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Bonding potential between two ^{12}C nuclei

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Abstract. The potential between two ^{12}C nuclei in linear chain configuration has been calculated microscopically using the Ali-Bodmer α-α potential. This potential shows a pocket and compares well in the tail region with the phenomenological potential extracted before, from the data on the quasi-molecular resonances of the $^{12}C+^{12}C$ system. This provides support to the diatomic like rotation-vibration picture of quasi-molecular states.

Keywords. Bonding potential; quasimolecular resonances.

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Eventhough more than twenty five years have passed since the discovery of the nuclear quasi-molecular states in the $^{12}C+^{12}C$ system in 1960 by Bromley *et al* (1960), the mechanism of the phenomena has not yet been clearly established. During these years, such states have been observed in many other combinations of projectile and target nuclei. The general features of these states are that they show resemblance to the rotation-vibration spectra generally observed in a diatomic molecule. It was first suggested by Iachello (1981) that dipole degree of freedom characterized by the distance between the two nuclei could play important role in the mechanism. So the internuclear potential $V(R)$, where R is the distance between the two centres of the nuclei would govern the physics of these states. The analysis of the experimental data shows that the moment of inertia is very large, which correspond to a stretched out configuration suggestive of a very long range potential. Such long range is not in any way compactible with the usual optical potential of the $^{12}C+^{12}C$ system. This has been a quite puzzling problem in this area of heavy-ion physics. Satpathy *et al* (1983, 1986) had attempted to obtain this potential by using the data of the quasi-molecular states in the framework of a quantum mechanical two-body problem. They had supposed that the effective potential between two ^{12}C ions comprising both Coulomb and nuclear part could be represented by a four parameter Morse type potential. By fitting the energies of more than 20 quasi molecular states with the eigenvalue expression, parameters of the potential were determined. This potential was found to have a range of the order of 15 fm. The potential, besides fitting the data, suggests a possible mechanism (Satpathy *et al* 1986) of the quasi molecular states, according to which, the two nuclei when come closer by overcoming Coulomb barrier, or by sub-barrier tunnelling, they form a composite system, and in the effort to separate, they undergo a strong prolate deformation in the exit channel. Because of this strong prolate deformation, a broad Coulomb barrier is formed, which inhibits separation. Hence, they undergo rotation and vibration like a diatomic molecule and generate molecular resonance states.

Finally they separate with restoration of original shape after a long time in the exist channel. Thus, it is desirable to see that such a long range potential between two ^{12}C nuclei found above, really originates at a more microscopic level. It is known that the first 0^+ excited state of ^{12}C at 7·65 MeV has the configuration in which three α-particles form a linear chain. Feshbach (1976) had suggested long back, that this linear chain state may play an important role in explaining the mechanism of the quasi-molecular states. Hence, it is very much desirable to see if, the potential between two ^{12}C in linear chain configuration has a pocket to sustain resonances, and if, it resembles the phenomenological potential extracted by Satpathy and Faessler (1983). Previously Faessler *et al* (1984) have calculated this potential in the framework of α-cluster model, using the Brink-Boeker and Volkov effective interactions. They have found that their calculated potential has a long range with a pocket and resembles the phenomenological potential in the tail region. Considering the importance of this problem, we intend to calculate this potential using the Ali-Bodmer (1969) α-α potential, which was obtained by fitting the experimental phase-shifts and had been widely used (Afzal *et al* 1969; Portilho and Coon 1979; Portilho *et al* 1983). The Ali-Bodmer α-α potential is given by

$$V_{\alpha\alpha}(r) = V_r \exp(-\mu_r^2 r^2) - V_a \exp(-\mu_a^2 r^2) \tag{1}$$

where $V_r = 475$ MeV, $V_a = 130$ MeV, $\mu_r^2 = 0{\cdot}49\,\text{fm}^{-2}$ and $\mu_a^2 = 0{\cdot}2256\,\text{fm}^{-2}$. To this potential, Portilho *et al* (1983) have added a three-body term given by

$$V_{\alpha\alpha\alpha}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = V_0[\exp(-\lambda(r_{12}^2 + r_{13}^2)) + \exp(-\lambda(r_{12}^2 + r_{23}^2)) + \exp(-\lambda(r_{13}^2 + r_{23}^2)] \tag{2}$$

where, $V_0 = -75$ MeV and $\lambda = 0{\cdot}21792\,\text{fm}^{-2}$. In our investigation, we have taken the nuclear part $V_N(r)$ as the combination of the above two potentials.

$$V_N(r) = V_{\alpha\alpha}(r) + V_{\alpha\alpha\alpha}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3). \tag{3}$$

Since we are considering only the linear chain state for the two ^{12}C nuclei, the configuration chosen is the one as shown in figure 1, where R denotes the separation between the centres of the two carbon nuclei and r denotes the separation between two adjacent α-particles.

