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An extended technicolour model for grand unification

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Abstract. An extended technicolour grand unification model based on the gauge group $E_6 \times SU(7)$ extended technicolour is presented. The symmetry-breaking based on extended technicolour theory is discussed. It is shown that the existing phenomenology is well explained by the model. The strangeness changing neutral currents may not be a problem with this model.

Keywords. Grand unified theory; extended technicolour.

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1. Arguments for technicolour

At present there are three grand unified models which explain most of the low energy phenomena including the value of Weinberg angle (Langacker 1981). They are based on the gauge groups (i) SU(5) (Georgi and Glashow 1974) (ii) SO(10) (Georgi 1974; Fritzsh and Minkowski 1975) and (iii) E_6 (Barbieri and Nanopoulos 1980) and on the Higgs mechanism for providing masses for the various vector bosons and fermions. They suffer from two serious defects (Langacker 1981) (i) the gauge hierarchy problem and (ii) the induced cosmological constant problem.

The hierarchy problem refers to the incredibly tiny ratio $M_W^2/M_X^2 \approx 10^{-24}$ that occurs in the above mentioned grand unified models. Here M_W and M_X are the masses of the W boson and X boson respectively that mediates proton decay. Such hierarchy masses are not natural features. If the masses are generated by the Higgs mechanism and even if we originally assume such a hierarchy, the mass of the W boson becomes almost of the same order of magnitude as that of the X boson due to radiative corrections to the Higgs potential. To prevent it one has to fine tune the parameters of the Higgs potential to 24 places of decimals. Such an unnatural hierarchy would imply that nature is based on a 24 figure accident that is impossible to believe.

The second serious difficulty with the idea of Higgs mechanism concerns the non-zero value of the Higgs potential $V(\phi)$ when it is evaluated at the minimum of the Higgs field $\phi = v$. As pointed out by Zeldovich (1968) and Langacker (1981) such a vacuum self-energy term must be interpreted as a cosmological constant $\Lambda = 8\pi G_N V(v)/C^4$ in Einstein's equations for general relativity where G_N is the gravitational constant. For the Glashow-Weinberg-Salam model this corresponds to a cosmological constant 52 orders of magnitude larger than the observed limit on cosmological constant term. For the above mentioned grand unified models this is more than 100 orders of magnitude too large (Langacker 1981). This discrepancy can be corrected only by introducing into Einstein's equations a primordial cosmological constant term which is fine-tuned to hundred places of decimals to cancel the induced cosmological constant term due to the Higgs potential. The above two difficulties viz. the gauge hierarchy and the cosmological constant problems, find a natural solution in the extended technicolour (ETC) approach to grand unified models (Langacker 1981). In ETC theories there is no need to fine-tune parameters to obtain gauge hierarchy and there is no induced cosmological constant term.

2. Arguments for $E_6 \times \mathrm{SU}(7)$ group

We choose as the gauge group for our grand unified model based on dynamical symmetry breaking the semi-simple group $E_6 \times SU(7)^{\mathrm{ETC}}$. We assign the fermions to transform as the fundamental irrep $\underline{27}$ under E_6 and the fundamental irrep $\underline{7}$ under $\mathrm{SU}(7)^{\mathrm{ETC}}$. The choice of the gauge group is not ad hoc, but based on definite reasons. We choose E_6 as the basic component of the gauge group for the following reasons (Barbieri and Nanopoulos 1980)

(i) automatic absence of anomalies (ii) all the basic components of one generation transforming as the fundamental irrep ($\underline{27}$) of the group (iii) the possibility of discussing the pattern of symmetry breaking, involving at least two steps:

$$G \to \mathrm{SU}(3) \times \mathrm{SU}(2) \times U(1) \equiv G_1 \to \mathrm{SU}(3) \times U(1) \equiv G_2$$

in terms of fermion-fermion operator, or in other words the operator transforming as ($\underline{27} \times \underline{27}$). This third condition is necessary for constructing a satisfactory model based on dynamical symmetry breaking. This is a crucial point responsible for choosing E_6 and ruling out the other two viable groups SU(5) and SO(10).

