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Confinement of tensor gluons – a classical approach

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Abstract. We look at the confinement of tensor gluons ($f^{(c)}_{\mu\nu}$ field) in a strong gravity background and find that the strong gravity provides a trap for the confinement of colour waves of selected frequencies. We assume that the tensor $f^{(f)}_{\mu\nu}$ field (mediating quanta: tensor 2^+f-meson) satisfies Einstein-like equations with a cosmological constant. The colour field satisfy equations resembling Maxwell form of the linear theory of gravitation and see the effect of $f^{(f)}_{\mu\nu}$ field as playing the role of a medium having space dependent dielectric permeabilities. The solution of colour field equations resemble harmonic oscillator type wave functions with equispaced energy levels (no continuum) leading to confinement.

Keywords. Confinement; tensor gluons; strong gravity.

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1. Introduction

Recently a classical model for confining quarks, photons and gluons has been proposed (Jena and Pradhan 1981, 1984; Khare and Pradhan 1983; Khadkikar and Kumar 1987). We, in this paper, propose another mechanism and try to understand the confinement of quarks and gluons in a different way. The motivation behind such a classical approach is two fold. In conventional QCD, the dynamics of vector gluons is determined from an action modelled after Maxwell's theory of electromagnetism. However, it is known (Coleman and Smarr 1977) that in such sourceless non-abelian gauge theories there are no classical glueballs which otherwise would be an indication for the occurrence of confinement in the quantised theory. In our approach the confined modes are quantized and allow us to form colour singlet states of colour gluons called glueballs (Jaffe and Johnson 1976). Secondly in all classical models the dynamics of vector gluons are determined from equations patterned after Maxwell's equations of classical electrodynamics, in dielectric medium, with an assumed form of space dependent colour dielectric constant. In our approach we have an answer to this problem and guess that the nonlinear terms simulate a strong curvature which is described by a tensor field $f^{(f)}_{\mu\nu}$. The work related in this direction has been dealt in a separate paper.

In this paper we adopt the strong gravity formalism. The strong gravity approach to strong interaction is well known (Salam and Strathdee 1976; Sivaram and Sinha 1977, 1979) and the connection between Regge theory and strong gravity has been illucidated by Biswas and Das (1983). It has also been established that quarks with colour degrees of freedom play a fundamental role in the understanding of hadronic interaction. Hence colour aspects should also be incorporated in strong gravity formalism. To incorporate the colour scheme one introduces (Mielke 1980; Mielke and Scherzer 1980) in the space-time structure the group

$$G = GL(8,C)^f \otimes GL(8,C)^c$$
$$\supset U(4)^f_L \otimes U(4)^f_R \otimes U(4)^c_L \otimes U(4)^c_R.$$

as gauge group. Here the superscripts f and c denote flavor and colour degree of freedom, L and R correspond to left and right helicities of fermion respectively. One then constructs a gauge invariant metric

$$f_{\mu\nu} = \tfrac{1}{2}(f^{(f)}_{\mu\nu} \oplus f^{(c)}_{\mu\nu})$$

and with this $f_{\mu\nu}$ one writes down the Einstein type field equation for the solitonic solutions of classical field equations are then viewed as hadrons. Black soliton mass formula for the Kerr-Newmann solution results in Gell-Mann-Okubo type mass formula for hadrons. In the so called strong gravity formalism the role of colour is not explicitly understood as the hadrons are identified as black holes. In this paper we investigate the role of the $f^{(f)}_{\mu\nu}$ field (flavor field) and the $f^{(c)}_{\mu\nu}$ (colour field) separately to understand the coupling between the two fields. In §2 we write the colour field equations like Maxwell form of equations in the linear theory of gravitation. Its coupling with a given gravitational background is done in an analogous way the Maxwell's equation of classical electrodynamics are written in three dimensional form in a gravitational background. In §3 we solve the Einstein-like field equations of $f^{(f)}_{\mu\nu}$ field. We obtain the solutions of colour fields in the strong gravity background in §4. In §5 we discuss the construction of glueballs states for vector gluons and end up with a concluding section.

