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An improved approximate solution of Altarelli-Parisi equations

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Abstract. We have presented here an improved solution to Altarelli-Parisi equations, and it is found to be in good agreement with SLAC-MIT data up to $x \geqslant 0{\cdot}1$.

Keywords. Quantum chromodynamics; structure function; Altarelli-Parisi equation.

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1. Introduction

Recently, phenomenological analysis of an approximate form of QCD structure function was reported (Choudhury and Saikia 1987). It was obtained as an approximate solution of Altarelli-Parisi (AP) equations (Altarelli and Parisi 1977). In the present paper, we report an improved version of the QCD model and compare it with the SLAC-MIT data (Bodek *et al* 1979). We also investigate the higher twist (HT) effect (Abbott *et al* 1980; Aubert *et al* 1981; Bollini *et al* 1981; Godbole and Roy 1982; Eisele *et al* 1982; Aubert *et al* 1985; Choudhury and Misra 1987).

2. Formalism

2.1 *Approximate solutions of AP equations*

In our previous work (Choudhury and Saikia 1987, hereafter referred to as I) we had neglected the variation of $F_2^S(x,t)$, $F_2^{NS}(x,t)$ and $G(x,t)$ on the right hand sides of AP equations (Altarelli and Parisi 1977) and obtained:

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0) + (4/25)\ln(t/t_0)\cdot\Big[(3+4\ln(1-x))F^{NS}(x,t_0) + 2\int_x^1 \frac{dw}{(1-w)}\cdot\{(1+w^2)F_2^{NS}(x/w,t_0) - 2F_2^{NS}(x,t_0)\}\Big], \quad (1)$$

and

$$F_2^S(x,t) = F_2^S(x,t_0) + (4/25)\ln(t/t_0)\Big[(3+4\ln(1-x))F_2^S(x,t_0) + 2\int_x^1 \frac{dw}{(1-w)}\{(1+w^2)F_2^S(x/w,t_0) - 2F_2^S(x,t_0)\} + 6\int_x^1 dw(w^2+(1-w)^2)G(x/w,t_0)\Big]. \quad (2)$$

In deriving (1) and (2), we defined $t = \ln Q^2/\Lambda^2$, $t_0 = \ln Q_0^2/\Lambda^2$ and used four flavours so that the running coupling constant $\alpha(t)$ has the structure

$$\alpha(t)/3 = \frac{4\pi}{25t}. \tag{3}$$

Using the relations,

$$F_2(x, Q^2) = \tfrac{5}{18} F_2^S(x, Q^2) + \tfrac{3}{18} F_2^{NS}(x, Q^2), \tag{4}$$

we then obtain the following representation of QCD structure function

$$F_2(x, t) = F_2(x, t_0) + \ln(t/t_0) H(x, t_0), \tag{5}$$

which was the main result of I (Choudhury and Saikia 1987). Here,

$$H(x, t_0) = \tfrac{5}{18} H^S(x, t_0) + \tfrac{3}{18} H^{NS}(x, t_0), \tag{6}$$

where

$$H^S(x, t_0) = (3 + 4\ln(1-x))\cdot F_2^S(x, t_0) + 2\int_x^1 \frac{dw}{(1-w)}\{(1+w^2)F_2^S(x/w, t_0) - 2F_2^S(x, t_0)\} + 6\int_x^1 dw(w^2 + (1-w)^2)G(x/w, t_0), \tag{7}$$

and

$$H^{NS}(x, t_0) = (3 + 4\ln(1-x))F_2^{NS}(x, t_0) + 2\int_x^1 \frac{dw}{(1-w)}\{(1+w^2)F_2^{NS}(x/w, t_0) - 2F_2^{NS}(x, t_0)\}. \tag{8}$$

2.2 *Improved solution of AP equations*

In deriving (1) and (2) one needs to assume

$$\begin{aligned} F_2^{NS}(x, t) &\approx F_2^{NS}(x, t_0), \\ F_2^S(x, t) &\approx f_2^S(x, t_0), \\ G(x, t) &\approx G(x, t_0), \end{aligned} \tag{9}$$

on the right hand sides of AP equations.

