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Stability of the S matrix pole

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Abstract. We show how the position and residue of the S matrix pole can remain stable under changes in the form of the parametrization of the S matrix elements. We also derive a relation among the shifts in the Breit–Wigner resonance parameters under the same changes and verify the relation numerically for the case of $\Delta(1232)$. Despite its stability, the pole does not provide a unique definition of the resonance because of the existence of shadow poles.

Keywords: Scattering theory; S matrix; resonance poles; Δ (1232) resonance; Breit–Wigner formula; shadow poles.

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1. Introduction

The problem of characterizing the properties of a resonance or an unstable particle has attracted much attention in the past. This problem is particularly relevant for hadrons many of which are very short-lived and so occur as resonances with large widths. A question of practical importance is how to define the mass of the resonance and its width. There is no unique set of values for the resonance mass and width. Depending on the resonance formula used to fit the scattering amplitude, one gets different values for these parameters. Thus, for instance, in the case of $\Delta(1232)$, the best known hadronic resonance, the mass and width are found to shift by as much as 10 MeV and 40 MeV respectively, when different parametrizations or different formulae are used to describe the resonant amplitude.

Some time ago it was pointed out by Ball *et al* (1972, 1973) that the position of the pole of the scattering amplitude in the second sheet of the complex energy plane is much less dependent on the resonance formula used. To a certain extent this was found to be true for the residue at the pole also. In the example of $\Delta(1232)$, the real and imaginary parts of the pole position were in fact found to change by less than 1 MeV. This stability property of the pole is now well-known and the Particle Data Groups (Aguilar-Benitez *et al* 1986; Yost *et al* 1988) quote the real and imaginary parts of the pole (wherever known) in addition to the conventional mass and width of the hadron. This striking property of the pole of the scattering amplitude gives added support to the view that a resonance or an unstable particle must be defined by the pole of the S matrix.

Although the stability property of the pole is now well-known, so far it seems to be based only on empirical observation. Our purpose is to provide a theoretical foundation for it. We give a simple but general proof for the stability of the position and residue of the pole under certain conditions. Our method also allows us to obtain a relation between the shifts in the Breit–Wigner mass and width parameters. The shifts in the mass and the energy-dependent width function are found to be so correlated that the shift in the pole position is zero.

In §2, the proof of the stability of the position and residue of the S matrix pole is given, while §3 is devoted to the derivation of the relations between the shifts of the Breit–Wigner parameters. In §4, we discuss the examples of various forms of parametrization used for the case of the resonance $\Delta(1232)$ which illustrate the stability of the pole. The numerical comparison of the data with our relation between shifts of the Breit–Wigner parameters is also presented in this section. In the final section, the loss of the uniqueness of the S matrix pole because of the existence of "shadow poles" is briefly discussed.

2. Stability of the pole parameters

Let us define the T matrix in terms of the S matrix by

$$T = \frac{i}{2}(1 - S). \tag{1}$$

We consider two-body reactions in a single partial wave. Also, except in the final section where the discussion is relevant to multichannel scattering, in the rest of the paper we consider only single-channel scattering. However, the formulae will be written in such a way that they will be valid for multichannel case also. Let W be the energy in the c.m. system. If W_p is the position of a resonance pole in an unphysical sheet of the complex W plane, we may write

$$T(W) = \frac{R}{W - W_p} + C(W) \tag{2}$$

in the vicinity of the resonance. Here R is the residue matrix and $C(W)$ is the nonpole term which is finite at W_p. It is more convenient for our purposes to consider $T^{-1}(W)$ which has only a zero at W_p:

$$T^{-1}(W) = (W - W_p)\{R + (W - W_p)C\}^{-1}. \tag{3}$$

Consider now two different parametrizations $T_1(W)$ and $T_2(W)$ of the scattering amplitude given by two different formulae. Examples of such parametrizations will be given in §4. Since we may always put the amplitude in the form (2) in the neighbourhood of a resonance pole, we may write

$$\begin{aligned} T_1(W) &= \frac{R_1}{W - W_{p1}} + C_1(W) \\ T_2(W) &= \frac{R_2}{W - W_{p2}} + C_2(W). \end{aligned} \tag{4}$$