2. Maxwell form of the linear theory of gravitation for colour

We introduce two dyad fields $E^{(c)}_{ij} \equiv \tilde{E}$ and $B^{(c)}_{ij} \equiv \tilde{B}$ in analogy with the **E** and **B** fields of electromagnetism as

$$E^{(c)}_{ij} = -W_{0i0j} \tag{2}$$

$$B^{(c)}_{ij} = -\tfrac{1}{2}\varepsilon_{imn} W_{mn0j} \tag{3}$$

$$i, m, n = 1, 2, 3$$

where $W_{\mu\nu\rho\sigma}$ is the Weyl conformal tensor. We have taken only the spatial components of $W_{\mu\nu\rho\sigma}$. In empty space the Riemannian tensor $R_{\mu\nu\rho\sigma} = W_{\mu\nu\rho\sigma}$ and those dyad fields $\tilde{E}$ and $\tilde{B}$ satisfy Maxwell-like equations (Campbell and Morgan 1976; Good 1971)

$$\nabla \cdot \tilde{E} = 0 \tag{4a}$$

$$\nabla \cdot \tilde{B} = 0 \tag{4b}$$

$$\nabla \times \tilde{B} = \frac{\partial \tilde{E}}{\partial t} \tag{4c}$$

$$\nabla \times \tilde{E} = -\frac{\partial \tilde{B}}{\partial t}. \tag{4d}$$

In component form they read

$$\partial_i E_{ij}^{(c)} = 0 \quad (5a)$$

$$\partial_i B_{ij}^{(c)} = 0 \quad (5b)$$

$$\varepsilon_{ikl}\partial_k B_{ij}^{(c)} = \frac{\partial E_{ij}^{(c)}}{\partial t} \quad (5c)$$

$$\varepsilon_{ikl}\partial_k E_{ij}^{(c)} = \frac{\partial B_{ij}}{\partial t}\,. \quad (5d)$$

It should be noted that $E_{ij}^{(c)}$ and $B_{ij}^{(c)}$ are traceless 3×3 matrices and are dyad fields. Equations (5) correspond to massless spin 2 objects with an internal quantum number which in our case is colour. We call these as tensor gluons in analogy with vector gluons of QCD. Tensor gluons have already been discussed by many authors in connection with confinement of quark-gluon system (Mielke 1980; Mielke and Scherzer 1980).

It is well known that (Landau and Lifshitz 1975) Maxwell equations of electrodynamics can be written in a given gravitational background in three dimensional form. We adopt an analogous approach for $\tilde{E}$ and $\tilde{B}$ fields for $f_{\mu\nu}^{(f)}$ background. Let the $f_{\mu\nu}$ field be given by (henceforth we omit the superscript f)

$$\begin{aligned} ds^2 &= f_{\mu\nu}\,dx^\mu\,dx^\nu \\ &= f_{00}(dx^0 - f_i\,dx^i)^2 - dl^2 \end{aligned} \quad (6)$$

where

$$f_i = -f_{0i}/f_{00} \quad (7)$$

$$dl^2 = \gamma_{ij}\,dx^i\,dx^j \quad (8)$$

$$\gamma_{ij} = (-f_{ij} + f_{0i}f_{0j}/f_{00}). \quad (9)$$

With the assumption that the dyad fields $\tilde{E}$ and $\tilde{B}$ transform in the same way as the Maxwell equations of electrodynamics (Landau and Lifshitz 1975) equation (4) written in terms of the metric γ_{ij} read

$$\operatorname{div}\tilde{B} = 0 \quad (10a)$$

$$\operatorname{curl}\tilde{E} = -\frac{1}{(\gamma)^{1/2}}\frac{\partial}{\partial t}((\gamma^{1/2}\tilde{B}) \quad (10b)$$

$$\operatorname{div}\tilde{D} = 0 \quad (10c)$$

$$\operatorname{curl}\tilde{H} = \frac{1}{(\gamma)^{1/2}}\frac{\partial}{\partial t}((\gamma)^{1/2}\tilde{D}) \quad (10d)$$

where

$$\tilde{D} = \frac{\tilde{E}}{(f_{00})^{1/2}},\quad \tilde{B} = \frac{\tilde{H}}{(f_{00})^{1/2}} \quad (11)$$

$$(\operatorname{curl}\mathbf{a})^i = \frac{1}{2(\gamma)^{1/2}}e^{ijk}\left(\frac{\partial a_k}{\partial x^j} - \frac{\partial a_j}{\partial x^k}\right) \quad (12)$$

$$\operatorname{div} \mathbf{a} = \frac{1}{(\gamma)^{1/2}} \frac{\partial}{\partial x^i} (\sqrt{\gamma} a^i). \tag{13}$$

