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Instability of the plasma oscillations in finite temperature perturbative QCD

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Abstract. We use linear-response dielectric theory to show that the baryon-poor QCD plasma based on the perturbative vacuum is unstable, even at a high temperature. If deconfinement occurs in nuclear collisions or the early universe, it is not accompanied by the restoration of the perturbative vacuum.

Keywords. Quark gluon plasma; instability at finite temperature; perturbative QCD; linear response dielectric theory.

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1. Introduction

The argument for a deconfining phase transition in QCD at high temperature is based on the estimate that a plasma of quarks and gluons, interacting perturbatively in the bare vacuum of QCD, would have a lower free energy than a gas of hadrons at the same temperature T and chemical potential (Shuryak 1980; Gross *et al* 1981; Cleymans *et al* 1986). Lattice Monte Carlo simulation of pure gauge theory shows a first-order phase transition; the inclusion of quarks, however, complicates the calculations, and the question of phase transition has not been resolved satisfactorily. In order to determine whether or not this new state of matter is produced in ultra-relativistic nuclear collisions, several physical phenomena have been proposed as signatures. One of the most interesting suggestions, since it is based directly on a property of the plasma, is due to Matsui and Satz (1986). They predict that due to screening (in the plasma) there will be a suppression of J/psi particle production in the plasma phase relative to the hadronic phase. Experiments carried out at CERN (NA38 collaboration) (see for example Bussiere *et al* 1988) during the past two years are currently being analysed to find definite evidence for this signal of the quark-gluon plasma.

It is obvious that a comprehensive study of the collective properties of the plasma state is necessary for a detailed understanding of its behaviour. At high temperatures the running coupling constant is expected to become small and hence it is common practice to use perturbation theory to study the dynamics of the plasma. A lot of work (Kajantie and Kapusta 1985; Heinz *et al* 1987a, b) employing perturbative QCD at finite temperature, has been done to examine the plasma properties such as Debye screening of colour electric field, frequencies of longitudinal and transverse collective oscillations and their damping rates etc. A surprising result was obtained by Lopez *et al* (1985) and Lopez (1986) who found that to lowest order (g^2T) in perturbation expansion, the term that ought to give rise to the damping of the plasma oscillation had the "wrong" sign. The plasmon mode as a consequence would grow in time, instead of getting damped, and hence the system would be unstable. There has been a great deal of controversy surrounding these (Kajantie and Kapusta 1985; Heinz *et al* 1987a, b) studies largely because many of the results were gauge-dependent. Recently the "damping" of the plasma oscillation has been examined (Hansson *et al* 1987a, b; Nadkarni 1988) in gauge-invariant formulations and it has been shown conclusively that the plasma remains unstable to the lowest perturbative order. If this instability of the plasma persists in the presence of non-perturbative effects, it would imply that a third state of the system (different from the confined hadronic state or the deconfined plasma state) would be thermodynamically preferred.

The basic aim of this paper is to review the work on the evaluation of the "damping" rate of the plasma oscillations using perturbative QCD at finite temperature. We present in §2 the linear response theory results of Lopez *et al* (1985). The gauge-invariant calculations of Hansson and Zahed (1987) and Nadkarni (1988) are summarized in §3, with concluding remarks in §4.

2. Plasma oscillations and linear response theory

The collective behaviour of a many-body system is most easily examined within the framework of linear response theory. The general formulation can be found (for example) in Fetter and Walecka (1971) and its particularization for non-abelian field theories has been discussed by Kajantie and Kapusta (1985), Heinz (1986) and Heinz *et al* (1987).

The linear response function $\tilde{\chi}_{ab}(x-x')$ relates the induced current in the medium $\delta j^{\mu}_{a}(x)$ to a weak externally applied glue field $A^{b}_{\nu E}(x')$:

$$\delta j^{\mu}_{a}(x) = \int \mathrm{d}^4x' \, \tilde{\chi}^{\mu\nu}_{ab}(x-x') A^{b}_{\nu E}(x'). \tag{1}$$

