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Dispersion of electromagnetic waves in the presence of magnetic monopoles of electron mass in a uniform magnetic field

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Abstract. The dispersion relation of electromagnetic waves in the presence of magnetic monopoles of electron mass in a uniform magnetic field is obtained. The waves of the frequency ω in the range $\omega_{p_i} < \Omega_i < \omega < \Omega_e < \omega_{p_e}$ are analysed. It is shown that the monopole charges lead to observable effects. Finally, the results are applied to a typical pulsar.

Keywords. Magnetic monopoles; electromagnetic waves; pulsars; dispersion relation; dispersion measure; phase shift.

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1. Introduction

Ever since the magnetic field lines were identified and found to originate at two points (poles), the existence of magnetic monopoles (henceforth referred to as monopoles) has been speculated. On the basis of charge quantization Dirac argued that monopoles should be present (Jackson 1975) and which was further supported by Saha (1936, 1949). A hypothetical monopole passing through a superconducting loop also leads to the results obtained by Dirac (Goldhaber 1983).

In the presence of dually charged particle having both electric and monopole charge, the Maxwell's equations become symmetric and the charge quantization leads to units of magnetic monopole values for every unit of electric charge (Schwinger 1969). The monopole charge on a dually charged particle need not be an integral multiple of a unit electric charge e. The particle can posses the monopole charge $g = (n/2)g_d$ with $g_d = e/\alpha$ where α is the fine structure constant: n is an integer: e is the electronic charge. Monopole charge density, electric charge density and their respective currents can be expressed as components along two orthogonal axis using duality transformation. The angle ξ (Jackson 1975) is with respect to charge axis and it is real. But to account for the limits on the monopole fluxes set by experiments, we believe this angle ξ is very small but finite: which is proportional to the monopole flux. We assume that the universe is oriented along the axis having finite angle with respect to charge density; there is a finite density of monopoles. Though the charge quantization does not put a limit on the masses, the gauge theories do not expect monopoles of masses less than TeV but greater than 10^{16} GeV.

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The search for classical monopoles has been attempted at high energy accelerators in cosmic rays and bulk matters (Loh 1983). The most significant cosmic ray experiments were by Bartlett *et al* (1982) and Cabrera (1983). These experiments placed upper limit on the monopole fluxes F to be less than $10^{-10}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$. The limit obtained theoretically by Salpeter *et al* (1982), is $10^{-15}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ which is less than the above values but higher than $10^{-16}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$ obtained by Parker (1971). We believe that extremely high magnetic fields can produce monopoles and the pulsars can be sources of galactic monopoles.

In this paper, we assume a uniform and homogeneous plasma of electron density N_0 in the presence of a uniform magnetic field $\mathbf{B}_0$. This plasma contains a small number of monopoles of electron mass and moving with velocity $v_0 \ll c$. These are classical monopoles i.e. original Dirac monopoles and has density N_1.

In the presence of uniform magnetic field, the monopoles acquire finite, sometimes relativistic velocities depending on the magnitude of the magnetic field. As discussed in § 3, we have neglected the associated effects; (a) the energy exchange due to instability mechanism; (b) the effect on the wave propagation. The former effect will be useful to assess the magnitude of energy exchange while latter is responsible for the effect discussed in this paper.

We also assume $n = 1$ and an elementary charge to be of an electron. In that case, monopoles have charge $g = 3{\cdot}39 \times 10^{-8}$ CGS units and electric charge e. The oppositely charged heavy particles are assumed to provide the charge neutralising background. When the monopoles have finite velocities, it is shown that the refractive index becomes complex. Finally, we have analyzed the related aspects and the results are discussed with respect to a typical example like a pulsar.

2. Dispersion relation

The equation of motion of electrons of mass m and charge $-e$, of density N_0, in the presence of a uniform magnetic field $\mathbf{B}_0$ along z axis of the right hand coordinate system in the CGS units is given by

$$m\frac{\mathrm{d}V}{\mathrm{d}t} = -e\left(\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c}\right) \tag{1a}$$

and the equation of motion of monopoles of electron mass, electron charge $-e$ and monopole charge $-g$ of density $N_1 (N_1 \ll N_0)$ moving with velocity $\mathbf{V}_0 (11\mathbf{B}_0)$ is given by

$$m\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = -e\left(\mathbf{E} + \frac{\mathbf{V}\times\mathbf{B}}{c}\right) - g\left(\mathbf{B} - \frac{\mathbf{V}\times\mathbf{E}}{c}\right) \tag{1b}$$