In (11) we assume $f_i = 0$ and γ is the determinant calculated from quantities γ_{ij} and is given by

$$-f = \gamma f_{00} \tag{14}$$

where $f = \det f_{ij}$. Using (11)–(14), (10) can be converted in terms of flat space curl and divergence as (in static case, γ independent of time)

$$\nabla \cdot ((\gamma/f_{00})^{1/2} \tilde{H}) = 0 \tag{15a}$$

$$\nabla \times \vec{\mathbf{E}} = -\frac{\partial}{\partial t} ((\gamma/f_{00})^{1/2} \tilde{H}) \tag{15b}$$

$$\nabla \cdot ((\gamma/f_{00})^{1/2} \tilde{E}) = 0 \tag{15c}$$

$$\nabla \times \tilde{H} = \frac{\partial}{\partial t} ((\gamma/f_{00})^{1/2} \tilde{E}). \tag{15d}$$

Thus we see that with respect to its effect on the colour field a static strong gravitational field plays the role of a medium with electric and magnetic permeabilities (in colour space) with

$$\varepsilon = \mu = (\gamma/f_{00})^{1/2}$$

$$\tilde{B} = \tilde{H}/\varepsilon, \quad \tilde{D} = \tilde{E}/\varepsilon. \tag{16}$$

We take (15) as defining equations of tensor gluon field in the strong gravitational background.

3. Solution of Einstein field equations

We take the Lagrangian of $f_{\mu\nu}$ field in the form

$$L = \frac{(-f)^{1/2}}{2k_f} [R(f) - 2\Lambda_f + c_f \phi_{,\alpha} \phi_{,\beta} f^{\alpha\beta}]. \tag{17}$$

Here $R(f)$ is Ricci scalar constructed from $f_{\mu\nu}$ and Λ_f is the cosmological constant. K_f is the analogous of Newtonian constant for strong gravity, $K_f = 8\pi G_f/c^4$ where $G_f = 10^{38} G_N$ and $\Lambda_f \sim 10^{28}\,\mathrm{cm}^{-2}$. The scalar field ϕ represents massless quark. The occurrence of Λ_f suggests that the space is not empty and it is non-geometrical in origin and all the stresses have been rounded up in Λ_f. A concept of describing extended particles by means of strong internal curvature has already been suggested by Lanczos (1957). It is known that quark type fields satisfy Heisenberg-Pauli-Weyl (HPW) nonlinear spinor equation (Weyl 1950; Heisenberg 1966). The equation can be simulated from scalar quarks with nonlinear interaction so that the wave equation for scalar quarks resemble very much the squared version of HPW equation. Furthermore the origin of nonlinearity in HPW equation is from the coupling of spin with the space-time background so that (17) represents a description where the coupling is neglected. This admirably simplifies the solution of (17) for the massless quark case. We have taken only a single scalar field because the inclusion of other scale fields will not change the results of our calculation as will be seen below. In our approach we take the coupling of quark with the background in a different way. First we solve (17) for massless case and use the solution in massless Dirac equation in the same way as in §2 on the assumption that the massless Dirac equation can be cast in the Maxwell form. This has already been carried out in a separate paper.