Let us suppose that both parametrizations provide equally good fits to the experimental data according to some suitably chosen criterion. We may then expect that the differences

$$\begin{aligned}\delta T(W) &\equiv T_2(W) - T_1(W)\\ \delta W_p &\equiv W_{p_2} - W_{p_1}\\ \delta R &\equiv R_2 - R_1\\ \delta C(W) &\equiv C_2 - C_1\end{aligned} \tag{5}$$

are small enough so that second and higher orders of these differences may be ignored. In that case the differences may be computed as if they were ordinary differentials. We interpret $\delta W_p, \delta R, \delta C(W)$ as the changes in W_p, R and $C(W)$ respectively introduced by a change in the parametrization of the scattering amplitude.

For the sake of clarity, we shall first present our results in the form of two theorems

Theorem 1. *If the first order difference in $T^{-1}(W)$ is zero for a region near W_p, then so is the first order difference in the pole position W_p.*

Theorem 2. *If the first order differences in both $T^{-1}(W)$ and $(\partial/\partial W)T^{-1}(W)$ are zero, then so are the first order differences in W_p and the pole residue R.*

The proof is straightforward. From (3), the first order difference is

$$\delta T^{-1} = -\delta W_p\{R+(W-W_p)C\}^{-1} + (W-W_p)\delta\{R+(W-W_p)C\}^{-1} \tag{6}$$

where the differences are to be taken at fixed W. Putting

$$\delta T^{-1}(W_p) = 0 \tag{7}$$

we get

$$\delta W_p = 0 \tag{8}$$

which proves theorem 1.

To prove theorem 2, we differentiate (3) and get

$$\frac{\partial T^{-1}}{\partial W} = \{R+(W-W_p)C\}^{-1} + (W-W_p)\frac{\partial}{\partial W}\{R+(W-W_p)C\}^{-1}. \tag{9}$$

Hence the first order difference of the derivative of T^{-1} is

$$\begin{aligned}\delta\frac{\partial T^{-1}}{\partial W} &= \delta\{R+(W-W_p)C\}^{-1} - \delta W_p\frac{\partial}{\partial W}\{R+(W-W_p)C\}^{-1}\\ &\quad + (W-W_p)\delta\left[\frac{\partial}{\partial W}\{R+(W-W_p)C\}^{-1}\right].\end{aligned} \tag{10}$$

We now demand the vanishing of $\delta T^{-1}(W_p)$ as well as

$$\delta\frac{\partial T^{-1}}{\partial W}(W_p) = 0. \tag{11}$$

Since the condition $\delta T^{-1}(W_p) = 0$ led to the vanishing of δW_p, we get from (10)

$$[\delta\{R + (W - W_p)C\}^{-1}]_{W_p} = 0$$

which implies

$$[\delta\{R + (W - W_p)C\}]_{W_p} = 0.$$

This leads to

$$[\delta R - \delta W_p C + (W - W_p)\delta C]_{W_p} = 0.$$

Again making use of the vanishing of δW_p, we get the required result:

$$\delta R = 0. \tag{12}$$

We now come to the physical interpretation of the theorems. If the scattering amplitude in the neighbourhood of the resonance is known to sufficient accuracy from experimental data, and different parametrizations are used to fit the same data, then it is reasonable to suppose that $\delta T^{-1}(W_p)$ is vanishingly small for any two parametrizations which provide equally good fits to the amplitude. (Recall the meaning of the difference δT^{-1} given by (5).) Our theorem 1 then says that δW_p tends to zero, i.e., the pole position is well-determined.