We now demonstrate an alternative solution of AP equations without assuming (9). The AP equations have the standard structure

$$\frac{dF_2^{NS}}{dt}(x, t) = \frac{4}{25t}\left[(3 + 4\ln(1-x))\cdot F_2^{NS}(x, t) + 2\int_x^1 \frac{dw}{(1-w)}\{(1+w^2)F_2^{NS}(x/w, t) - 2F_2^{NS}(x, t)\}\right], \tag{10}$$

$$\frac{dF_2^S}{dt}(x, t) = \frac{4}{25t}\left[(3 + 4\ln(1-x))\cdot F_2^S(x, t)\right.$$

$$+2\int_x^1 \frac{dw}{(1-w)}\cdot\{F_2^S(x/w,t)\cdot(1+w^2)-2F_2^S(x,t)\}$$

$$+6\int_x^1 (w^2+(1-w)^2)\cdot G(x/w,t)\,dw\Bigg]. \tag{11}$$

We now assume that the x and t dependences of the structure functions are factorizable,

$$F_2^{NS}(x,t)=f^{NS}(x)h^{NS}(t), \tag{12}$$

$$F_2^S(x,t)=f^S(x)h^S(t), \tag{13}$$

and

$$G(x,t)=f^g(x)h^g(t), \tag{14}$$

so that (10) and (11) can be recast as

$$\frac{dF_2^{NS}}{dt}(x,t)-\frac{4}{25t}\Bigg[(3+4\ln(1-x))$$

$$+2\int_x^1 \frac{dw}{(1-w)}\left((1+w^2)\frac{f^{NS}(x/w)}{f^{NS}(x)}-2\right)\cdot F_2^{NS}(x,t)=0, \tag{15}$$

$$\frac{dF_2^S}{dt}(x,t)-\frac{4}{25t}\cdot\Bigg[3+4\ln(1-x)+2\int_x^1 \frac{dw}{(1-w)}\left((1+w^2)\frac{f^S(x/w)}{f^S(x)}-2\right)$$

$$+6\int_x^1 dw(w^2+(1-w)^2)\frac{f^g(x/w)h^g(t)}{f^s(x)h^s(t)}\Bigg]F_2^S(x,t)=0. \tag{16}$$

We assume that the t-dependent gluon evolution function is similar to that of the singlet case,

$$h^g(t)\approx h^S(t). \tag{17}$$

Defining

$$\int_x^1 \frac{dw}{(1-w)}\left((1+w^2)\frac{f^{NS}(x/w)}{f^{NS}(x)}-2\right)=I^{NS}(x), \tag{18}$$

$$\int_x^1 \frac{dw}{(1-w)}\left((1+w^2)\frac{f^S(x/w)}{f^S(x)}-2\right)=I^S(x), \tag{19}$$

and

$$\int_x^1 dw(w^2+(1-w)^2)\frac{f^g(x/w)}{f^S(x)}=g(x), \tag{20}$$

one has

$$\frac{dF_2^{NS}}{dt}(x,t)-\frac{4}{25t}(3+4\ln(1-x)+2I^{NS}(x))F_2^{NS}(x,t)=0, \tag{21}$$

and

$$\frac{dF_2^S}{dt}(x,t)-\frac{4}{25t}(3+4\ln(1-x)+2I^S(x)+6g(x))F_2^S(x,t)=0. \tag{22}$$

Equations (21) and (22) lead to the solutions (Hildebrand 1976)

$$F_2^{NS}(x,t) = F_2^{NS}(x,t_0)(t/t_0)(4/25)(3 + 4\ln(1-x) + 21^{NS}(x)), \tag{23}$$

and

$$F_2^{S}(x,t) = F_2^{S}(x,t_0)(t/t_0)(4/25)(3 + 4\ln(1-x) + 2I^{S}(x) + 6g(x)). \tag{24}$$