In (1a) and (1b), $\mathbf{E}(E_x, E_y, 0)$ is the electric field, $\mathbf{B}(B_x, B_y, B_0)$ is the magnetic field and $\mathbf{V}(V_x, V_y, V_0)$ is the velocity. The quantities with subscripts x, y are associated with electromagnetic waves propagating along the magnetic field. We linearize (1a) and (1b) using the perturbation approach and treat perturbations proportional to $\exp i(\omega t - kz)$ where $i = (-1)^{1/2}$. The perturbed velocities written as $V_{\pm}^{0,1} =$

$V_x^{0,1} \pm iV_y^{0,1}$ are found to be for (1a)

$$V_\pm^0 = \frac{ieE_\pm}{m(\omega \mp \Omega)} \tag{2a}$$

and for (1b) are

$$V_\pm^1 = \frac{ie}{m}\left\{\frac{E_\pm(1 \mp i\eta\beta) + B_\pm(\eta \pm i\beta)}{(\omega - kV_0 \mp \Omega)}\right\} \tag{2b}$$

Here $\Omega = eB_0/mc$ is electron gyrofrequency; $\eta = g/e$; $\beta = V_0/c$; $E_\pm = E_x \pm iE_y$ and $B_\pm = B_x \pm iB_y$. k is the wave number ω is corresponding frequency.

The Maxwell's equations

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} - \frac{4\pi}{c}\mathbf{J}_m$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J}_e + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} \tag{3}$$

when simplified for the waves of the form $\exp i(\omega t - kz)$ give

$$\mathbf{E}_\pm + \frac{ick}{\omega}\mathbf{B}_\pm = \frac{4\pi i}{\omega}\mathbf{J}_{e\pm},$$

$$\mathbf{B}_\pm - \frac{ick}{\omega}\mathbf{E}_\pm = \frac{4\pi i}{\omega}\mathbf{J}_{m\pm}. \tag{4}$$

The currents for the electric charge component is given by

$$\mathbf{J}_{e\pm} = -N_0 e\mathbf{V}_\pm^0 - N_1 e\mathbf{V}_\pm^1, \tag{5}$$

and for the monopole charge component is

$$\mathbf{J}_{m\pm} = -N_1 g\mathbf{V}_\pm^1. \tag{6}$$

Equations (2), (4), (5) and (6) give for $\mathbf{E}_+$ and $\mathbf{B}_+$ as

$$\begin{vmatrix} \mathscr{E}_{11}^+ & \mathscr{E}_{12}^+ \\ \mathscr{E}_{21}^+ & \mathscr{E}_{22}^+ \end{vmatrix} \begin{vmatrix} \mathbf{E}_+ \\ \mathbf{B}_+ \end{vmatrix} = 0, \tag{7}$$

where

$$\mathscr{E}_{11}^+ = 1 - A_0 - A_1(1 - i\eta\beta),$$

$$\mathscr{E}_{12}^+ = \frac{ick}{\omega} - A_1(\eta + i\beta),$$

$$\mathscr{E}_{21}^+ = -\frac{ick}{\omega} - A_1\eta(1 - i\eta\beta),$$

$$\mathscr{E}_{22}^+ = 1 - A_1\eta(\eta + i\beta),$$

$$A_0 = \frac{\omega_{\rho o}^2}{\omega(\omega - \Omega)}; \quad A_1 = \frac{\omega_{\rho_1}^2}{\omega(\omega - kV_0 - \Omega)}. \tag{8}$$

Here ω_{p_0} and ω_{p_1} respectively are plasma frequencies corresponding to N_0 and N_1.
Similarly, the relation for $\mathbf{E}_-$ and $\mathbf{B}_-$ is found to be

$$\begin{vmatrix} \mathscr{E}_{11}^- & \mathscr{E}_{12}^- \\ \mathscr{E}_{21}^- & \mathscr{E}_{22}^- \end{vmatrix} \begin{vmatrix} \mathbf{E}_- \\ \mathbf{B}_- \end{vmatrix} = 0, \tag{9}$$

where

$$\mathscr{E}_{11}^- = 1 - D_0 - D_1(1 + i\eta\beta),$$

$$\mathscr{E}_{12}^- = -\frac{ick}{\omega} - D_1(\eta - i\beta),$$

$$\mathscr{E}_{21}^- = \frac{ick}{\omega} - D_1\eta(1 + i\eta\beta),$$

$$\mathscr{E}_{22}^- = 1 - D_1\eta(\eta - i\beta), \tag{10}$$

with

$$D_0 = \frac{\omega_{p_0}^2}{\omega(\omega + \Omega)}; \quad D_1 = \frac{\omega_{p_1}^2}{\omega(\omega - kV_0 + \Omega)}.$$