If further the experimental data are so accurately known and the parametrizations provide such good fits that even the first derivative of the scattering amplitude in the neighbourhood of the resonance is well represented (so that $\delta(\partial T^{-1}/\partial W)\,(W_p)$ can be set equal to zero), then, according to our theorem 2, the residue also is well determined. Because of the more stringent condition for the stability of the residue, it is clear that the stability of the pole position will be on a better footing when one looks at the results of actual fits to the data.

3. Shift of the Breit-Wigner parameters

Since in almost all phenomenological work on resonances, the Breit–Wigner formula, or some modification thereof, is utilized, it will be useful and instructive to deduce the consequences of the theorems of the last section for the Breit-Wigner parameters. The shifts in the Breit–Wigner mass and width parameters caused by a change in the type of parametrization will be found to be related.

The Breit–Wigner formula for the T matrix element near a resonance can be written as

$$T(W) = \frac{\frac{1}{2}F(W)}{W_0 - W - (i/2)\Gamma(W)} + B(W). \tag{13}$$

Here W_0 is the resonance energy or mass taken to be real. In general, F and Γ are energy-dependent quantities. B is a background term. In the multichannel case F and B are to be interpreted as matrices in the channel space.

Let us now relate the Breit–Wigner parameters W_0 and Γ to the pole parameters of the last section. The pole position W_p is determined by the equation

$$W_0 - W_p - \frac{i}{2}\Gamma(W_p) = 0. \tag{14}$$

The residue of the right hand side of (13) at this pole is given by

$$R = \frac{\frac{1}{2}F(W_p)}{1 + (i/2)\Gamma'(W_p)} \tag{15}$$

where the prime denotes differentiation with respect to W:

$$\Gamma'(W_p) = [\partial \Gamma(W)/\partial W]_{W_p}.$$

Calculating the first order differences from (14) and (15), we get the formulae

$$\delta W_p = \{\delta W_0 - \tfrac{1}{2} i \delta \Gamma(W_p)\}\{1 + \tfrac{1}{2} i \Gamma'(W_p)\}^{-1} \tag{16}$$

and

$$\delta R = \{1 + (i/2)\Gamma'(W_p)\}^{-2}[\tfrac{1}{2}\{1 + (i/2)\Gamma'(W_p)\}\delta F(W_p) - (i/4)\delta\Gamma'(W_p)F(W_p) + \{\tfrac{1}{2}(1 + (i/2)\Gamma'(W_p))F'(W_p) - (i/4)\Gamma''(W_p)F(W_p)\}\delta W_p]. \tag{17}$$

Using (16) and (17), it is now easy to state the theorems of the last section in terms of Breit–Wigner parameters. If $\delta T^{-1}(W_p) = 0$, then

$$\delta W_0 = \tfrac{1}{2} i \delta \Gamma(W_p). \tag{18}$$

If, in addition, $\delta(\partial T^{-1}/\partial W)\,(W_p) = 0$, then

$$\{1 + (i/2)\Gamma'(W_p)\}\delta F(W_p) = (i/2)\delta\Gamma'(W_p)F(W_p). \tag{19}$$

Eqs (18) and (19) are the desired relations between the shifts in the Breit–Wigner parameters.

In spite of their simplicity these relations are quite remarkable. Especially interesting is the appearance of i in both relations. This emphasizes the known fact that $\Gamma(W)$ evaluated at the complex pole W_p is complex. Since δW_0 is real, (18) implies that the shift in the width $\delta\Gamma(W_p)$ caused by any change in the type of parametrization is necessarily purely imaginary.

In the next section we shall study how well the relation (18) is satisfied when applied to various parametrizations that have been used to fit the $\Delta(1232)$ resonance.

4. Application to $\Delta(1232)$

In this section we apply our formulae to the $\Delta(1232)$ resonance and provide concrete illustration of the ideas of the foregoing sections.