The calculation has been performed in two steps. In the first step, we determine the wavefunction and energy of an isolated C-nucleus in this linear chain state. We choose a variational wavefunction which includes correlation such that it does not allow overlap of any pair. This is given by

$$\phi(x_1, x_2, x_3) = S\Big[(x_1 - x_2)^4 (x_1 - x_3)^4 (x_2 - x_3)^4 \times \exp(-\alpha(x_1 - x_2)^2 - \alpha(x_2 - x_3)^2 - \frac{\alpha}{4}(x_1 - x_3)^2\Big]. \tag{4}$$

Since α-particles are bosons, therefore, S is the symmetrization operator, which ensures that ϕ remains symmetric under the exchange of co-ordinates of any two α-particles.

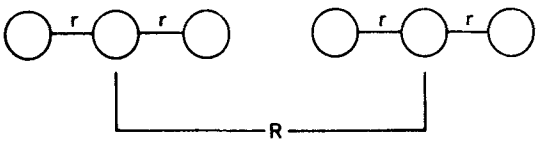

Figure 1. A schematic diagram of the linear chain configuration for the nuclear molecular resonance in ^{24}Mg formed by a ^{12}C-^{12}C collision.

This wave function has one parameter α, which was determined by minimizing the energy

$$E = \langle \phi | H | \phi \rangle / \langle \phi | \phi \rangle \tag{5}$$

where H is the Hamiltonian defined as

$$H = T - T_{cm} + V_N + V_C \tag{6}$$

where V_C is the Coulomb interaction and T_{cm} is the centre of mass energy. The values of E and α so obtained are 1·03 MeV and 0·19 respectively. If we take the ground state energy to be 7·25 MeV calculated by Portilho *et al* (1983) using the same potential which had agreed quite well with the experiment, then the energy of the first excited 0^+ states comes to 8·28 MeV in our calculation. This agrees reasonably well with the experimental value of 7·65 MeV. Portilho *et al* (1983) obtained a value of 9·33 MeV for this state using the same potential. Using our wavefunction we calculated the mean square distance $\langle (x_1 - x_2)^2 \rangle^{1/2}$ between two adjacent α-particles to be 3·31 fm. Thus the length of the C chain comes out to be 6·62 fm. In the study of this state by Friedrich *et al* (1971) using the Brink-Boeker force in the framework of α-cluster model, the length of the chain was obtained as about 7·7 fm.

In the second step of our calculation we determine the interaction potential between two C-ions, which is given by

$$V(R) = \langle \psi(R) | H | \psi(R) \rangle - \langle \psi(R = \infty) | H | \psi(R = \infty) \rangle. \tag{7}$$

Here $\psi(R)$ represents the wavefunction of the two carbon nuclei separated by a distance R. $\psi(R)$ in the frozen density approximation has been taken to be

$$\psi(R) = [\phi(x_1, x_2, x_3)\phi(x_4, x_5, x_6)]\delta \times \left[R - \frac{x_1 + x_2 + x_3}{3} - \frac{x_4 + x_5 + x_6}{3} \right] \tag{8}$$

where the ϕ's are defined in (4). In figure 2, we have compared our calculated potential with the phenomenological potential of Satpathy *et al* (1983, 1986) and the results of Faessler *et al* (1984). The solid curve is the pure nuclear part obtained in references [Satpathy and Faessler 1983; Satpathy *et al* 1986]. The broken curve is the nuclear part of the potential calculated by Faessler *et al* 1984 using the Brink–Boeker force in the framework of α-cluster model. In our calculation we have switched off the Coulomb term in (6) to get the corresponding nuclear part which has been shown as dotted curve in figure 2. Thus our potential agrees with the phenomenological potential in the tail region. It is also quite similar to the potential obtained by Faessler *et al* (1984). In figure 3, we have presented our results obtained with the inclusion of the