Here, $\mu\nu$ denote Lorentz indices and ab are colour indices, while $x=(t, \mathbf{x})$ and in Minkowski metric $g_{\mu\nu} = g^{\mu\nu}$ $(1, -1, -1, -1)$. The polarizability χ_{ab} $(k_0, \mathbf{k})$ is the Fourier transform of the response function, and is related to the polarization tensor $\Pi^{\mu\nu}_{ab}$ (the expectation value of the time-ordered product of field operators) by the fluctuation-dissipation theorem (Heinz 1986)

$$\mathrm{Re}\, \chi^{\mu\nu}_{ab}(k_0, \mathbf{k}) = -\mathrm{Re}\, \Pi^{\mu\nu}_{ab}(k_0, \mathbf{k}), \tag{2a}$$

$$\mathrm{Im}\, \chi^{\mu\nu}_{ab}(k_0, \mathbf{k}) = -\tanh(k_0/2T)\, \mathrm{Im}\, \Pi^{\mu\nu}_{ab}(k_0, \mathbf{k}), \tag{2b}$$

where

$$K^2 = k_0^2 - \mathbf{k}^2.$$

The real part of the polarizability χ determines the conservative forces acting on the external field whereas the imaginary part determines the rate at which energy is absorbed or emitted by the system (Siemens *et al* 1987). For example, in QED the collective plasmon mode shows up as a peak in the absorption rate when the frequency and wavelength of the external field match those of the plasmon. The plasmon-

Figure 1. Gluon self-energy to $0(g^2)$. The lines correspond to: wavy-gluons, dashed ghosts, solid-quarks.

collective excitation does not appear in lowest order (one-loop) perturbation, but emerges when the loops are iterated in the gluon propagator; in non-relativistic many-body theory the iterated-loop method is known as the random phase approximation (RPA).

To determine $\chi_{ab}^{\mu\nu}$ in RPA, we evaluate $\Pi_{\mu\nu}^{ab,\mathrm{RPA}}$. This first requires calculating the lowest-order loop $\Pi_{\mu\nu}^{ab,0}$, and then summing an infinite series of "ring" diagrams. We perform the basic 1-loop computation for finite T in real-time formalism (Dolan and Jackiw 1974; Weldon 1982). This has the advantage that the one-loop diagram is the sum of a "vacuum" contribution $\Pi_{\mu\nu}^{ab,V}$ which is independent of temperature, and a "medium" contribution $\Pi_{\mu\nu}^{ab,M}$ which vanishes at $T=0$:

$$\Pi_{\mu\nu}^{ab,0}=\Pi_{\mu\nu}^{ab,V}+\Pi_{\mu\nu}^{ab,M}. \tag{3}$$

This instructive separation is lost in the usual imaginary-time formalism, as indeed also in the iterated-loop RPA.

Choosing the Feynman gauge, $\Pi_{\mu\nu}^{ab,0}$ involves the diagrams in figure 1. Following Dolan and Jackiw (1974) and Weldon (1982), we introduce the transverse and longitudinal polarization operators, $\Pi_T^{ab}(k_0 k)$ and $\Pi_L^{ab}(k_0,k)$, defined by

$$\Pi_L^{ab}(k_0,\mathbf{k})=-\frac{K^2}{\mathbf{k}^2}\Pi_{00}^{ab}(k_0,\mathbf{k}),$$

$$\Pi_T^{ab}(k_0,\mathbf{k})=\tfrac{1}{2}\left[\frac{K^2}{\mathbf{k}^2}\Pi_{00}^{ab}+g^{\mu\nu}\Pi_{\mu\nu}^{ab}\right].$$

More generally, in a covariant gauge, $\Pi_{\mu\nu}$ is written in terms of four unknown functions of k_0 and k (see Gross *et al* 1981; Kajantie and Kapusta 1985). For the vacuum, $\Pi_L=\Pi_T$.

Since we are mainly interested in Im χ, we give only the expressions for Im Π. For a plasma with N_F flavours and N colours we find (using $k=|\mathbf{k}|$)

$$\operatorname{Im}\Pi_{00}^{ab,V}(k_0,k)=-\delta^{ab}\frac{g^2}{48\pi}(5N-2N_F)k^2\theta(k_0^2-k^2), \tag{4}$$

$$\begin{aligned}\operatorname{Im}\Pi_{00}^{ab,M}(k_0,k)=\delta^{ab}\frac{g^2}{8\pi k}N_F\int_0^\infty \mathrm{d}p\bigg[&\left(2p^2+2k_0p+\frac{K^2}{2}\right)\\ &\times\langle n_F(p)+n_F(p+k_0)-2n_F(p)n_F(p+k_0)\rangle\\ &\times(\theta_++\theta_-)+(k_0\to -k_0)\bigg]\\ &+\delta^{ab}\frac{g^2}{8\pi k}\cdot\frac{N}{2}\int_0^\infty \mathrm{d}p[(4p^2+4k_0p-k^2+K^2)\\ &\times\langle n(p)+n(p+k_0)+2n(p)n(p+k_0)\rangle\\ &\times[(\theta_++\theta_-)+(k_0\to -k_0)],\end{aligned} \tag{5}$$