For comparison with our formulae, only those fits satisfying the following two criteria should be chosen: (i) the fits should be to the same data, and (ii) the fits should be equally good, in the sense that χ^2 per degree of freedom should be comparable. We have chosen the following parametrizations of the best studied hadronic resonance, the Δ^{++} (1232):

(a) ***Breit–Wigner I*** (*Söding et al* 1972; *Barbaro-Galtieri* 1973): In this parametrization, the P_{33} partial wave amplitude in the pion-nucleon channel is taken to be a resonant term plus a nonresonant background:

$$T = \left(\frac{W_0\Gamma}{W_0^2 - W^2 - iW_0\Gamma}\right)\exp(2i\delta_B) + \exp(2i\delta_B)\sin\delta_B \tag{20}$$

with

$$\Gamma = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \left(\frac{1 + q_0^2 r^2}{1 + q^2 r^2}\right) \tag{21}$$

$$\delta_B = \tan^{-1} a q^3. \tag{22}$$

W and q are the c.m. energy and momentum respectively. W_0, Γ_0, r and a are adjustable parameters.

(b) *Breit–Wigner II* (*Cheng and Lichtenberg* 1973). This has the same expression for T as Breit–Wigner I, but with

$$\Gamma = \Gamma_0 \left(\frac{q}{q_0}\right)^3 \left(\frac{W_0}{W}\right)^2 \tag{23}$$

$$\delta_B = \tan^{-1} a \left(\frac{q}{W}\right)^3 \tag{24}$$

(c) *Polynomial* (*Söding et al* 1972; *Barbaro–Galtieri* 1973): In this fit the amplitude is written as

$$T = \frac{q^3}{q^3 \cot \delta - i q^3} \tag{25}$$

with

$$q^3 \cot \delta = \sum_{n=1}^{4} a_n q^{2n-2} \tag{26}$$

(d) *Extended Chew-Low* (*Vasan* 1976): This parametrization is defined by

$$T = q^3 \{\mu'^3 (1 - A\omega) - i q^3 + \frac{\omega}{\lambda'} (1 - A\omega)(1 + B\omega + C\omega^2)\}^{-1} \tag{27}$$

where $\omega = W - M\ (1 - \frac{1}{2}\beta)$, M is the proton mass and $\beta = 0{\cdot}00277$. Despite its appearance, this form has only three adjustable parameters A, B and C since it is semitheoretical. The parameters μ' and λ' are related to the π^+ mass μ and the pseudovector pion-nucleon coupling constant f by the following equations

$$\mu' = \mu \left(1 - \frac{\mu^2}{4M^2} + \frac{1}{2}\beta\right)^{1/2},$$

$$\lim_{\omega \to 0} \frac{\omega T}{q^3} = \frac{4 f^2}{3\mu^2} \left(1 + \frac{19}{20} \frac{\mu^2}{M^2}\right)$$

(*e*) *Breit–Wigner III* (*Vasan* 1976): T and Γ are as in (a) but δ_B as in (b).

For all the parametrizations above, the resonance energy or mass W_0 is defined to be the energy at which the resonant part of the amplitude T is equal to i. The width parameter Γ_0 is defined by the relation

$$\Gamma_0 = -2 \left\{\frac{\partial}{\partial W} (T^{-1} + i)\right\}^{-1}_{W = W_0}. \tag{28}$$

We have considered fits to two different sets of πN P_{33} phase shift data. One set consists of 14 data points from Carter 71 partial wave analysis (Carter *et al* 1971). The other set consists of 11 points from the more accurate data of Carter 73 analysis (Carter *et al* 1973). The parametrizations (a) to (c) above were fitted to the Carter 71 data while (d) and (e) were, to the Carter 73 data. The values of the parameters in the various fits are as follows:

Fits to Carter 71 *data*:

Breit–Wigner I: $W_0 = 1234\,\text{MeV}, \Gamma_0 = 120\,\text{MeV},$

$r = 0{\cdot}89\,\text{f}, a = 0{\cdot}028\,\text{fm}^3$

Breit–Wigner II: $W_0 = 1243{\cdot}3\,\text{MeV}, \quad \Gamma_0 = 152{\cdot}2\,\text{MeV},$

