiority over the traditional probes.

6.3. Heat transfer by convection and radiation in the proposed technique is minimum. In this method a low heat flux ($q=I^2R/L$) is required for obtaining a measurable rise in temperature between the sensors, where R is the resistance of the probe heater and L being its length. So a low current (I = 45 mA) is needed during an experimental run. This causes a rise in temperature of nearly 3°C in the vicinity of the probe. Whereas in parallel wire method a current (I = 120 mA) was required to obtain a measurable temperature rise at the sensor. This gives rise to a temperature increase as high as 15°C in the probe surroundings. Luikov (1966) has shown that when temperature difference is small, the transfer of heat by convection and radiation in the pores of the substances is negligibly small in comparison with the heat transfer by conduction. It is evident that, convective and radiative heat transfer in parallel wire method is on higher side which is not desirable. Such errors are minimum in the proposed method.

These errors are reflected in the graphs above 50°C where the plotted points deviate from the normal trend, for the transient probe and parallel wire methods.

6.4. In the present method the temperature difference measured by two sensors never exceed beyond 2°C, so the same step microvoltmeter range is needed over the entire temperature range.

6.5. In the literature, much data are not available for diffusivity of substances investigated. The present method can be relied upon for diffusivity measurements. If for a set of observations, sum of error square is least in data fitting, thermal conductivity is precise, then it is likely that value of diffusivity is also precise.

6.6. The present probe with its accessory instruments can be calibrated for direct reading of conductivity and diffusivity.

7. Calibration for direct measurement

7.1 *Conductivity determination*

From (3) one finds that for large time, the temperature difference i.e. microvoltmeter reading approaches a constant value (say ϕ) given by

$$\phi = \frac{q}{2\pi K} \ln\left(\frac{r_2}{r_1}\right),$$

so for a constant value of current in the heater wire the final reading of microvoltmeter

$$\phi \propto \frac{1}{K},$$

so the microvoltmeter can be calibrated for direct measurements.

Experimentally it takes about 7 to 8 min for attainment of almost a constant microvoltmeter reading.

7.2 *Diffusivity measurement*

Diffusivity is the property of a substance undergoing transient changes. For reading it directly from the instrument, a time, say 90 s is specified. At such time values, the effect of second order term is almost negligible. Hence the (4) reduces to

$$(\theta_1 - \theta_2) = \frac{q}{2\pi K} \ln\left(\frac{r_2}{r_1}\right) - \frac{q}{2\pi K} \frac{(r_2^2 - r_1^2)}{720\alpha}$$

or

$$\alpha = \frac{p \cdot q}{2\pi K} \cdot \frac{(r_2^2 - r_1^2)}{720} \cdot \left[\frac{1}{\frac{p \cdot q}{2\pi K} \ln\left(\frac{r_2}{r_1}\right) - \phi}\right],$$

where $\phi = (\theta_1 - \theta_2)$ is being measured in μV. p is the calibration constant of the thermocouple. In our case $p = 39{\cdot}45\,\mu\mathrm{VK}^{-1}$. Now an initial bias emf of $= (p \cdot q/2\pi K)\ln(r_2/r_1)\,\mu\mathrm{V}$ is applied to the microvoltmeter, to give a constant reading. If the ends of the thermocouples are connected to the microvoltmeter, such that thermo emf produced oppose the bias emf its reading at 90 ths will give the thermal diffusivity. The instrument can be calibrated to read it directly.

It is obvious that this calibration will hold good for a particular value of current for a particular instant of time. For the present experimental set-up for glycerine (current $I = 0{\cdot}043$ mA, time -90 s) the expression works out to be

$$\alpha = \frac{1{\cdot}03 \times 10^{-6}}{(23{\cdot}1 - \phi)}\,\mathrm{m}^2\mathrm{s}^{-1}.$$

The electronic circuitry can be so designed to produce a single unit to give constant current supply to the probe heater, bias emf to microvoltmeter and register observation at required instant of time.

Such an instrument in association with differential temperature sensors probe will be a direct reading instrument for obtaining thermal characteristics of substances.

It might prove to be a useful handy apparatus for in situ measurements.

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References

Agrawal M P and Bhandari R C 1970 *Appl. Sci. Res.* **23** 113

Carslaw H S and Jaeger J C 1959 *Conduction of heat in solids* (Oxford: Clarendon) 2 ed., p. 261

Chaudhary D R 1968 *Some problems of heat transfer in disperse and porous media: Rajasthan desert sand.* Ph.D. thesis University of Rajasthan, Jaipur, India

Inaba H 1983 *Trans. ASME* **105** 268

Kerrisk J F 1971 *J. Appl. Phys.* **42** 268

Lozano J E, Urbicain M and Rostein E 1979 *J. Food. Sci.* **44** 198

Luikov A V 1966 *Heat and mass transfer in capillary-porous media.* (Translated by Harrison P W B, Translation (ed) W M Pun) (Oxford: Pergamon Press) First English ed., p. 263

Penner E, Johnsion G H, Goodrich L E 1975 *Can. Geotech. J.* **12** 271

Sharma R G, Pandey R N and Chaudhary D R 1984 *Indian J. Pure Appl. Phys.* **22** 658

Singh A K, Singh R, Beniwal R S and Chaudhary D R 1988a *Natl. Acad. Sci. Lett.* (India) **11** 261

Singh A K, Singh R and Chaudhary D R 1988b *Pramāṇa – J. Phys.* **31** 523

Singh R, Beniwal R S, Chaudhary D R 1985a *Indian J. Pure Appl. Phys.* **23** 630

Singh R, Saxena N S and Chaudhary D R 1985b *J. Phys.* **D18** 1

Wechsler A E 1966 *Development of thermal conductivity for Soil and Insulation.* Technical Report 182 U S Army Material Command, Cold Region Research and Engineering Laboratory Hanover, New Hampshire. p. 4