where n_F and n are the Fermi and Bose distributions, respectively, and the step functions

$$\theta_\pm = \theta\left(-1 \pm \frac{2kp}{k_0^2 - k^2 + 2k_0 p}\right) \tag{6}$$

restrict the p integrations. Expressions similar to (5) are found for $\mathrm{Im}\, g^{\mu\nu}\Pi^{ab,M}_{\mu\nu}$ and for the real parts of Π^M. We evaluate these integrals numerically to obtain Π_L and Π_T. The iterated loop summation of these one-loop contributions then yields the RPA functions

$$\Pi^{ab,\mathrm{RPA}}_{T,L}(K) = \Pi^{ab,0}_{T,L}(K)/(1 - \Pi^{ab,0}_{T,L}(K)/K^2). \tag{7}$$

The imaginary part of (7) may then be used to find $\chi^{\mu\nu,\mathrm{RPA}}_{ab}$, from (2b).

We have carried out computations for zero baryon chemical potential, assuming massless gluons and quarks. Figure 2 shows the kinematic regions where $\mathrm{Im}\,\chi^{\mathrm{RPA}}_L$ is negative, for the case $T = 0{\cdot}25$ GeV and $\alpha_s = g^2/4\pi = 2{\cdot}2$, chosen to agree with hadron phenomenology in the bag model. Similar results were found for $\alpha_s = 0{\cdot}2$, and for $T = 1$ GeV with each value of α_s. A negative $\mathrm{Im}\,\chi_{L,T}$ is identified with energy-emitting processes or unstable systems (Landau and Lifshitz 1961; Siemens *et al* 1987).

The negative sign of $\mathrm{Im}\,\chi^{\mathrm{RPA}}_L$ may be traced to the imaginary part of the one-loop polarization Π^0_L, whose physical interpretation depends on whether the external glue field is above ($k_0 > k$) or below ($k_0 < k$) the mass shell (light cone), see figure 3. For $k > k_0$, the external gluon is absorbed by a particle in the medium (figure 3a); in this region, the change from emission (region 2 of figure 2) to absorption (region 3) of energy with increasing external momentum may be traced to the tendency of the plasma toward equipartition of energy with the external field.

Above the mass shell ($k_0 > k$), the external gluon produces a pair of gluons (or ghosts) or quarks, figure 3b. For all external frequencies (region 1 in figure 2), $\mathrm{Im}\,\chi^{\mathrm{RPA}}_L$

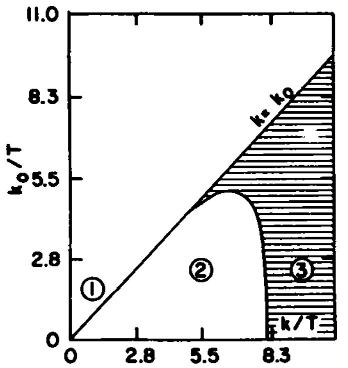

Figure 2. Regions where $\mathrm{Im}\,\chi^{\mathrm{RPA}}_L$ is positive (shaded). Here $\alpha_s = 2{\cdot}2$ and $T = 0{\cdot}25$ GeV.

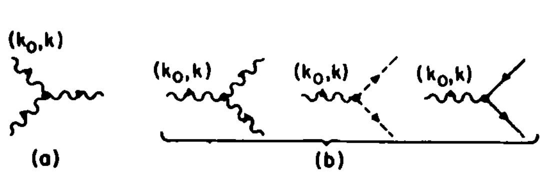

Figure 3. Scattering processes (**a**) below and (**b**) above mass shell.

is again negative, but for a different reason than in region 2. Here, $\mathrm{Im}\,\Pi_L^0$ contains a positive contribution from the vacuum, which is not present for $k_0 < k$. The medium's contribution to $\mathrm{Im}\,\Pi_L^0$ is also positive, and dominates the vacuum contribution for $k_0 < 4T$, but when k_0 is large enough, the contributions from the medium die away and the vacuum's contribution dominates. Thus we can trace the negative sign of $\mathrm{Im}\,\chi$ primarily to the instability of the vacuum.