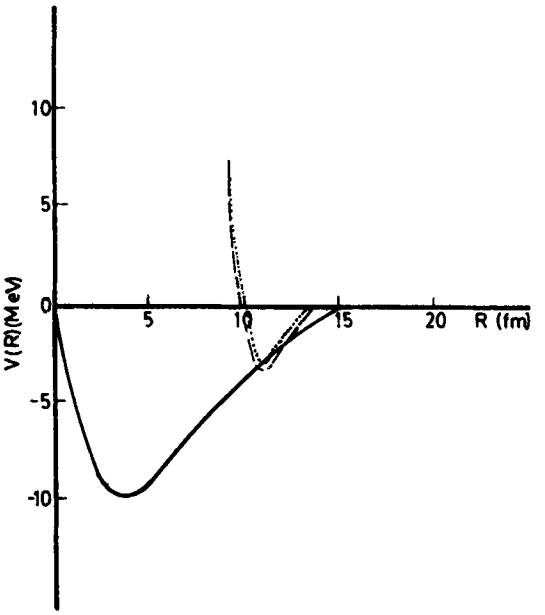

Figure 2. The interaction potential between two ^{12}C nuclei calculated using the Ali-Bodmer α-α potential (dotted curve) together with the phenomenological potential of Satpathy and Faessler (full curve) and the calculated potential of Faessler *et al* using the Brink-Boeker force (broken curve).

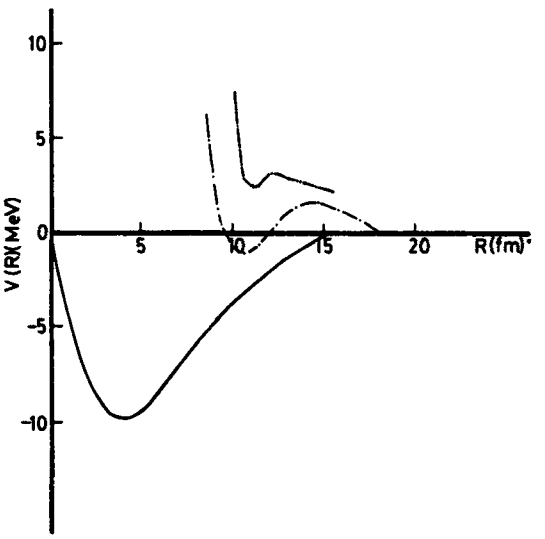

Figure 3. The full curve is as in figure 2. The chain curve is calculated by Faessler *et al* using the Volkov force together with Coulomb potential and the dotted curve is the present result obtained using the α-α potential along with the Coulomb interaction.

Coulomb potential represented by the dotted curve. The chain curve is the result obtained by Faessler *et al* (1984) using the Volkov force and the Coulomb interaction. Although somewhat shallower, we find a pocket in our potential. This potential is positive throughout, however because of the presence of the pocket it can have, in principle, bound states as well as resonances.

The general feature of our potential in the figure has a hard core of long range followed by an attractive part. The hard core has a range of about 8 fm, which is mainly due to the frozen density approximation. But the total range of the potential is more than 13 fm found in our calculation. Besides this long range, it is quite interesting to note that the potential does show a pocket which is very essential for the sustenance of quasi-molecular resonances.

The quasi-molecular states are manifested in low energy experiment, in which the projectile has energy whose value is about the Coulomb barrier of the projectile and target nuclei. The outer region of the potential is explored in the process. So, the physics of the phenomena is mainly governed by that region of the potential. Hence, our potential calculated here, although differ very substantially from the phenomenological one found by Satpathy *et al* (1986) in the core region, provides support for the mechanism.

In conclusion we would like to say that the mechanism of quasi-molecular states which has emerged in our previous study is being corroborated by the present calculation. It is quite satisfying that starting from a totally different approach and using α-α potential determined from experimental data, we obtain an interaction potential between two ^{12}C nuclei in linear chain configuration, which is quite similar to that of Faessler *et al* (1984) calculated with effective nucleon-nucleon potentials. This enhances our confidence on the diatomic like rotation-vibration mechanism of the quasi-molecular states in the $^{12}C + ^{12}C$ system proposed earlier.

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