On carrying out variations of (17) with respect to $f_{\mu\nu}$ and ϕ, we get the set of field equation

$$R^{(f)}_{\mu\nu} - \tfrac{1}{2}R(f)f_{\mu\nu} + \Lambda_f f_{\mu\nu} = -k_f c_f[\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{2}f_{\mu\nu}\phi_{,\alpha}\phi^{,\alpha}] \tag{18}$$

and

$$f^{\mu\nu}\phi_{;\mu\nu} = 0 \tag{19}$$

where semicolon represents a convariant differentiation and

$$\phi^{;\alpha}_{;\alpha} = \frac{1}{(-f)^{1/2}}\frac{\partial}{\partial x_\alpha}\left(f^{\alpha\beta}(-f)^{1/2}\frac{\partial}{\partial x_\beta}\right) \tag{20}$$

represent the convariant d'Alembertion. Let us assume a conformal solution for $f_{\mu\nu}$

$$f_{\mu\nu} = \exp(2\lambda)\cdot\eta_{\mu\nu} \tag{21}$$

$\eta_{\mu\nu}$ being the flat (Lorentz) metric and λ is a function of space coordinate. Introducing tensor

$$E_{\mu\nu} = R_{\mu\nu} - \tfrac{1}{2}R(f)\cdot f_{\mu\nu} + \Lambda_f f_{\mu\nu} \tag{22}$$

(18) can be written as

$$E_{\mu\nu} = -k_f c_f[\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{2}f_{\mu\nu}\phi_{,\alpha}\phi_{,\beta}f^{\alpha\beta}]. \tag{23}$$

With the metric given in (21), we calculate E_{00}, E_{11}, E_{22}, E_{33} from (22) and (23) and adding the four sets of equations we get

$$2\lambda'' + \frac{4\lambda'}{r} + 4(\lambda')^2 + 2\Lambda_f \exp\cdot(2\lambda) = 0. \tag{24}$$

If we choose a mass term in (17), the RHS of (24) will contain term $\sim m^2\phi^2$ dependent upon γ, θ, φ through the solution of ϕ equation (19). Whereas the LHS of (24) is function of r only. Hence we get an inconsistency. This is a common problem faced in solving Einstein equations. Avoidance of mass term and other nonlinear terms remove this inconsistency. On making the substitution

$$F = r\exp\cdot(2\lambda) \tag{25}$$

(24) reduces to

$$F'' + 2\Lambda_f F^2/r = 0. \tag{26}$$

In the absence of cosmological term

$$F'' = 0. \tag{27}$$

This has the general solution

$$F = ar + b. \tag{28}$$

Now we look at the solution at $r \to 0$ so that $\exp(2\lambda) \to 1$. This is a confinement ansatz so that the solution does not blow up at origin and the quarks are almost free (see (19)). So we put $a = 1$ and $b = 0$ in (28). Assuming this limiting behaviour $F \to r$ when $r \to 0$ in (26) we get the solution

$$F = r - \Lambda_f r^3/3 \tag{29}$$

and

$$\exp(2\lambda) = 1 - \Lambda_f r^2/3. \tag{30}$$

In normal gravitational theory $(1/\Lambda)^{1/2}$ is related to the size of the universe and plays a negligible role in local problems, whereas $(1/\Lambda_f)^{1/2}$, in our case of strong gravity, will be related to the size of a typical hadron considered as a microcosmos.

We will find in subsequent discussions that $\exp(2\lambda)$ acts like a dielectric constant for Maxwell-like fields. In this respect our approach is a breakthrough and may be looked upon seriously. We now proceed with (30) to find solutions of $\tilde{E}$ and $\tilde{B}$ fields.

4. Solution of colour field equations

We write (15a) with $\varepsilon = (\gamma/f_{00})^{1/2}$ as

$$\nabla\cdot(\varepsilon\tilde{E}) = \varepsilon\nabla\cdot\tilde{E} + \frac{\partial\varepsilon/\partial r}{r}\mathbf{r}\cdot\tilde{E} = 0. \tag{31}$$

Assuming transversality of colour waves, i.e., $\vec{\mathbf{r}}\cdot\tilde{E} = 0$ we get

$$\nabla\cdot\tilde{E} = 0. \tag{32}$$

Using (15d), (15b) can be cast into the form

$$\nabla\times\nabla\times\tilde{E} - \frac{\partial\varepsilon/\partial r}{\varepsilon r}(\mathbf{r}\times\nabla\times\tilde{E}) = -\frac{\partial^2}{\partial t^2}(\varepsilon^2\tilde{E}). \tag{33}$$