For the waves of finite amplitudes, i.e. when $E_\pm \neq 0$ and $B_\pm \neq 0$ the determinant in (7) and (9) should vanish, giving the condition

$$\mathscr{E}_{11}^\pm \mathscr{E}_{22}^\pm - \mathscr{E}_{12}^\pm \mathscr{E}_{21}^\pm = 0. \tag{11}$$

The relation (11) differs from the relation discussed in the plasma physics books (e.g. Krall and Trivelpiece 1973). Here we see that $\mathscr{E}_{11}^\pm \neq \mathscr{E}_{22}^\pm$ and $\mathscr{E}_{12}^\pm \neq \mathscr{E}_{21}^{\pm *}$ (* indicates the complex conjugate) which are otherwise equal. Therefore, the dispersion relation is complex. We analyze (7) and (9) separately.

Case 1. When the relation $\mathscr{E}_{11}^+ \mathscr{E}_{22}^+ - \mathscr{E}_{12}^+ \mathscr{E}_{21}^+ = 0$ is analyzed, we get the dispersion relation

$$\frac{c^2k^2}{\omega^2} - \frac{ck}{\omega}A_1\beta(1 + \eta^2) + A_0 + A_1(1 + \eta^2) - A_0A_1\eta(\eta + i\beta) - 1 = 0. \tag{12}$$

The relation (12) describes the right circularly polarized waves. When $\beta = 0$, i.e. monopoles are at rest, then we get

$$\frac{c^2k^2}{\omega^2} = 1 - A_0 - A_1(1 + \eta^2) - A_0A_1\eta^2.$$

This reduces to the relation obtained by Kulkarni (1983) when $N_0 = 0$ and reduces to the relation for the right circularly polarized waves when monopole density is zero (e.g. Krall and Trivelpiece 1973). For finite β, the imaginary part vanishes when either $A_0 = 0$ or $A_1 = 0$. This shows the plasma consisting of only electron charges or only of the monopole charges has similar behaviour (Kulkarni 1983; Kulkarni and Bhokare 1984).

In the limit when $\omega - kV_0 \neq \Omega$ i.e. away from the gyroresonance region and also when $N_1/N_0 \ll 1$, we approximate $k = k_r + ik_i$ (k_r is the real part and k_i is the imaginary part) with the condition $k_i/k_r < 1$. The real part of the relation is

$$\frac{c^2k_r^2}{\omega^2} = 1 - A_0 - A_1(1+\eta^2) + A_0A_1\eta^2 + \frac{ck_r}{\omega}A_1\beta(1+\eta^2) \tag{13a}$$

and the imaginary part is given by

$$k_i = \frac{A_0A_1\eta\beta}{\left[2c^2k_r/\omega^2 - \frac{cA_1\beta}{\omega}(1+\eta^2)\right]} \tag{13b}$$

This value of k_i is important and we would like to analyze it further.

Case 2. For the left circularly polarized waves, i.e. the sense of rotation of electric field of the waves is in the direction of gyration of positive ions, we solve the equation (10) to get

$$\frac{c^2k^2}{\omega^2} = 1 - D_0 - D_1(1+\eta^2) + D_0D_1\eta(\eta - i\beta) + \frac{ck}{\omega}D_1(1+\eta^2). \tag{14}$$

When the monopole are at rest, i.e. when $\beta = 0$, we get

$$\frac{c^2k^2}{\omega^2} = 1 - D_0 - D_1(1+\eta^2) + D_0D_1\eta^2.$$

This reduces to the relation for the left circularly polarized waves discussed in Krall and Trivelpiece (1973) when the monopoles are absent. In the frequency limit as discussed earlier, we get the real part as

$$\frac{c^2k_r^2}{\omega^2} = 1 - D_0 + D_1(1+\eta^2) - D_0D_1\eta^2 + \frac{ck_r}{\omega}D_1\beta(1+\eta^2) \tag{15a}$$

The imaginary part is given by

$$k_i = \frac{-D_0D_1\eta\beta}{\left[\frac{2c^2k_r}{\omega^2} - \frac{cD_1\beta}{\omega}(1+\eta^2)\right]} \tag{15b}$$

3. Discussions

From the relations (12) and (14) we see that the wave propagation is altered by the presence of monopoles. Since the monopole charge is greater than that of the electric charge, the behaviour of the waves is different in that the waves having weak electric fields will now have relatively stronger electric fields.

