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Comment on a solution of Altarelli Parisi equations

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Abstract. We show that the assumptions of factorizability of the structure function $F(x,t)$ in x and t and the equality of t evolution of the singlet and gluon distributions are not necessary to obtain the approximate solutions of Altarelli–Parisi equations derived by us recently.

Keywords. Structure function; Altarelli–Parisi equations.

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In a recent paper (Choudhury and Saikia 1989), we have shown that the Altarelli–Parisi (AP) equations (Altarelli and Parisi 1977) can be recasted as

$$\frac{\partial F_2^{NS}}{\partial t}(x,t)=\frac{4}{25t}\left[(3+4\ln(1-x))F_2^{NS}(x,t)+2\int_x^1\frac{dw}{(1-w)}\{(1+w^2)F_2^{NS}(x/w,t)-2F_2^{NS}(x,t)\}\right] \tag{1}$$

and

$$\frac{\partial F_2^{S}}{\partial t}(x,t)=\frac{4}{25t}\left[(3+4\ln(1-x))F_2^{S}(x,t)+2\int_x^1\frac{dw}{(1-w)}\{(1+w^2)F_2^{S}(x/w,t)-2F_2^{S}(x,t)\}+6\int_x^1(w^2+(1-w)^2)G(x/w,t)dw\right] \tag{2}$$

where we considered the number of flavours to be four and $t=\ln Q^2/\Lambda^2$.

Let us consider the trial solutions of (1) and (2) to be

$$F_2^{NS}(x,t)=F_2^{NS}(x,t_0)\left(\frac{t}{t_0}\right)^{\beta(x)} \tag{3}$$

and

$$F_2^{S}(x,t)=F_2^{S}(x,t_0)\left(\frac{t}{t_0}\right)^{\gamma(x)} \tag{4}$$

respectively. Here $t_0=\ln Q_0^2/\Lambda^2$ and $\beta(x)$ and $\gamma(x)$ are some functions of x only.

Putting (3) in (1), we obtain

$$\beta(x)F_2^{NS}(x,t) = \frac{4}{25}\left[(3 + 4\ln(1-x))F_2^{NS}(x,t) + 2\int_x^1 \frac{dw}{(1-w)}\{(1+w^2)F_2^{NS}(x/w,t) - 2F_2^{NS}(x,t)\}\right]$$

or,

$$\left[\frac{25}{4}\beta(x) - (3 + 4\ln(1-x))\right]F_2^{NS}(x,t) = 2\int_x^1 \frac{dw}{(1-w)}\{(1+w^2)F_2^{NS}(x/w,t) - 2F_2^{NS}(x,t)\}. \tag{5}$$

Solving the above equation, at $t = t_0$ we obtain

$$\beta(x) = \frac{4}{25}\left[2\int_x^1 \frac{dw}{(1-w)}\left\{(1+w^2)\frac{F_2^{NS}(x/w)}{F_2^{NS}(x)} - 2\right\} + 3 + 4\ln(1-x)\right] \tag{6}$$

where $F_2^{NS}(x) = F_2^{NS}(x, t_0)$.

The integral which occurred in (6) is same as $I^{NS}(x)$ defined in (18) of our previous work (Choudhury and Saikia 1989)

$$\beta(x) = \tfrac{4}{25}[3 + 4\ln(1-x) + 2I^{NS}(x)]$$

which is nothing but $H^{NS}(x)$ of our previous work (eq(26)).

Similarly, putting (4) in (2), we can show that $\gamma(x)$ is equal to $H^S(x)$ of the previous work (eq (27)).

Hence, without assuming (12), (13), (14), (17) of the previous work we arrive at the same result, considering $\beta(x)$ and $\gamma(x)$ to be dependent on x only. Thus neither the factorizability of the structure function nor the equal rate of t-evolution of the singlet and gluon distributions is necessary to derive our previous result. We however note that the trial solutions (3) and (4) are valid only in leading logarithmic approximation (LLA) but not with higher order effects. With such effects, $\beta(x)$ and $\gamma(x)$ should have t-dependence as well.

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