Assuming $\tilde{E} = \tilde{e}_0(r)\exp(-i\omega t)$, we write (33) as

$$\nabla^2\tilde{e}_0 - \frac{\partial\varepsilon/\partial r}{\varepsilon}\frac{\partial\tilde{e}_0}{\partial r} + \omega^2\varepsilon^2\tilde{e}_0 = 0. \tag{34}$$

To obtain (34) we have assumed $\nabla\cdot\tilde{E} = 0$ and this condition also simplifies

$$(\mathbf{r}\times\nabla\times\tilde{e}_0) \text{ to } -r\frac{\partial\tilde{e}_0}{\partial r}.$$

We write $\tilde{e}_0$ as dyad product

$$\tilde{e}_0 = \mathbf{AM}. \tag{35}$$

where $\mathbf{A}$ is a constant vector. In this case (34) reduces to

$$\nabla^2 \mathbf{M} - \frac{\partial\varepsilon/\partial r}{\varepsilon}\frac{\partial \mathbf{M}}{\partial r} + \omega^2\varepsilon^2\mathbf{M} = 0. \tag{36}$$

For a given dyad $\tilde{e}_0$ one can always find a vector $\mathbf{M}$ for a constant non null vector $\mathbf{A}$ using the property $\mathbf{A}\cdot\tilde{e}_0 = \mathbf{A}\cdot(\mathbf{AM}) = \mathbf{M}(\mathbf{A}\cdot\mathbf{A}) = \mathbf{M}A^2$ so that $\mathbf{M} = \mathbf{A}\cdot\tilde{e}_0/A^2$. The solution of (36) is now carried out in standard way (Stratton 1941). It is given by

$$\mathbf{M} = \nabla \times (\mathbf{r}u) \tag{37}$$

where u satisfies the radial equation

$$\nabla^2 u(r,\theta,\varphi) - \frac{\partial\varepsilon/\partial r}{\varepsilon}\cdot\frac{\partial u(r,\theta,\varphi)}{\partial r} + \omega^2\varepsilon^2 u(r,\theta,\varphi) = 0. \tag{38}$$

Knowing $\mathbf{M}$ one calculates $\tilde{E}$ by (25) and $\tilde{B}$ from

$$\tilde{B} = -\frac{i}{\omega}\nabla \times \tilde{E}. \tag{39}$$

Putting $u(r,\theta\varphi) = R_l(r)\, Y_{lm}(\theta,\varphi)$ the radial part R_L satisfies the equation

$$R_l'' + \left(\frac{2}{r} - \frac{\varepsilon'}{\varepsilon}\right)R_l' + \left[\omega^2\varepsilon^2 - \frac{l(l+1)}{r^2}\right]R_l = 0. \tag{40}$$

Using (30) and (14) we get

$$\varepsilon = (\gamma/f_{00})^{1/2} = (1 - \Lambda_f r^2/3)$$

and (40) reduces to

$$R_l'' + \left(\frac{2}{r} - \frac{2\Lambda_f r}{(1-\Lambda_f r^2/3)}\right)R_l' + \left[\omega^2(1-\Lambda_f r^2/3)^2 - \frac{l(l+1)}{r^2}\right]R_l = 0. \tag{41}$$

As the form (30) is valid around origin, we solve (41) for $(\Lambda_f)^{1/2} r \ll 1$ regions. Making the substitution

$$R_l = \exp(-m^2r^2/8)u_l \tag{42}$$

where

$$m^2 = \tfrac{4}{3}\Lambda_f \tag{43}$$

and neglecting O (r^3) term, we get

$$u_l'' + \frac{2}{r}u_l' + \left(\omega^2 - \tfrac{1}{16}m^4r^2 - \tfrac{1}{2}m^2\omega^2r^2 - \frac{l(l+1)}{r^2}\right)u_l = 0. \tag{44}$$

It can be cast into harmonic oscillator form with the substitution

$$\lambda^2 = \frac{m^4}{16} + \tfrac{1}{2}m^2\omega^2 \tag{45a}$$

$$\omega^2/2\lambda = \mu \tag{45b}$$

$$u_l = \chi_l(r)/r \tag{45c}$$

and we get

$$\frac{d^2\chi_l}{dr^2} + \left(\omega^2 - \lambda^2 r^2 - \frac{l(l+1)}{r^2}\right)\chi_l = 0. \tag{46}$$