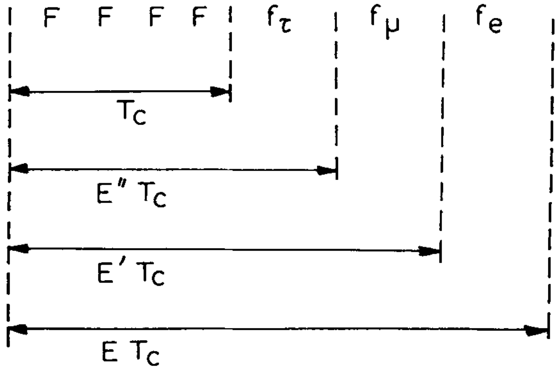
There is also a definite reason for choosing SU(7) as the extended technicolour (TC) component of the gauge group. It is reasonable to assume that the TC group grows in the infra-red limit faster than QCD. This demands that the TC group must at least be SU(4). We choose this minimum group as the TC group. If we assume that the present-day phenomenology has only three generations of fermions as a fact of nature, then from the extended technicolour theory (Dimopoulos and Susskind 1979; Eichten and Lane 1980) it follows that the ETC component of the gauge group must be SU(7). Here it must be remembered that a semi-simple group is chosen instead of a simple group because of the mathematical fact that $E_6 \times \mathrm{SU}(7)$ cannot be embedded in a reasonable simple group.

3. Description of the theory

To give masses to the three families with each family having 27 fermions, we need to break the ETC group down to TC group in three successive stages.

$$\mathrm{SU}(7)^{\mathrm{ETC}} \xrightarrow[\mu]{} \mathrm{SU}(6)^{\mathrm{E'TC}} \xrightarrow[\mu']{} \mathrm{SU}(5)^{\mathrm{E''TC}} \xrightarrow[\mu'']{} \mathrm{SU}(4)^{\mathrm{TC}}$$

with the fermions transforming as a fundamental irrep $\underline{7}$ of ETC as shown here:



According to the concept of tumbling (Farhi and Susskind 1981) because of asymptotic freedom which is present in the theory the fermions form scalar bound states in various stages as we come down in energy. They are responsible according to the ETC theory (Dimopoulos and Susskind 1979; Eichten and Lane 1980) for breaking the symmetries and giving mass to the fermions in three successive stages, first to the electron family, then to the muon family and finally to the τ-lepton family. Ultimately the gauge coupling constant runs according to the SU(4) β-function. The $SU(4)^{TC}$ bound states are given by the SU(4) irreps $(\underline{4}\times\underline{4})=\underline{6}+\underline{10}$ where $\underline{6}$ is a real irrep and hence the tumbling stops (Farhi and Susskind 1981) and $SU(4)^{TC}$ remains unbroken. This model is expected to explain all particle phenomena in which gravitation can be neglected in terms of two fundamental coupling constants.

4. Technicolour and masses of vector bosons

The E_6 group has 78 vector bosons most of which have to be given a superheavy mass except 12 bosons which have to be left almost massless. In our present theory the bosons acquire a mass according to the technicolour theory (Weinberg 1976; Susskind 1979) through pseudo-Goldstone bosons formed by the technifermions. They are given by the irreps

$$\underline{27}\times\underline{27}=(\underline{27}+\underline{351})_S+\underline{351}'_A. \quad (1)$$

The mass of any vector boson is given by

$$M_B\sim\tfrac{1}{2}gF_\pi. \quad (2)$$

Where g is the coupling constant of the vector boson to the technifermions and F_π the technipion decay constant of the pseudo-Goldstone boson which becomes the longitudinal component of the vector boson. The E_6 group undergoes the following successive reductions

$$E_6\supset SO(10)\times U(1)\supset SU(5)\times U(1)\times U(1).$$

Under these reductions the fermion irrep $\underline{27}$ breaks up as follows (Slansky 1981).

$$\underline{27}=\underline{16}+\underline{10}+\underline{1}\ \text{(under SO(10))}$$

$$\underline{16}=\underline{10}+\underline{\bar{5}}+\underline{1}\ \text{(under SU(5))}$$

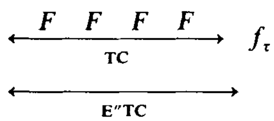
$$\underline{10}=\underline{5}+\underline{\bar{5}}\ \text{(under SU(5))}.$$

In the following discussion of symmetry breaking we will closely follow the paper by Barbieri and Nanopoulos (1980) who first showed that E_6 symmetry-breaking can be done by scalar bosons of the ($\underline{27} \times \underline{27}$) irreps. They even suggested that the symmetry breaking may be dynamical, but they did not propose any explicit model of dynamical symmetry-breaking.