Using (4), we obtain the following representation of the structure function

$$F_2(x,t) = \tfrac{5}{18}F_2^{S}(x,t_0)(t/t_0)^{HS(x)} + \tfrac{3}{18}F_2^{NS}(x,t_0)(t/t_0)^{H^{NS}(x)}, \tag{25}$$

with

$$H^{NS}(x) = \tfrac{4}{25}(3 + 4\ln(1-x) + 2I^{NS}(x)) \tag{26}$$

and

$$H^{S}(x) = \tfrac{4}{25}(3 + 4\ln(1-x) + 2I^{S}(x) + 6g(x)) \tag{27}$$

which is our main result.

For quantitative analysis, we need to numerically evaluate $I^{NS}(x)$, $I^{S}(x)$ and $g(x)$ as defined in (18)–(20). Further, we also need the shapes of $F_2^{S}(x,t_0)$, $F_2^{NS}(x,t_0)$ and $G(x,t_0)$. The $F_2(x,Q^2)$ obtained from the approximate solution is compared with the $F_2(x,Q^2)$ obtained by starting with the same input but exactly solving the AP equations.

To do so we use the inputs of $F_2^{S}(x,t_0)$, $F_2^{NS}(x,t_0)$ and $G(x,t_0)$ is given by Gluck *et al* (1982). Using the approximate solutions developed in (25), we compare our prediction with that of Glück *et al* (1982). For HT effect we use the parametrization of Aubert *et al* (1985),

$$F_2(x,Q^2) = F_2^{QCD}(x,Q^2)\left(1 + \frac{\mu_4^2 x^{\alpha}}{(1-x)Q^2}\right) \tag{28}$$

where

$$\mu_4^2 = 1{\cdot}7\,\mathrm{GeV}^2 \text{ and } \alpha = 3{\cdot}1.$$

3. Results

Let us test the structure function (25) with SLAC-MIT data (Bodek *et al* 1979) and compare the prediction with that of Glück *et al* (1982). Since the crucial issue of discussion is the Q^2 dependence than the x dependence of $F_2(x,Q^2)$, we present $F_2(x,Q^2)$ as a function of Q^2 at representative x values viz $x = 0{\cdot}15, 0{\cdot}25, 0{\cdot}4, 0{\cdot}5, 0{\cdot}67$ and $0{\cdot}75$ covered by the data (figure 1(a–f)). Figures 1(a, b) represent $Q^2 < 4\,\mathrm{GeV}^2$ where the parametric equations of Glück *et al* (1982) for Q^2 evolution cannot be applied. The values with HT and without HT are represented by solid and dotted lines respectively. In figure 1(c–f), the dotted curves represent the predictions of the present formalism and the dashed ones represent those of Glück *et al* (1982). The difference between the two is negligibly small. It starts with zero difference at $4\,\mathrm{GeV}^2$ and slowly increases up to 3%. The prediction of Glück *et al* (1982) is above ours. With HT, our result is within 12% of the SLAC-MIT data in the range $1 \leqslant Q^2 \leqslant 16$ GeV^2. We also observe that while for the earlier version, the agreement was good for $x \geqslant 0{\cdot}25$, the present one works up to $x \geqslant 0{\cdot}1$. An additional advantage of our method is that one can extend it to $t < t_0$.

The problem of proper choice of Q_0^2 for QCD phenomenology still remains. There are various studies with $Q_0^2 = 1{\cdot}8$ (Buras and Gaemers 1978), 5 (Benvenuti *et al* 1989),