The solution regular at the origin is given by

$$\chi_l \sim r^{l+1} \exp\left(-\frac{\lambda}{2}r^2\right) {}_1F_1\,[\tfrac{1}{2}(l+\tfrac{3}{2}-\mu), l+\tfrac{3}{2};\ \lambda r^2]. \tag{47}$$

Since

$${}_1F_1(a.c,z) \rightarrow \frac{\Gamma(c)}{\Gamma(a)} \exp(Z) Z^{a-c},$$

for large Z and becomes divergent for $Z = \lambda r^2$, we put the parameter $a = -n_r$ with $n_r = 0, 1, 2, \ldots$ thus transforming the series into polynomial of degree n_r Hence

$$\tfrac{1}{2}(l + \tfrac{3}{2} - \mu) = -n_r$$

or,

$$\mu = (2n_r + l + \tfrac{3}{2}). \tag{48}$$

The solution is then

$$R_l \sim r^l \exp\cdot\left(-\frac{\lambda}{2}r^2 - m^2 r^2/8\right) {}_1F_1(-n_r, -2n_r + \mu;\ \lambda r^2). \tag{49}$$

Thus we see that energy spectrum are quantised having no continuum spectrum. Further as n_r increases the states characterised by R_l become closer and closer to the centre i.e., higher energy states are closer to the centre than the lower ones. For an easy insight to the confinement aspect we write (48) and (49) for the case $\omega^2(1 - \frac{1}{2}m^2r^2) \simeq \omega^2$ in (41). The result is

$$\omega_\Lambda = \frac{m^2}{2\sqrt{2}}(\Lambda + \tfrac{3}{2})^{1/2}; \quad \Lambda = 2n_r + l \tag{50}$$

and

$$R_l = C_1 r^l \exp\cdot(-m^2 r^2/4)_1 F_1(-n_r, -2n_r + \mu;\ \lambda r^2). \tag{51}$$

Thus we see that $\tilde{E}$ and $\tilde{B}$ fields are mostly confined in the region $(\Lambda_f)^{1/2} r \ll 1$ i.e., for $r \ll 10^{-14}$ cm for $\Lambda_f \sim 10^{28}\,\mathrm{cm}^{-2}$. Thus we see that the strong gravity provides a trap for the confinement of colour waves. The region $(\Lambda_f)^{1/2} r \gg 1$ is not relevant to confinement problem. However, it may be interesting to look at the solution for $(\Lambda_f)^2\ r \gg 1$.

5. Construction of glueball states

Colour objects are not seen free in nature. This suggests that apart from $q\bar{q}$ and qqq states one expects the colour singlet states of colour gluons, called glueballs. We consider here only the case for vector gluons. We assume that vector gluons satisfy Maxwell-like equations

$$\nabla\cdot(\varepsilon\mathbf{E}) = 0 \tag{52a}$$

$$\nabla\cdot\mathbf{B} = 0 \tag{52b}$$

$$\nabla\times\mathbf{H} = \partial D/\partial t \tag{52c}$$

$$\nabla\times\mathbf{E} = -\partial\mathbf{B}/\partial t \tag{52d}$$

in strong gravity background as in previous sections. We express the field strength in terms of the quasi-gluon potential $(\mathbf{A}, \varphi)$ as Khadkikar and Kumar 1987)

$$\mathbf{E} = -\frac{\partial\mathbf{A}}{\partial t} - \nabla\varphi$$

$$\mathbf{B} = \nabla\times\mathbf{A}.$$

Assuming a time variation of the field as $\exp(-i\omega t)$ we write (52c) as

$$\nabla\times(\mathbf{B}/\varepsilon) = \frac{\partial}{\partial t}(\varepsilon\mathbf{E})$$

because of the fact that $\varepsilon = \mu$ and $\mathbf{D} = \varepsilon\mathbf{E}$, $\mathbf{H} = \mathbf{B}/\varepsilon$ [see (15)]. Using (53) we get

$$\nabla\times\nabla\times\mathbf{A} = \frac{\partial\varepsilon/\partial r}{\varepsilon r}(\mathbf{r}\times\nabla\times\mathbf{A}) = -\ \varepsilon^2\frac{\partial^2\mathbf{A}}{\partial t^2} - \varepsilon^2\nabla\varphi$$

or

$$\nabla^2\mathbf{A} + \frac{\partial\varepsilon/\partial r}{\varepsilon r}(\mathbf{r}\times\nabla\times\mathbf{A}) + \omega^2\varepsilon^2\mathbf{A} = \nabla(\nabla\cdot\mathbf{A} - i\omega\varepsilon^2\nabla\varphi). \tag{55}$$