The various pseudo-Goldstone boson components of ϕ_{351}, $\chi_{351'}$, and H_{27} of (1) are formed by the TC fermions and are responsible for the symmetry-breaking of E_6. The various reductions of $\underline{351}$, $\underline{351}'$, and $\underline{27}$ under SO(10) and SU(5) can be obtained from standard tables (Slansky 1981). The ϕ_{351} and χ_{351} are responsible for superlarge breaking (Barbieri and Nanopoulos 1980) with $\underline{16}$ of SO(10) breaking E_6 down to SU(5) and the $\underline{24}$ of SU(5) breaking SU(5) down to SU(3) × SU(2) × $U(1)$. The $\underline{5}$ under SU(5) of $\underline{27}$ are responsible for the last stage of symmetry breaking to SU(3) × $U(1)$ and we reproduce the usual relation $M_W = M_Z \cos\theta_W$ in this model also (Weinberg 1976; Susskind 1979). The theory also predicts a large number of physically observable pseudo-Goldstone bosons whose masses are enhanced by the extended technicolour interactions.

5. Extended technicolour and heavy fermions

To give masses to the fermions we must produce effective Yukawa couplings between ordinary fermions and technifermions. For simplicity let us consider the τ-family



The extended technicolour (E″TC) breaks down to TC at some energy scale μ''. The ε'' vector boson which is in E″TC but not in TC acquires a mass $M_{\varepsilon''} \approx g_{E''TC}\mu''$ and couples to currents of the form $\bar{F}\gamma_\mu f_\tau$. This leads to an effective mass for the fermions according to the ETC theory (Dimopoulos and Susskind 1979; Eichten and Lane 1980) given by

$$m_{\tau\text{-family}} = \frac{g^2_{E''TC}}{M^2_{\varepsilon''}} \langle \bar{F}F \rangle_0. \tag{3a}$$

Similarly masses for muon and electron family are respectively given by

$$m_{\mu\text{-family}} = \frac{g^2_{E'TC}}{M^2_{\varepsilon'}} \langle \bar{F}F \rangle_0 \tag{3b}$$

$$m_{e\text{-family}} = \frac{g^2_{ETC}}{M^2_{\varepsilon}} \langle \bar{F}F \rangle_0. \tag{3c}$$

To discuss the masses of the fermions let us have the following self-explanatory notation for the $\underline{27}$ multiplet ψ of left-handed fermions

(i) the $\underline{16}$ under SO(10):

$(\nu, u_i, e, d_i, d^c_i, e^c, u^c_i, \nu^c)$ $(i = R, B, W)$

(ii) the $\underline{10}$ under SO(10):

$(D_i, N, E, E^c, N^c, D_i^c)$

(iii) the $\underline{1}$ under SO(10) denoted by L.

The quark D_i and the leptons N and E acquire superheavy masses (Barbieri and Nanopoulos 1980). Let $\phi(a,b)$ ($\chi(a,b)$) denote the component ϕ_{351} ($\chi_{351'}$) which is $\underline{a}$ under SO(10) and $\underline{b}$ under SU(5). Then from our theory it follows that the masses of D, N and E are given by (Dimopoulos and Susskind 1979; Eichten and Lane 1980; Barbieri and Nanopoulos 1980)

$$m_D = \frac{g_{\mathrm{ETC}}^2}{M_\varepsilon^2}[\langle\chi(45,1)\rangle_0 + \langle\chi(45,24)\rangle_0 + \langle\phi(54,24)\rangle_0] \tag{4a}$$

$$m_E = m_N = \frac{g_{\mathrm{ETC}}^2}{M_\varepsilon^2}[\langle\chi(45,1)\rangle_0 - \tfrac{3}{2}\langle\chi(45,24)\rangle_0 - \tfrac{3}{2}\langle\phi(54,24)\rangle_0]. \tag{4b}$$

Similar expressions can be deduced for the muon and τ-lepton families. It is worthwhile noting that m_E may be much smaller than m_D because of the negative signs in (4b). The above expressions for m_N does not correspond to the observed mass due to mixing. In $Q = 0$ sector mixing leads to two superheavy Majorana particles, a heavy Dirac particle and a nearby massless neutrino (Barbieri and Nanopoulos 1980).

6. Masses of light fermions

The light fermions receive their masses due to the vacuum expectation value of H_{27}. They are given by (Dimopoulos and Susskind 1979; Eichten and Lane 1980; Barbieri and Nanopoulos 1980)

$$m_d = m_e = \frac{g_{\mathrm{ETC}}^2}{M_\varepsilon^2}\langle H(10,\bar{5})\rangle_0 \tag{5a}$$

$$m_u = \frac{g_{\mathrm{ETC}}^2}{M_\varepsilon^2}\langle H(10,5)\rangle_0. \tag{5b}$$