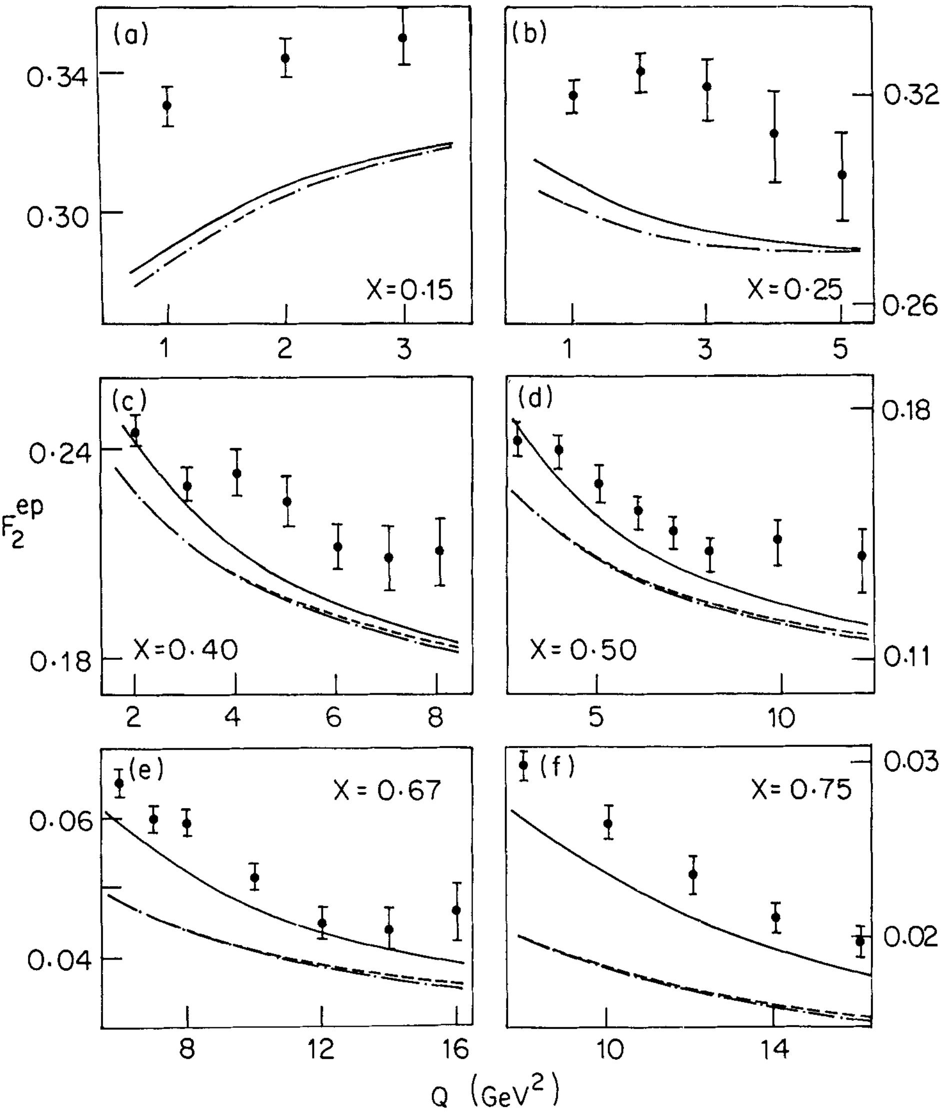

Figure 1. $F_2^{ep}(x, Q^2)$ vs Q^2 for $x = 0{\cdot}15$, $0{\cdot}25$, $0{\cdot}40$, $0{\cdot}50$, $0{\cdot}67$ and $0{\cdot}75$ respectively. Dotted curves represent the prediction without H T, while solid curves represent with H T. Dashed curves represent the prediction of Glück *et al* (1982). Data are taken from Bodek *et al* (1979).

10 (Diemoz *et al* 1988) and $30{\cdot}5\,\mathrm{GeV}^2$ (Abbott *et al* 1980). Q_0^2 being the boundary between perturbative and non-perturbative domain, one is tempted to think $Q_0^2 \sim 1\,\mathrm{GeV}^2$ as in some recent studies (Bednyakov 1984). Hence, it is important to find a universal criterion to determine Q_0^2 so that Q^2 evolution can be properly computed.

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