The sign of $\mathrm{Im}\,\Pi^v$, which is crucial to our argument, is obtained by the choice of Riemann sheets for the logarithm in the familiar calculation for the running coupling constant of QCD. That we have made the correct choice is verified by performing a similar analysis for QED. Since (4) shows that the Fermionic and gluonic parts of Π^v have opposite signs, we find that QED (which has no bosonic coupling) has a stable vacuum in the entire kinematic plane. Indeed, we note that the sign difference in $\mathrm{Im}\,\Pi^v$ which causes QCD to be unstable while QED is stable is exactly the same as that which, in $\mathrm{Re}\,\Pi^v$, produces asymptotic freedom. For finite-temperature QED, the contributions from the medium favour the stimulated emission of energy on both sides of the light cone, in agreement with our argument of the tendency toward equipartition of energy. We cannot expect that our simple perturbative treatment gives an adequate account of the true long-wavelength instabilities of the plasma, whether in QED or QCD. In each case, however, the high-frequency behaviour is dominated by the properties of the vacuum, because the medium contribution vanishes due to the thermal weighting factors.

It is not surprising that the perturbative vacuum of QCD is unstable; indeed, this instability is essential to the mechanism of colour confinement. It is, however, remarkable that the instability already shows up in lowest order perturbation. One objection, that the result might be an artefact of the non-physical degrees of freedom in Feynman gauge needs to be seriously examined. This has been done by Hansson and Zahed (1987) and Nadkarni (1988). We discuss their results in the next section. Another important question is whether higher orders of perturbation will restore the plasma's stability. A naive argument would suggest that this seems unlikely since the thermal excitations will never be able to influence properties at momenta and frequency much bigger than the temperature. It is crucial therefore to evaluate $\mathrm{Im}\,\chi_L$ to higher orders. Nadkarni (1988) recently tried to calculate it but no numerical results are yet available.

3. Gauge-invariant approaches to the damping constant

Hansson and Zahed (1987) adopt the background field method to define a manifestly gauge-invariant effective action $\Gamma(A,\alpha)$. Here A_μ is the classical background field and α is the gauge-fixing parameter. They further argue that (at least upto one-loop level) the gauge $\alpha = 0$ (Landau) gives physically meaningful results. For the "damping" of the plasma oscillation they obtain (order 1-loop) the expression

$$\gamma = -\frac{Ng^2 T}{24\pi}\left[11 + \frac{1}{4}(\alpha - 1)^2\right], \tag{8}$$

which has an explicit α dependence. Clearly for all values of α, the constant γ is negative, although as mentioned before, only the value $\alpha = 0$ is physically relevant.

A different approach has been considered by Nadkarni (1988). He uses the scheme developed by Cornwall (1982) and Cornwall *et al* (1985) where certain vertex diagrams provide propagator corrections which in turn define a gauge-invariant transverse polarization tensor $\hat{\Pi}_{\mu\nu}$. As discussed in the previous section, the linear response of the plasma is generally given by the polarization tensor which in this case is the gauge-invariant quantity $\hat{\Pi}_{\mu\nu}$. The value of the "damping" constant (to 1-loop) obtained by Nadkarni (1988) is

$$\gamma = -\frac{11}{24\pi} N g^2 T, \tag{9}$$

which corresponds to the Feynman ($\alpha = 1$) and not Landau ($\alpha = 0$) gauge value (equation (8)) of Hansson and Zahed (1987). The reasons for this discrepancy are not known.

In any case these authors (Hansson *et al* 1987; Nadkarni 1988) confirm with gauge-invariant calculations, the earlier result of Lopez *et al* (1985) that the "damping" constant has the wrong sign in 1-loop calculations.

Going beyond the bare 1-loop level, Nadkarni (1988) obtains an expression for the damping constant in the long wavelength limit, which is given by

$$\gamma_{sc}(|\mathbf{k}|) \approx -\left[\frac{C_{\mathrm{el}}}{m_{\mathrm{el}}^2} + \frac{C_{\mathrm{mag}}}{m_{\mathrm{mag}}^2}\right] N g^2 T \omega_p |\mathbf{k}|. \tag{10}$$

Here m_{el} and m_{mag} are electric and magnetic screening masses and ω_p is the plasma frequency. C_{el} and C_{mag} are constants whose values have not been computed as yet. The sign of $\gamma_{sc}(|\mathbf{k}|)$ depends on the relative magnitudes and the signs of these two quantities.

4. Summary and concluding remarks

Gauge-invariant calculations upto bare 1-loop level suggest an instability of the quark-gluon plasma. It is not known what the result would be when non-perturbative effects are included. If the sign of the "damping" constant continues to be negative in such calculations then one would have to conclude that the QCD plasma built on the perturbative vacuum still has the essential instability of that vacuum. It cannot therefore be the thermodynamically favoured state, even at a large temperature. Obviously the perturbative techniques cannot predict what that most favoured state will be.

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