As seen by the relations (12) and (14) monopoles having finite velocity strongly affect both circularly polarized waves. For discussions we study the relation (12) in detail. The waves in the frequency range $\omega_{p_i} < \Omega_i < \omega < \omega_{p_e}$, Ω_e undergo very high dispersion. From the relation (12) the monopoles moving with finite β will give finite

k_i. Depending on the resonance situations, we can discuss the energy exchange between waves and particles. This energy exchange can be attributed to the existence of monopoles.

The energy travels with the group velocity which is frequency dependent. It is possible to evaluate the frequency drift rate and the dispersion measure (DM) which has been used to evaluate the distance of pulsars (Terizen 1972; Michael 1982). Detecting monopoles with the help of the DM is rather difficult because the transit time can change due to inhomogeneities in the magnetic field and density fluctuations.

Therefore, we suggest a possible alternative approach. We are discussing the transverse waves. The electric and magnetic fields are perpendicular to one another which follow fairly accurately

$$k \times \mathbf{E} = \frac{\omega}{c}\mathbf{B}$$

Since k is complex, it can be written as $k \exp(i\phi)$, $k = (k_r^2 + k_i^2)^{1/2}$, $\phi = \tan^{-1}(k_i/k_r)$ (e.g., Marion 1965). The phase difference which is normally 90° is now different. This difference can be due to the growth, damping and due to the presence of monopoles. In the absence of growth and damping, the monopoles will add to the angle ϕ.

When the monopole density is very small the phase lag between **E** and **B** is directly proportional to the monopole density N_1 and its velocity β. For $\beta \approx 1$ the phase velocity leads to the phase lag ϕ which will depend on N_1. In case $\phi \ll 1$ the integrated effect may be observable. Please note that this effect is absent in the absence of monopoles.

Parker has estimated, based on the steady state of the galactic magnetic fields, the monopole flux F to be less than $10^{-16}\,\mathrm{cm^{-2}\,sr^{-1}\,s^{-1}}$. Cabrera (1982) has reported from experiments observing monopoles using the superconducting rings to the SQUID magnetosphere, the flux values should be less than $1{\cdot}2 \times 10^{-10}\,\mathrm{cm^{-2}\,sr^{-1}\,s^{-1}}$.

In the presence of very large magnetic fields, especially like pulsars, the monopoles will acquire the relativistic velocities. We could expect the streaming instabilities. But for the waves considered here, the resonance condition must be satisfied. For the left circularly polarized waves, when the monopoles and waves are moving in the same direction, there is a positive doppler shift. This makes impossible to satisfy the resonance condition $\omega - kV_{11} = \Omega$. For the right circularly polarized waves the resonance conditions need very small velocity; monopoles which may not be available. Therefore, the streaming or kinetic instabilities discussed in the textbooks (e.g. Krall and Trivelpiece 1973) will play not so dominant roles and therefore we have not included them. However, the monopoles will certainly affect the wave propagation as discussed earlier.

One of the sources of galactic monopoles could be pulsars which have large magnetic fields. Assuming these pulsars to be uniformly distributed over the spherical shell, the observed flux will be from a single pulsar. This gives the monopole density on the surface of typical pulsar to be

$$N_1 = \frac{FR^2}{cr_p^2}$$

where r_p is the pulsar radius and R is the pulsar distance. For $r_p \approx 10\,\mathrm{km}$ and $R \approx 1\,\mathrm{kpc}$ the density N_1 will be in the range 10^{11}–$10^{5}\,\mathrm{cm^{-3}}$. When the lower limit is taken, this value is small as compared to the electron densities in the pulsars. Due to extremely large magnetic fields and also presence of relativistically moving electrons, on the surface of pulsars, the present study needs to include associated relativistic effects. Therefore, the present study is suitable and applicable with approximations as the relativistic effects are not included.

4. Conclusions

The propagation of electromagnetic waves in the presence of monopoles of electron mass is strongly affected. Monopoles could even dominate the wave propagation. From the limit of the monopole fluxes, monopole densities on the surface of pulsars could be comparable and significant. The monopoles could lead to the phase difference between electric and magnetic field component of the electromagnetic waves, which may be identifiable. In this presentation the relativistic effects are neglected. The derivation and study of dispersion relation with relativistic effects is taken in continuations. In conclusion, the study of plasma with monopole component is interesting and needs further analysis.

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