$a = 27{\cdot}67$

Polynomial: $a_1 = 1{\cdot}78\,\text{f}^{-3}, \quad a_2 = -0{\cdot}41\,\text{f}^{-1}, \quad a_3 = -0{\cdot}53\,\text{f},$

$a_4 = -0{\cdot}13\,\text{f}^3, \quad W_0 = 1231{\cdot}4\,\text{MeV}, \quad \Gamma_0 = 112\,\text{MeV}$

Fits to Carter 73 *data*:

Extended Chew–Low: $A = 3{\cdot}41\,\text{GeV}^{-1}, \quad B = -2{\cdot}35\,\text{GeV}^{-1},$

$C = 8{\cdot}28\,\text{GeV}^{-2}, \quad W_0 = 1230{\cdot}6\,\text{MeV},$

$\Gamma_0 = 114{\cdot}5\,\text{MeV}, \quad \mu' = 139{\cdot}3\,\text{MeV},$

$\frac{1}{\lambda'} = 0{\cdot}187\,\text{GeV}^2$

Breit–Wigner III: $W_0 = 1234{\cdot}0\,\text{MeV}, \quad \Gamma_0 = 124\,\text{MeV},$

$r = 0{\cdot}842\,\text{f}, \quad a = 9{\cdot}23.$

In table 1, the values of W_0 and Γ_0 for the various parametrizations are collected together for convenience. The first three lines a) to c) refer to fits to Carter 71 data while the last two lines d) and e) refer to fits to Carter 73 data. These fits satisfy the two criteria mentioned in the beginning of this section. The rather large shifts in W_0 and Γ_0 as one goes from one parametrization to another can be noted. The pole position W_p is also presented in the same table. The remarkable stability in both the

Table 1. Mass, width and pole positions for various parametrizations of Δ (1232).

Parametrization	χ^2/d.f	W_0 (MeV)	Γ_0 (MeV)	W_p (MeV)	$\Gamma(W_p)$ (MeV)
a) B-W I	9·7/10	1234·0	120	1211·2–50·0i	98·1–47·3i
b) B-W II	16/11	1243·3	152·2	1210·7–50·7i	98·9–66·3i
c) Poly.	12·2/10	1231·4	112	1211·3–50·2i	98·7–42·0i
d) E.C-L	5·4/8	1230·6	114·5	1209·5–50·5i	99·2–43·8i
e) B-W III	5·0/7	1234·0	124	1209·7–50·3i	98·7–50·3i

Table 2. Comparison of the mass shift with the shift in the width function.

Parametrization	δW_0 (MeV)	$\frac{i}{2}\delta\Gamma(W_p)$ (MeV)
Breit–Wigner I, Breit–Wigner II	9·3	$9{\cdot}5+0{\cdot}4i$
Polynomial, Breit–Wigner II	11·9	$12{\cdot}2+0{\cdot}1i$
Extended Chew-Low, Breit–Wigner III	3·4	$3{\cdot}2+0{\cdot}2i$

real and imaginary parts of W_p is obvious. It is interesting to note that the pole position is apparently stable even when the data are changed! In the last column of this table, we give the value of the width function at the pole, $\Gamma(W_p)$, which will be needed for verifying our relation (18).

The verification of the relation (18) is presented in Table 2. To minimize unknown effects in the numerical comparison of the fits to our formulae, we have compared two fits (to the same data) having appreciably different values for the parameters W_0 and Γ_0. The shifts δW_0 and $\delta\Gamma(W_p)$ are calculated as differences between the parametrizations indicated in the first column of table 2. A look at the second and third columns shows that the relation $\delta W_0 = i\delta\Gamma(W_p)/2$ is well satisfied. It is seen that the large imaginary part of $\delta\Gamma(W_p)$ is taken care of by the extra factor i. As δW_p is not zero, though very small, we cannot expect strict equality of the two sides of the relation. As seen from table 1, the pole position W_p is determined at best with an error of 0·5 MeV. Hence, the agreement between the two columns of table 2 should be considered quite satisfactory. This good agreement of our relation (18) with data supports the arguments made in proving the stability of the pole position since both derivations are based essentially on the same logic.