Where as above $H(a,b)$ denotes a component of H_{27} which is $\underline{a}$ under SO(10) and $\underline{b}$ under SU(5). Relations similar to $m_d = m_e$ such as $m_b = m_\tau$ and $m_s = m_\mu$ also follow. Actually a unit KM matrix is obtained if only ϕ_{351} symmetric representation gets superlarge vacuum expectation values since in this case (Barbieri and Nanopoulos 1980)

$$M^{-1/3} = (a\cos\theta + b\sin\theta)UM^{2/3} \tag{6a}$$

$$M^{-1} = (a\cos\theta + b\sin\theta)M^{2/3}U^T \tag{6b}$$

Here $M^{-1/3}$, $M^{2/3}$ and M^{-1} are respectively mass matrix corresponding to $Q = -1/3$, $2/3$ and -1, U is a unitary matrix in the generation space and a and b are complex numbers related to the vacuum expectation values of the pseudo-Goldstone bosons in H_{27}. One is then led to speculate that a perturbative deviation from this situation would introduce the relatively small KM angles and at the same time would correct the relations $m_b = m_\tau$, $m_s = m_\mu$ and $m_d = m_e$ by an increasing amount for the lighter generations as is phenomenologically required (Barbieri and Nanopoulos 1980). Such perturbation may be due to radiative corrections because of heavy fermions (D, E) or due to relatively small vacuum expectation values of $\chi_{351'}$-antisymmetric representation. It is worth recalling here that precisely three families (or four at most) are needed to renormalize $m_b = m_\tau$ from superheavy energies to the observed value $m_b = 2{\cdot}5\, m_\tau$ at the bottomonium mass (Buras *et al* 1978; Barbieri and Nanopoulos 1980).

Finally (6a) and (6b) imply the relation

$$m_t : m_\tau = m_c : m_\mu = m_u : m_e \tag{7}$$

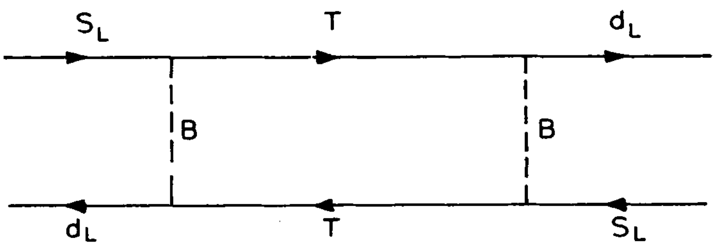
These relations will also get increasing corrections for the lighter generations. Assuming they are true for the second and third generations we get

$$m_t = (m_\tau / m_\mu) m_c \tag{8}$$

Taking into account finite renormalization effects this implies $m_t \approx 20\,\mathrm{GeV}$ for a mass of the charmed quark $m_c \approx 1{\cdot}5\,\mathrm{GeV}$ (Barbieri and Nanopoulos 1980).

Strangeness changing neutral currents

Usually ETC theories are plagued by strangeness changing neutral current effects due to single gauge-boson exchange, double gauge-boson exchange, pseudo-Goldstone boson exchange, etc. (Dimopoulos and Ellis 1981). As an example consider the following diagram to the K_L, K_S mass difference. In the figure s, d, T represents



the s-quark, d-quark and the technicolour quark respectively. B is an extended technicolour boson. It is important to note that according to our model the B-boson in the figure has to be the heaviest ETC boson which gives mass to the d-quark (e-family). In the case when we replace the B-boson by the weak boson and the T-quark by an u-quark there is GIM mechanism to cancel the leading piece, but in our case of B-boson, there is no such suppression. The effective four Fermi operator which leads to K_L, K_S mass difference can be written as (Farhi and Susskind 1981; Dimopoulos and Ellis 1980).

$$\frac{g^4}{32\pi^2 m_B^2} \bar{s}\gamma_\mu(1-\gamma_5)\mathrm{d}\bar{s}\gamma_\mu(1-\gamma_5)\mathrm{d} \tag{9}$$

here m_B is the mass of the gauge boson. To be consistent with the observed K_L,

K_S mass difference the coefficient in (9) must be less than $5 \times 10^{-13}\,\text{GeV}^{-2}$ (Gaillard and Lee 1974; Farhi and Susskind 1981) and so m_B/g^2 is greater than 80 TeV. Since $m_B \approx g\ \Lambda_E$ this becomes

$$\Lambda_{E/g} > 80\,\text{TeV}$$

In our type of models the scale of interaction Λ_E required to give one GeV quark mass is around 7 TeV (Farhi and Susskind 1981). Note in our model the B boson is that boson responsible for giving mass to the d-quark. Taking the mass of d-quark as around one MeV, this gives a value for $\Lambda_E \approx 200\,\text{TeV}$ which is consistent with the above limit if $g \approx 1$. It is hoped similar calculations for other strangeness violating processes also will be consistent with experimental limits.

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