Because of the spatial dependence of the $\varepsilon(r)$ (55) cannot be homogeneous for $\mathbf{A}$ as in bag model cavity eigenmodes. The gauge condition enforced is

$$\nabla\cdot\mathbf{A} - i\omega\varepsilon^2\nabla\varphi = 0. \tag{56}$$

We have already seen that the metric is Lorentzian at $r\to 0$ and hence Lorentz condition is satisfied in this region. Equation (56) supports the fact. To define the eigenmodes we work in the region $r^2\to 3/\Lambda_f$ so that

$$\nabla\cdot\mathbf{A} = 0. \tag{57}$$

Then in this region the transverse eigenmodes can be defined as

$$\mathbf{A}^{\mathrm{TE}} = \mathbf{L}\psi_{nlm} \tag{58}$$

and

$$\mathbf{A}^{\mathrm{TM}} = \nabla\times\mathbf{L}\psi_{nlm} \tag{59}$$

where TE and TM represent the transverse electric and magnetic modes, L is the angular momentum operator. ψ_{nlm} is the solution of scalar part of wave equation

$$\nabla^2 \mathbf{A} - \frac{\partial\varepsilon/\partial r}{\varepsilon}\frac{\partial \mathbf{A}}{\partial t} + \omega^2\varepsilon^2\mathbf{A} = 0. \tag{60}$$

Construction of glueball states can now be carried out along the lines used by Khadkikar and Kumar (1987). The structure of (36) and (60) suggests that the construction of glueball states for tensor gluons can also be carried out in analogous fashion.

6. Discussion

The way in which the confinement is achieved in our model is very similar to the colour dielectric models proposed by various authors (Jena and Pradhan 1981, 1984; Khadkikar and Kumar 1987). The nonlinear gluon field equations can be cast into Maxwell-like form in dielectric medium. The nonlinear terms in the field equation are treated as a super-current $J_\mu \sim \theta_{\mu\nu}A_\nu$ analogous to Ginzburg-Landau theory of superconductivity. The form of J_μ is so chosen (Khadkikar and Kumar 1987) that the resulting quasi gluon field equation assumed the form (52) as if the nonlinear terms simulate the effect of a dielectric medium having space dependent permeabilities. Moreover there is vacuum polarisation which can be treated classically by considering the permeabilities as field quantities (Dicke 1957). Looking back to the modification of Coulomb's law in dielectric medium, we can interprete the appearance of space dependent dielectric constant as a scaling transformation such that $r^2 \to \varepsilon(r)r^2$, i.e; $\eta_{ij} \to \varepsilon\eta_{ij}$. We have now a curved space-time and ε is treated as scalar or tensor fields depending upon the model. The behaviour of non-abelian gluon can now be described by an abelian quasi gluon moving in the curved space-time. The curved space-time takes into consideration the effects of nonlinear terms and vacuum polarisation. We have developed a model (Biswas and Kumar 1988) along these lines considering $\varepsilon(r)$ as a scalar field. In the model the field equations corresponding to quark field and gluon field are simulated from a single Lagrangian with the field equations like (15) and with $\varepsilon(r) \simeq 1 - \Lambda r^2$. The field equation corresponding to quark is analogous to Maxwell-like equation with the identification $\varphi \equiv \mathbf{E}$; $\chi = i\mathbf{B}$. It may be pointed out that the condition $\varepsilon(r)\,\mu(r) = 1$ is not satisfied because the nonlinear terms contribute also to $\varepsilon(r)$ and $\mu(r)$ thus destroying the property of polarized vacuum. Some subtle questions regarding this approach is discussed in Dicke's paper.

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