We have not displayed any numerical verification of relation (19) (which would indicate that δR tends to zero) because we do not feel that the fits determine the residue R well enough for the application of our first order formula to be meaningful. It may be recalled that relation (19) implies that the fits are a good representation of not only the amplitude T but also the derivative $\partial T/\partial W$.

5. Discussion

The aim of our work has been to demystify the pole and provide a simple understanding of the stability of the pole, under different parametrizations of the scattering amplitude. We have shown that the pole position has a first order stability under variations of the forms of parametrizations, provided the experimental data in the neighbourhood of the pole are equally well fitted by the various parametrizations. Under the same conditions we have also derived a relation between the shifts of the Breit–Wigner parameters and this relation is found to agree well for the actual shifts found in the parametrizations fitted to $\Delta(1232)$.

Because of its stability, the position of the pole seems to be the best way of characterizing the properties of the resonant state – the real and imaginary parts of the pole position playing the roles of the conventional mass and width of the resonance. However, here we would like to draw attention to a basic issue. There is in fact no unique correspondence between a resonance and a single pole of the multichannel S matrix. It has been known for quite some time that a single pole of the S matrix in a particular Riemann sheet of the complex energy plane is in general followed by a retinue of poles, called shadow poles, existing in the other Riemann sheets (Dalitz and Rajasekaran 1963; Eden and Taylor 1963; Ross 1963; Amati 1963). Experimental evidence for the existence of these shadow poles has recently been found. Shadow poles exist and their positions have been determined in the case of $J^P = \frac{3}{2}^+$ resonance in the $(n\alpha, dt)$ channels of nuclear physics (Hale *et al* 1987) and in the case of the isoscalar $J^P = 0^+$ state $f_0(975)$ (formerly S (975)) in the ($\pi\pi$, $K\bar{K}$) channels of particle physics (Au *et al* 1987). Thus, in principle, a resonance is represented by a system of poles in various Riemann sheets and often there is no a priori justification for choosing one among them for characterizing the resonance.

In many physical situations, one of the poles may be nearest to the physical region where the experimental data on the scattering exist. Since our stability argument depended on the nearness of the pole to the region where experimental data are fitted, it is clear that only this nearby pole is expected to be stable and thus, for most practical purposes, the real and imaginary parts of this pole do have some significance. But, from a fundamental point of view, all the poles of the S matrix on the various Riemann sheets are equal manifestations of the resonance phenomenon and none is to be singled out as having a closer relationship with the resonance.

There does exist a pole which is of special significance and that is the pole of the K matrix. A single and unique pole of the K matrix in fact replaces or represents the whole system of shadow poles of the S matrix (Rajasekaran 1965). However, the determination of the K matrix and its pole from the experimental data being more indirect, the position of the K matrix pole is not expected to be so stable.

Finally, we must point out that there are certain important exceptional situations where shadow poles do not exist (Rajasekaran 1972). The question of the existence or nonexistence of shadow poles is in fact intimately connected to hadron dynamics and contains information about the nature of the hadron: whether it is a normal bound state of quarks or whether it is a "molecule" of other hadrons. The nucleon and $\Delta(1232)$ are examples of the former while deuteron and possibily Λ (1405) are examples of the latter (Rajasekaran 1972; Dalitz 1982). A fuller discussion of this problem is outside the scope of the present paper and is reserved for a future publication.

To sum up, poles of the S matrix may be the best candidates for the definition of unstable particles. One can even mathematically prove the stability of the poles under changes in the parametrization of the scattering amplitude. But, because of the existence of the shadow poles there is no unique correspondence between particle and pole. For each particle in general, there may be a retinue of poles in various Riemann sheets. The existence or nonexistence of shadow poles is related to deeper questions of dynamics.

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